The Enlightenment and Social Choice Theory*

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Abstract

The Enlightenment was a philosophical project to constuct a rational society, without the need for a supreme being. It opened the way for the Industrial Revolution, and for the creation of market democracy and rapid economic growth. At the same time economic growth has created the possibility of climate change, and we are now becoming aware that this growth may destroy our civilization. This paper surveys the results in topological social choice theory and suggests that social choice can be deeply chaotic, and impossible to predict. We consider two variants of such models, the prisoner's dilemma model of cooperation and the arrovian theory of preference aggregation. We argue that the standard model of market equilibrium also exhibits chaos. It is suggested that the unpredictability of these dynamical models means that the Enlightenment project is unsustainable. Perhaps our socity need a new moral compass, that is compatible with new understanding of our evolutionary background and the likely consequence of climate change. "

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1 lIntroduction

In this essay I shall consider what Israel (2001, 2012) calls the *Radical Enlightenment*, the program to establish rationality as the basis for society, opposed to monarchy, religion and the church. Radical enlighteners included Thomas Jefferson, Thomas Paine and James Madison. They believed that society could be based on rational constitutional principles , leading to the "probability of a fit choice." Implicit in the Radical enlightenment was the belief, originally postulated by Spinoza, that individuals could find moral bases for their choices without a need for a divine creator. An ancilliary belief was that the economy would also be rational and that the principles of the radical enlightenment would lead to material growth and the eradication of poverty and misery. This enlightenment philosophy has recently has had to face two troubling propositions. First are the results of Arrovian social choice theory. These very abstract results suggest that no process of social choice can be rational Second, recent events suggest that the market models that we have used to guide our economic actions are deeply flawed. Opposed to the Radical enlighteners, David Hume and Burke believed that people would need religion and nationalism to provide a moral compass to their lives. As Putnam and Campbell (2010) have noted religion is as important as it has ever been in the US. Recent models of US Elections (Schofield and Gallego, 2011) show that religion is a key dimension of politics that divides voters one from anotherequilibrium models of the political economy, and attempt a discussion of why equilibrium may collapse into chaos. A consequence of the Industrial Revolution, that followed on from the Radical Enlightenment, has been the unintended consequence of climate change. Since this is the most important policy dimension that the world economy currently faces, this paper will address the question whether we are likely to be able to make wise social choices to avoid future catastophe.

1.1 The Radical Enlightenment

It was no accident that the most important cosmologist after Ptolemy of Alexandria was Nicolaus Copernicus (1473 – 1543), born only a decade before Martin Luther. Both attacked orthodoxy in different ways.[1] Copernicus formulated a scientifically based heliocentric cosmology that displaced the Earth from the center of the universe. His book, *De revolutionibus orbium coelestium* (*On the Revolutions of the Celestial Spheres*, 1543), is often regarded as the starting point of the Scientific Revolution.

Margolis (2002) noted that something very significant occurred in the years after Copernicus. His ideas influenced many scholars: the natural philosopher, William Gilbert, who wrote on magnetism in *De Magnete* (1601); the physicist, mathematician, astronomer, and philosopher, Galileo Galilei (1564 – 1642); the mathematician and astronomer, Johannes Kepler (1571 –1630).

[1] Weber (1904) speculated that there was a connection between the values of Protestantism and Capitalism. It may be that there are connections between the preference for scientific explanation and protestant belief about the relationship between God and humankind.

Philosophiæ Naturalis Principia Mathematica (1687), by the physicist, mathematician, astronomer and natural philosopher, Isaac Newton (1642 – 1726) is considered to be the most influential book in the history of science.[2] Margolis (2002) argues that, after Newton, a few scholars realized that the universe exhibits laws that can be precisely written down in mathematical form. Moreover, we have, for some mysterious reason, the capacity to conceive of exactly those mathematical forms that do indeed govern reality. This mysterious connection between mind and reality was the basis for Newton's philosophy. While celestial mechanics had been understood by Ptolemy to be the domain most readily governed by these forms, Newton's work suggested that *all* reality was governed by mathematics. The influence of Newton can perhaps be detected in the work of the philosopher, mathematician, and political scientist, Marie Jean Antoine Nicolas de Caritat, Marquis de Condorcet (1743 – 1794), known as Nicolas de Condorcet. His work in formal social choice theory (Condorcet ([1785],1994) was discussed in Schofield (2006) in connection with the arguments about democracy by Madison and Jefferson. The work on Moral Sentiment by the Scottish Enlightenment writers, Francis Hutcheson (1694–1746), David Hume (1711–1776), Adam Smith (1723–1790) and Adam Ferguson (1723–1816), also influenced Jefferson and Madison. Between Copernicus and Newton, the writings of Thomas Hobbes (1588 – 1679), René Descartes (1596– 1650), John Locke (1632– 1704), Baruch Spinoza (1632– 1677), and Gottfied Liebnitz (1646– 1716) laid down foundations for the modern search for rationality in life.[3] Hobbes was more clearly influenced by the scientific method, particularly that of Galileo, while Descartes, Locke, Spinoza, and Liebniz were all concerned in one way or another with the imperishability of the soul.[4] The mathematician, Liebniz, in particular was concerned with an

> [E]xplanation of the relation between the soul and the body, a matter which has been regarded as inexplicable or else as miraculous.

Without the idea of a soul it would seem difficult to form a general scheme of ethics.[5] Indeed, the progress of science and the increasing secularization of society have caused many to doubt that our society can survive. Hawking and Mlodonow (2010) argue for a strong version of this universal mathmatical principle, called *model-dependent realism*,

[2] See Feingold (2004).

[3] For Hobbes, see Rogow (1986). For Descartes, see Gaukroger (1995). For Spinoza and Liebnitz see Stewart (2006) and Goldstein (2006). See also Israel (2012) for thr development of the Radical Enlightenment

[4] It is of interest that the English word "soul" derives from Old English *sáwol* (first used in the 8th century poem, *Beowulf.*

[5] Hawking and Mlodinow (2010) assert that God did not create the Universe, perhaps implying that the soul does not exist. However they do say that they understand Isaac Newton's belief that God did "create" and "conserve" order in the universe. See other books by Dawkins (2008) and Hitchens (2007) on the same theme, as well as Wright (2009) on the evolution of the notion of God and Lilla (2007) on political theology.

citing its origins in Pythagoras (580 BCE to 490 BCE), Euclid (383 -323 BCE and Archimedes (287-212 BCE), and the recent developments in mathematical physics and cosmology.

They argue that it is only through a mathematical model that we can properly perceive reality. However, this mathematical principle faces two philosophical difficulties. One stems from the Godel(1931)-Turing (1937) undecidability theorems. The first theorem asserts that mathematics cannot be both complete and consistent, so there are mathematical models that in principle cannot be verified. Turing's work, though it provides the basis for our computer technology also suggests that not all programs are computable. The second problem is associated with the notion of *chaos* or *catastrophe.* These mathematical ideas stem from Poincaré's work in celestial mechanics in the late nineteenth century, and developed later by Smale (1966), Zeeman and Lorenz (1962). Chaos refers to the possibility that a deterministic dynamical system can be impossible to study because of extreme sensitivity to the parameters of the system. As we mention below Lorenz showed that a very simple model of weather could be *structurally unstable* in the sense that an infinitesimally small shift of parameter can change the qualitative nature of the model.[6] I shall use the term *chaos* to mean that the trajectory taken by the dynamical process can wander anywhere.[7] One aspect of chaos is that a chaotic dynamical system can go through a *tipping point.* In the context of climate change it is possible that very sudden and large changes in temperature and climate have wiped out earlier human societies Guterl (2012) gives examples of such potential catastrophic tipping points, including the disappearance of the Asian monsoon, the collapse of the Greenland and Antartica ice sheets. The effects of small shifts in the past have had major effects on human civilization (Fagan, 1999, 2008)

Kauffman (1993) commented on "chaos" or the failure of "structural stability" in the following way.

> One implication of the occurrence or non-occurrence of structural stability is that, in structurally stable systems, smooth walks in parameter space must [result in] smooth changes in dynamical behavior. By contrast, chaotic systems, which are not structurally stable, adapt on uncorrelated landscapes. Very small changes in the parameters pass through many interlaced bifurcation surfaces and so change the behavior of the system dramatically.

Chaos is generally understood as sensitive dependence on initial conditions whereas *structural stability* means that the qualitative nature of the dynamical system does not change as a result of a small perturbation.[8] I shall use the

[6] The theory of chaos or complexity is rooted in Smale's fundamental theorem (Smale (1966) that structural stability of dynamical systems is not "generic" or typical whenever the state space has more than two dimensions.

[7] In their early analysis of chaos, Li and Yorke (1975) showed that in the domain of a chaotic transformation f it was possible for almost any pair of positions (x, y) to transition from x to $y = f^r(x)$, where f^r means the r times reiteration of f.

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term *chaos* to mean that the trajectory taken by the dynamical process can wander anywhere.[9] Since the early work of Garrett Hardin (1968) the "tragedy of the commons" has been recognised as a global prisoner' dilemma. In such a dilemma no agent has a motivation to provide for the collective good. In the context of the possibility of climate change, the outcome is the continued emission of greenhouses gases like carbon dioxide into the atmosphere and the acidification of the oceans. There has developed an extensive literature on the n-person prisoners' dilemma in an attempt to "solve" the dilemma by considering mechanisms that would induce cooperation.[10]

The problem of cooperation has also provided a rich source of models of evolution, building on the early work by Trivers (1971) and Hamilton (1964, 1970). Nowak (2011) provides an overview of the recent developments.Indeed, the last twenty years has seen a growing literature on a game theoretic, or mathematical, analysis of the evolution of social norms to maintain cooperation in prisoners' dilemma like situations. Gintis (2000, 2003), for example, provides evolutionary models of the cooperation through strong reciprocity and internalization of social norms.[11] The anthropological literature provides much evidence that, from about 500KYBP years ago, the ancestors of *homo sapiens* engaged in cooperative behavior, particularly in hunting and caring for offspring and the elderly.[12] On this basis we can infer that we probably do have very deeply ingrained normative mechanisms that were crucial, far back in time, for the maintenance of cooperation, and the fitness and thus survival of early hominids.[13] These normative systems will surely have been modified over the long span of our evolution. However human behavior displays a complex mix of both cooperation and conflict. In the following sections I shall review aspects of social choice theory that suggest that human behavior tends to be chaotic rather than structurally stable.

Current work on climate change has focussed on how we should treat the future. For example Stern (2007, 2009), Collier (2010) and Chichilnisky (2009a,b) argue essentially for equal treatment of the present and the future. Dasguta (2005) points out that how we treat the future depends on our current estimates of economic growth in the near future.

The fundamental problem of climate change is that the underlying dy-

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[10] See for example Hardin (1971, 1982), Taylor (1976, 1982), Axelrod and Hamilton (1981), Axelrod (1981, 1984), Kreps et al. (1982),Margolis (1982).

[11] Strong reciprocity means the punishment of those who do not cooperate.

[12] Indeed, White et al.(2009) present evidence of a high degree of cooperation among very early hominids dating back about 4MYBP (million years before the present). The evidence includes anatomical data which allows for inferences about the behavioral characteristics of these early hominids.

[13] Gintis cites the work of Robson and Kaplan (2003) who use an economic model to estimate the correlation between brain size and life expectancy (a measure of efficiencey). In this context, the increase in brain size is driven by the requirement to solve complex cooperative games against nature.

namic system is extremely complex, and displays many positive feedback mechanisms.[14] The difficulty can perhaps be illustrated by Figure 1. It is usual in economic analysis to focus on Pareto optimality. Typically in economic theory, it is assumed that preferences and production possibilities are generated by convex sets. However, climate change could create non-convexities. In such a case the Pareto set will exhibit stable and unstable components. Figure 1 distinguishes between a domain A, bounded by stable and unstable components P_1^s and P^u, and a second stable component P_2^s. If our actions lead us to an outcome within A, whether or not it is Paretian, then it is possible that the dynamic system generated by climate could lead to a catastrophic destruction of A itself. More to the point, our society would be trapped inside A as the stable and unstable components merged together to create a catastrphe (Zeeman, 1977).

[Insert Figure 1 here]

Our society has recently passed through a period of economic disorder, where "black swan" events, low probability occurrences with high costs, have occurred with some regularity. Recent discussion of climate change has also emphasized so called "fat-tailed climate events" again defined by high uncertainty and cost.[15] The catastophic change implied by Figure 1 is just such a black swan event. The point to note about Figure 1 is everything would appear normal until the evaporation of A.

Cooperation could in principle be attained by the action of a hegemonic leader such as the United States as suggested by Kindleberger (1973) and Keohane and Nye (1977). In Section 2 we give a brief exposition of the prisoners' dilemma and illustrate how hegemonic behavior could facilitate international cooperation. However, the analysis suggests that in the present economic climate, such hegemonic leadership is unlikely.

Analysis of games such as the prisoner's dilemma usually focus on the existence of a Nash equilibrium, a vector of strategies with the property that no agent has an incentive to change strategy. Section 3 considers the family of equilibrium models based on the Brouwer (1912) fixed point theorem, or the more general result known as the Ky Fan theorem (Fan1961) as well as the application by Bergstrom (1975, 1992) to prove existence of a Nash equilibrium and market equilibrium.

Section 4 considers a generalization of the Ky Fan Theorem, and argues that the general equilibrium argument can be interpreted in terms of particular properties of a preference field, H, defined on the tangent space of the joint strategy space. If this field is continuous, in a certain well-defined sense, and *"half open"* then it will exhibit a equilibrium. This half open property is the same as the non empty intersection of a family of dual cones. We mention a Theorem by Chichilnisky (1995) that a necessary and sufficient condition for market equilibrium is that a family of dual cones also has non-empty intersection.

[14] See the discussion in Schofield (2011).

[15] Weitzman (2009) and Chichilnisky (2010). See also Chichilnisky and Eisenberger (2010) on other catastophic events such as collision with an asteroid.

However, preference fields that are defined in terms of coalitions need not satisfy the half open property and thus need not exhibit equilibrium. For coalition systems, it can be shown that unless there is a collegium or oligarchy, or the dimension of the space is restricted in a particular fashion, then there need be no equilibrium. Earlier results by McKelvey (1976), Schofield (1978), McKelvey and Schofield (1987) and Saari (1997) suggested that voting can be "non-equilibriating" and indeed "chaotic."[16]

An earlier prophet of uncertainty was, of course, Keynes (1936) whose ideas on "speculative euphoria and crashes" would seem to be based on understanding the economy in terms of the qualitative aspects of its coalition dynamics.[17] An extensive literature has tried to draw inferences from the nature of the recent economic events. A plausible account of market disequilibrium is given by Akerlof and Shiller (2009) who argue that

> the business cycle is tied to feedback loops involving speculative price movements and other economic activity — and to the talk that these movements incite. A downward movement in stock prices, for example, generates chatter and media response, and reminds people of longstanding pessimistic stories and theories. These stories, newly prominent in their minds, incline them toward gloomy intuitive assessments. As a result, the downward spiral can continue: declining prices cause the stories to spread, causing still more price declines and further reinforcement of the stories.

It would seem reasonable that the rise and fall of the market is due precisely to the coalitional nature of decision-making, as large sets of agents follow each other in expecting first good things and then bad. A recent example can be seen in the fall in the market after the earthquake in Japan, and then recovery as an increasing set of investors gradually came to believe that the disaster was not quite as bad as initially feared.

Since investement decisions are based on these uncertain evaluations, and these are the driving force of an advanced economy, the flow of the market can exhibit singularities, of the kind that recently nearly brought on a great depression. These singularities associated with the bursting of market bubbles are time-dependent, and can be induced by endogenous belief-cascades, rather than by any change in economic or political fundmentals (Corcos et al. 2002).

Similar uncertainty holds over political events. The fall of the Berlin Wall in 1989 was not at all foreseen. Political scientists wrote about it in terms of "belief cascades"[18] as the coalition of protesting citizens grew apace. As the very recent democratic revolutions in the Middle East and North Africa suggest,

[16] See Schofield (1977, 1980a,b). In a sense these voting theorems can be regarded as derivative of Arrow's Impossiblity Theorem (Arrow, 1951). See also Arrow (1986).

[17] See Minsky (1975, 1986) and Keynes's earlier work in 1921.

[18] Karklins and Petersen (1993); Lohmann (1994). See also Bikhchandani, Hirschleifer, and Welsh (1992).

these coalitional movements are extremely uncertain.[19] In particular, whether the autocrat remains in power or is forced into exile is as uncertain as anything Keynes discussed. Even when democracy is brought about, it is still uncertain whether it will persist.[20]

Section 5 introduces the Condorcet (1994, [1795]) Jury Theorem. This theorem suggests that majority rule can provide a way for a society to attain the truth when the individuals have common goals. Schofield (2002, 2006) has argued that Madison was aware of this theorem while writing Federalist X (Madison, 1999, [1787]) so it can be taken as perhaps the ultimate justification for democracy. However, models of belief aggregation that are derived from the Jury Theorem can lead to belief cascades that bifurcate the population. In addition, if the aggregation process takes place on a network, then centrally located agents, who have false beliefs, can dominate the process.[21]

In section 6 we introduce the idea of a belief equilibrium, and then go on to consider the notion of "punctuated equilibrium" in general evolutionary models. Again however, the existence of an equilibrium depends on a fixed point argument, and thus on a half open property of the "cones" by which the developmental path is modeled. This half open property is equivalent to the existence of a social direction gradient defined everywhere.We conclude by suggesting that the ubiquity of chaotic uncertainty means that the Enlightenment project of creating a rational basis for society will depend on constructing a social choice theory that explicitly incorporates a model of human morality.

2 The Prisoners' Dilemma, Cooperation and Morality

> For before constitution of Sovereign Power ... all men had right to all things; which necessarily causeth Warre. (Hobbes,2009 [1651]).

Kindleberger (1973) gave the first interpretation of the international economic system of states as a "Hobbesian" prisoners' dilemma, which could be solved by a leader, or "hegemon."

> A symmetric system with rules for counterbalancing, such as the gold standard is supposed to provide, may give way to a system with each participant seeking to maximize its short-term gain. ...But a world of a few actors (countries) is not like [the competitive system envisaged by Adam Smith]. ... In advancing its own economic good by a tariff, currency depreciation, or foreign exchange control, a country may worsen the welfare of its partners by more than its gain. Beggar-thy-neighbor tactics may lead to retaliation so that

[19] The response by the citizens of these countries to the demise of Osama bin Laden on May 2, 2011, is in large degree also unpredictable.

[20] See for example Carothers (2002) and Collier (2009).

[21] Golub and Jackson, 2010.

> each country ends up in a worse position from having pursued its own gain ...
>
> This is a typical non-zero sum game, in which any player undertaking to adopt a long range solution by itself will find other countries taking advantage of it ...

In the 1970s, Robert Keohane and Joseph Nye (1977) rejected “realist” theory in international politics, and made use of the idea of a hegemonic power in a context of “complex interdependence” of the kind envisaged by Kindleberger. Although they did not refer to the formalism of the prisoners’ dilemma, it would appear that this notion does capture elements of complex interdependence. To some extent, their concept of a hegemon is taken from realist theory rather than deriving from the game-theoretic formalism.

However, it is very easy to adapt the notion of a symmetric prisoners’ dilemma, so as to clarify the concept of a hegemon. A non-symmetric n-agent prisoners’ dilemma (nPD) can be constructed as follows. We define the strategy of the i^{th} country to be $d_i = 0$ when i defects, and $d_i = 1$ when i cooperates. We can also denote mixed strategies by letting $d_i \in [0,1]$. Each country was a weight (proportional to its GDP), a_i say. The *total collective good* of the system, N, of states is defined to be:

$$B(N) = \sum_{j=1}^{n} a_j d_j \tag{1}$$

The payoff u_i to state i, when it adopts strategy d_i is

$$u_i(d_i) = \frac{r}{n} B(N) - d_i. \tag{2}$$

To construct a prisoners’ dilemma, we assume $1 < r < n$. From the above, the term involving d_i in $u_i(d_i)$ is

$$\frac{r}{n}(a_i d_i) - d_i. \tag{3}$$

Clearly if

$$\frac{r}{n}(a_i) < 1, \tag{4}$$

then u_i is maximized at $d_i = 0$, and takes the value $\frac{r}{n}B(N) = \frac{r}{n}\sum_{j \neq i} a_j d_j$. If

$$\frac{r}{n}(a_i) > 1, \tag{5}$$

then $u_i$is maximized at d_i= 1.

In the *symmetric* game, $a_i = 1$ for all i, so the “rational” strategy for each country is to defect, by choosing $d_i = 0$. In this case $u_i = 0$. On the other hand if everyone chooses the *irrational* strategy $d_i = 1$, then $B(N) = r > 1$, and so $u_i(d_i = 1) > 0$.

On the other hand, if

$$a_i > \frac{n}{r}, \tag{6}$$

then $a_i > 1$, and this country,i, rationally must *cooperate*, irrespective of the strategies of other countries. To keep things simple, suppose $a_j = 1$ for all j other than this hegemon, i. In this very trivial formulation, some things are obvious. If more states join the game (so n increases, while r *remains constant*), it becomes more "difficult" for a_i to be large enough for cooperation. The coefficient,r, is the "rate of return on cooperation." As r falls it becomes more difficult again for i to remain the cooperative hegemon. In this formulation the term hegemon is something of a misnomer, since i is simply a rational cooperator. However, if coalitions are possible and a hegemonic power, called i, leads a coalition M of states, dictating policy to these states, then the optimality condition for the joint cooperation of the states in the coalition M is

$$\sum_{i \in M} a_i > \frac{n}{r}. \tag{7}$$

The collective benefits of the coalition M can then be redistributed by the hegemon in some way, to keep the coalition intact. The essence of the theory of hegemony in international relations is that if there is a degree of inequality in the strengths of nation states then a hegemonic power may maintain cooperation in the context of an n-country prisoners' dilemma. Clearly, the British Empire in the 1800's is the role model for such a hegemon (Ferguson, 2002).

Hegemon theory suggests that international cooperation was maintained after World War II because of a dominant cooperative coalition. At the core of this cooperative coalition was the United States; through its size it was able to generate collective goods for this community, first of all through the Marshall Plan and then in the context first of the post-world war II system of trade and economic cooperation, based on the Bretton Woods agreement and the Atlantic Alliance, or NATO. Over time, the United States has found it costly to be the dominant core of the coalition In particular, as the relative size of the U.S. economy has declined, so that $\sum_{i \in M} a_i$ has fallen, then cooperation will become very difficult, especially if r also falls. Indeed, the global recession of 2008-10 suggests that problems of debt could induce "begger thy neighbor strategies", just like the 1930's.

The future utility benefits of adopting policies to ameliorate these possible changes depend on the discount rates that we assign to the future. Dasgupta (2005) gives a clear exposition of how we might assign these discount rates. Obviously enough, different countries will in all likelihood adopt very different evaluations of the future. Developing countries like the BRICs (Brazil, Russia, India and China) will choose growth and development now rather than choosing consumption in the future.

There have been many attempts to "solve" the prisoners' dilemma in a general fashion. For example Binmore (2005) suggests that in the iterated nPD there are many equilibria with those that are *fair* standing out in some fashion. However, the criterion of "fairness" would seem to have little weight with regard to climate change. It is precisely the poor countries that will suffer from climate change, while the rapidly growing BRICS believe that they have a right to choose their own paths of development.

An extensive literature over the last few years has developed Adam Smith's ideas as expressed in the *Theory of Moral Sentiments* (1984 [1759]) to argue that human beings have an inate propensity to cooperate. This propensity may well have been the result of co-evolution of language and culture (Boyd and Richerson, 2005; Gintis, 2000).

Since language evolves very quickly (McWhorter, 2001; Deutcher, 2006), we might also expect moral values to change fairly rapidly, at least in the period during which language itself was evolving. In fact there is empirical evidence that cooperative behavior as well as notions of fairness vary significantly across different societies.[22] While there may be fundamental aspects of morality and "altruism," in particular, held in common across many societies, there is variation in how these are articulated. Gazzaniga (2008) suggests that moral values can be described in terms of various *modules*: reciprocity, suffering (or empathy), hierarchy, in-group and outgroup coalition, and purity/ disgust. These modules can be combined in different ways with different emphases. An important aspect of cooperation is emphasized by Burkart, Hrdy and Van Schaik (2009) and Hrdy (2011), namely cooperation between man and woman to share the burden of child rearing.

It is generally considered that hunter-gatherer societies adopted egalitarian or "fair share" norms. The development of agriculture and then cities led to new norms of hierarchy and obedience, coupled with the predominence of military and religious elites (Schofield, 2010).

North (1990), North et al (2009) and Acemoglu and Robinson (2006) focus on the transition from such oligarchic societies to open access societies whose institutions or "rules of the game", protect private property, and maintain the rule of law and political accountability, thus faciltating both cooperation and economic development. Acemoglu et al. (2009) argue, in their historical analyses about why "good" institutions form, that the evidence is in favor of "critical junctures."[23] For example, the "Glorious Revolution" in Britain in 1688 (North and Weingast, 1989), which prepared the way in a sense for the agricultural and industrial revolutions to follow (Mokyr, 2005, 2010; Mokyr and Nye, 2007) was the result of a sequence of historical contingencies that reduced the power of the elite to resist change. Recent work by Morris (2010), Fukuyama (2011), Ferguson (2011) and Acemoglu and Robinson, (2011) has suggested that these fortuitous circumstances never occurred in China and the Middle East, and as a result these domains fell behind the West. Although many states have become democratic in the last few decades, oligarchic power is still entrenched in many

[22] See Henrich et al. (2004, 2005), which reports on experiments in fifteen "small-scale societies," using the game theoretic tools of the "prisoners' dilemma," the "ultimatum game," etc.

[23] See also Acemoglu and Robinson, 2008.

parts of the world.[24]

At the international level, the institutions that do exist and that are designed to maintain cooperation, are relatively young. Whether they suceed in facilitating cooperation in such a difficult area as climate change is a matter of speculation. As we have suggested, international cooperation after World War II was only possible because of the overwheming power of the United States. In a world with oligarchies in power in Russia, China, and in many countries in Africa, together with political disorder in almost all the oil producing counties in the Middle East, cooperation would appear unlikely.

To extend the discussion, we now consider more general theories of social choice.

3 Topological Social Choice

The above discussion has considered a very simple version of the prisoner's dilemma.The more general models of cooperation typically use variants of evolutionary game theory, and in essence depend on proof of existence of Nash equilibrium, using some version of the Brouwer's fixed point theorem (Brouwer 1912).

Brouwer's theorem asserts that any continuous function $f : B \to B$ from the finite dimensional ball,B (or indeed any compact convex set in $\mathbb{R}^w$) into itself, has the *fixed point property*. That is, there exists some $x \in B$ such that $f(x) = x$.

We will now consider the use of variants of the theorem, to prove existence of an equilibrium of a general choice mechanism. We shall argue that the condition for existence of an equilibrium will be violated if there are cycles in the underlying mechanism.

Let W be the set of alternatives and let X be the set of all subsets of W. A *preference correspondence*, P, on W assigns to each point $x \in W$, its *preferred set* $P(x)$. Write $P\colon W \to X$ or $P\colon W \twoheadrightarrow W$ to denote that the image of x under P is a set (possibly empty) in W. For any subset V of W, the restriction of P to V gives a correspondence $P_V\colon V \twoheadrightarrow V$. Define $P_V^{-1}\colon V \twoheadrightarrow V$ such that for each $x \in V$,

$$P_V^{-1}(x) = \{y : x \in P(y\} \cap V.$$

$P_V^{-1}(x) = \{y : x \in P(y\} \cap V$. The sets $P_V(x), P_V^{-1}(x)$ are sometimes called the *upper* and *lower* preference sets of P on V. When there is no ambiguity we delete the suffix V. The *choice* of P from W is the set

$$C(W, P) = \{x \in W\colon P(x) = \Phi\}.$$

Here Φ is the empty set. The choice of P from a subset, V, of W is the set

$$C(V, P) = \{x \in V\colon P_V(x) = \Phi\}.$$

[24] The popular protests in N.Africa and the Middle East in 2011 were in opposition to oligarchic and autocratic power.

Call C_P a *choice function* on W if $C_P(V) = C(V, P) \neq \Phi$ for every subset V of W. We now seek general conditions on W and P which are sufficient for C_P to be a choice function on W. Continuity properties of the preference correspondence are important and so we require the set of alternatives to be a topological space.

Definition 1.

Let W, Y be two topological spaces. A correspondence $P\colon W \twoheadrightarrow Y$ is

(i) *Lower demi-continuous* (*ldc*) iff, for all $x \in Y$, the set

$$P^{-1}(x) = \{y \in W : x \in P(y)\}$$

is open (or empty) in W.

(ii) *Acyclic* if it is impossible to find a cycle $x_t \in P(x_{t-1}), x_{t-1} \in P(x_{t-2}), .., x_1 \in P(x_t)$.

(iii) *Lower hemi-continuous* (*lhc*) iff, for all $x \in W$, and any open set $U \subset Y$ such that $P(x) \cap U \neq \Phi$ there exists an open neighborhood V of x in W, such that $P(x') \cap U \neq \Phi$ for all $x' \in V$.

Note that if P is ldc then it is lhc.

We shall use lower demi-continuity of a preference correspondence to prove existence of a choice.

We shall now show that if W is compact, and P is an acyclic and ldc preference correspondence $P\colon W \twoheadrightarrow W$, then $C(W, P) \neq \Phi$. First of all, say a preference correspondence $P\colon W \twoheadrightarrow W$ satisfies the *finite maximality property* (FMP) on W iff for every finite set V in W, there exists $x \in V$ such that $P(x) \cap V = \Phi$.

Lemma 1 (Walker, 1977).

If W is a compact, topological space and P is an ldc preference correspondence that satisfies FMP on W, then $C(W, P) \neq \Phi$.

This follows readily, using compactness to find a finite subcover, and then using FMP.

Corollary 1.

If W is a compact topological space and P is an acyclic, ldc preference correspondence on W, then $C(W, P) \neq \Phi$.

As Walker (1977) noted, when W is compact and P is ldc, then P is acyclic iff P satisfies FMP on W, and so either property can be used to show existence of a choice. A second method of proof is to show that C_P is a choice function is to substitute a convexity property for P rather than acyclicity.

Definition 2.

(i) If W is a subset of a vector space, then the *convex hull* of W is the set, $\mathrm{Con}[W]$, defined by taking all convex combinations of points in W.

(ii) W is *convex* iff $W = \mathrm{Con}[W]$. (The empty set is also convex.)

(iii) W is *admissible* iff W is a compact, convex subset of a topological vector space.

(iv) A preference correspondence $P\colon W \twoheadrightarrow W$ is *semi-convex* iff, for all $x \in W$, it is the case that $x \notin \mathrm{Con}(P(x))$.

Fan (1961) has shown that if W is admissible and P is ldc and semi-convex, then $C(W, P)$ is non-empty.

Choice Theorem (Fan, 1961, Bergstrom, 1975).

If W is an admissible subset of a Hausdorff topological vector space, and $P\colon W \twoheadrightarrow W$ a preference correspondence on W which is ldc and semi-convex then $C(W, P) \neq \Phi$.

The proof uses the KKM lemma due to Knaster, Kuratowski and Mazurkiewicz (1929).

The original form of the Theorem by Fan made the assumption that $P\colon W \twoheadrightarrow W$ was *irreflexive* (in the sense that $x \notin P(x)$ for all $x \in W$) and *convex*. Together these two assumptions imply that P is semi-convex. Bergstrom (1975) extended Fan's original result to give the version presented above.[25]

Note that the Fan Theorem is valid without restriction on the dimension of W. Indeed, Aliprantis and Brown (1983) have used this theorem in an economic context with an infinite number of commodities to show existence of a price equilibrium. Bergstrom (1992) also showed that when W is finite dimensional then the Fan Theorem is valid when the continuity property on P is weakened to lhc and used this theorem to show existence of a Nash equilibrium of a game $G = \{(P_1, W_1), .P_i, W_i), ..(P_n, W_n) : i \in N\}$. Here the i^{th} stategy space is finite dimensional W_i and each individual has a preference P_i on the joint strategy space $P_i : W^N = W_1 \times W_2 ... \times W_n \twoheadrightarrow W_i$. The Fan Theorem can be used, in principle to show existence of an equilibrium in complex economies with externalities. Define the Nash improvement correspondence by $P_i^* : W^N \twoheadrightarrow W^N$ by $y \in P_i^*(x)$ whenever $y = (x_1, ..x_{i-1}, x_i^*, ..., x_n)$, $x = (x_1, .., x_{i-1}, x_i, .., x_n)$, and $x_i^* \in P_i(x)$ The joint Nash improvement correspondence is $P_N^* = \cup P_i^* : W^N \twoheadrightarrow W^N$. The Nash equilibrium of a game G is a vector $\mathbf{z} \in W^N$ such that $P_N^*(\mathbf{z}) = \Phi$. Then the Nash equilibrium will exist when P_N^* is ldc and semi-convex and W^N is admissible.

4 Dynamical Choice Functions

We now consider a *generalized preference field* $H : W \twoheadrightarrow TW$, on a manifold W. TW is the tangent bundle above W, given by $TW = \cup\{T_x W : x \in W\}$, where $T_x W$ is the tangent space above x. If V is a neighborhood of x, then $T_V W = \cup\{T_x W : x \in V\}$ which is locally like the product space $\mathbb{R}^w \times V$. Here W is locally like $\mathbb{R}^w$.

At any $x \in W$, $H(x)$ is a *cone* in the tangent space $T_x W$ above x. That is, if a vector $v \in H(x)$, then $\lambda v \in H(x)$ for any $\lambda > 0$. If there is a smooth curve,

[25] See also Shafer and Sonnenschein (1975).

$c : [-1,1] \to W$, such that the differential $\frac{dc(t)}{dt} \in H(x)$, whenever $c(t) = x$, then c is called an *integral curve* of H. An integral curve of H from x=$c(o)$ to y =$\lim_{t\to 1} c(t)$ is called an H -*preference curve* from x to y. In this case we write $y \in \mathbb{H}(x)$. We say y is reachable from x if there is a piecewise differentiable $H-$preference curve from x to y, so $y \in \mathbb{H}^r(x)$ for some reiteration r.The preference field H is called *S-continuous* iff the inverse relation $\mathbb{H}^{-1}$ is ldc. That is, if x is reachable from y, then there is a neighborhood V of y such that x is reachable from all of V. The *choice* $C(W, H)$ of H on W is defined by

$$C(W, H) = \{x \in W : H(x) = \Phi\}.$$

Say $H(x)$ is semi-convex at $x \in W$, if either $H(x) = \Phi$ or $0 \notin \mathrm{Con}[H(x)]$ in the tangent space T_xW. In the later case, there will exist a vector $v' \in T_xW$ such that $(v' \cdot v) > 0$ for all $v \in H(x)$. We can say in this case that there is, at x, a *direction gradient* d in the cotangent space T^*_xW of linear maps from T_xW to $\mathbb{R}$ such that $d(v) > 0$ for all $v \in H(x)$. If H is S-continuous and half-open in a neighborhood,V, then there will exist such a continuous direction gradient $d : V \to T^*V$ on the neighborhood V [26]

We define

$$Cycle(W, H) \;=\; \{x \;\in W : H(x) \neq \Phi, 0 \in \mathrm{Con}\, H(x)\}.$$

An alternative way to characterize this property is as follows.

Definition 3 The *dual* of a preference field $H : W \twoheadrightarrow TW$ is defined by $H^* : W \twoheadrightarrow T^*W : x \twoheadrightarrow \{d \in T^*_xW : d(v) > 0$ for all $v \in H(x) \subset T_xW\}$. For convenience if $H(x) = \Phi$ we let $H^*(x) = T_xW$. Note that if $0 \notin \mathrm{Con}\, H(x)$ iff $H^*(x) \neq \Phi$. We can say in this case that the field is *half open* at x.

In applications, the field $H(x)$ at x will often consist of some family $\{H_j(x)\}$. As an example , let $u : W \twoheadrightarrow \mathbb{R}^n$ be a smooth utility profile and for any coalition $M \subset N$ let

$$H_M(u)(x) \;=\; \{v \;\in\; T_xW : (du_i(x)(v) >\; 0,\; \forall\, i \;\in\; M\}.$$

If $\mathbb{D}$ is a family of *decisive* coalitions, $\mathbb{D} = \{M \subset N\}$, then we define

$$H_{\mathbb{D}}(u) = \cup H_M(u) : W \twoheadrightarrow TW$$

Then the field $H_{\mathbb{D}}(u) : W \twoheadrightarrow TW$ has a dual $[H_{\mathbb{D}}(u)]^* : W \twoheadrightarrow T^*W$ given by $[H_{\mathbb{D}}(u)]^*(x) = \cap[H_M(u)(x)]^*$ where the intersection at x is taken over all $M \in \mathbb{D}$ such that $H_M(u)(x) \neq \Phi$. We call $[H_M(u)(x)]^*$ the *co-cone of* $[H_M(u)(x)]^*$ .It then follows that at $x \in Cycle(W, H_{\mathbb{D}}(u))$ then $0 \in \mathrm{Con}[H_{\mathbb{D}}(u)(x)]$ and so $[H_{\mathbb{D}}(u)(x)]^* = \Phi$. Thus

$$Cycle(W, H_{\mathbb{D}}(u)) \;=\; \{x \;\in W : [H_{\mathbb{D}}(u)]^*(x) = \Phi\}.$$

The condition that $[H_{\mathbb{D}}(u)]^*(x) = \Phi$ is equivalent to the condition that $\cap[H_M(u)(x)]^* = \Phi$ and was called the *null dual condition* (at x). Schofield

[26] *ie* $d(v) > 0$ for all $v \in H(x)$, for all $v \in H(x)$, whenever $H(x) \neq \Phi$.

(1978) has shown that $Cycle(W, H_{\mathbb{D}}(u))$ will be an open set and contains cycles so that a point x is reachable from itself through a sequence of preference curves associated with different coalitions. This result was an application of a more general result.

Dynamical Choice Theorem (Schofield 1978).

For any S-continuous field H on compact, convex W, then

$$Cycle(W, H) \cup C(W, H) \neq \Phi.$$

If $x \in Cycle(W, H) \neq \Phi$ then there is a *piecewise differentiable* H *-preference cycle* from x to itself. If there is an open path connected neighborhood $V \subset Cycle(W, H)$ such that $H(x')$ is open for all $x' \in V$ then there is a *piecewise differentiable* H *-preference curve* from x to x'.□

(Here piecewise differentiable means the curve is continuous, and also differentiable except at a finite number of points). The proof follows from the previous choice theorem. The trajectory is built up from a set of vectors $\{v_1, ...v_t\}$ each belonging to $H(x)$ with $0 \in \text{Con}[\{v_1, ...v_t\}]$. If $H(x)$ is of full dimension, as in the case of a voting rule, then just as in the model of chaos by Li and York(1975), trajectories defined in terms of H can wander anywhere within any open path connected component of $Cycle(W, H)$.

Existence of Nash Equilibrium

Let $\{W_1, ..., W_n\}$ be a family of compact, contractible, smooth, strategy spaces with each $W_i \subset \mathbb{R}^w$. A smooth profile $u{:}W^N = W_1 \times W_2 ... \times W_n \twoheadrightarrow \mathbb{R}^n$. Let $H_i : W_i \twoheadrightarrow TW_i$ be the induced i-preference field in the tangent space over W_i. If each H_i is S-continuous and half open in TW_i then there exists a *critical Nash equilibrium,* $\mathbf{z} \in W^N$ such that $H^N(\mathbf{z}) = (H_1 \times ..H_n)(\mathbf{z}) = \Phi$.

This follows from the choice theorem because the product preference field, H^N, will be half-open and S-continuous. Below we consider existence of *local* Nash equilibrium.[27] With smooth utility functions, a local Nash equilibrium can be found by checking the second order conditions on the Hessians. (See Schofield, 2007, for an application of this technique)

Example 1.

To illustrate the Choice Theorem, define the preference relation $P_{\mathbb{D}}{:}\, W \twoheadrightarrow W$ generated by a family of *decisive* coalitions, $\mathbb{D} = \{M \subset N\}$, so that $y \in P_{\mathbb{D}}(x)$ whenever all voters in some coalition $M \in \mathbb{D}$ prefer y to x. In particular consider the example due to Kramer (1973), with $N = \{1, 2, 3\}$ and $\mathbb{D} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}$Suppose further that the preferences of the voters are characterized by the direction gradients

$$\{du_i(x){:}\ i = 1, 2, 3\}$$

as in Figure 2. In the figure, the utilities are assume to be "Euclidean," derived from distance from a preferred point, but this assumption is not important.

[Insert Figure 2 here]

[27] That is a critical Nash equilibrium which is an attractor of the integral curves.

As the figure makes evident, it is possible to find three points $\{a, b, c\}$ in W such that

$$\begin{aligned} u_1(a) &> u_1(b) = u_1(x) > u_1(c) \\ u_2(b) &> u_2(c) = u_2(x) > u_2(a) \\ u_3(c) &> u_3(a) = u_3(x) > u_3(b). \end{aligned}$$

That is to say, preferences on $\{a, b, c\}$ give rise to a *Condorcet cycle.* Note also that the set of points $P_{\mathbb{D}}(x)$, preferred to x under the voting rule, are the shaded "win sets" in the figure. Clearly $x \in \mathrm{Con}\, P_{\mathbb{D}}(x)$, so $P_{\mathbb{D}}(x)$ is not semi-convex. Indeed it should be clear that in *any* neighborhood V of x it is possible to find three points $\{a', b', c'\}$ such that there is *local* voting cycle, with $a' \in P_{\mathbb{D}}(b'), b' \in P_{\mathbb{D}}(c'), c' \in P_{\mathbb{D}}(a')$. We can write this as

$$a' \to c' \to b' \to a'.$$

Not only is there a voting cycle, but the Fan theorem fails, and we have no reason to believe that $C(W, P_{\mathbb{D}}) \neq \Phi$.

We can translate this example into one on preference fields by considering the preference field

$$H_{\mathbb{D}}(u) = \cup H_M(u) : W \twoheadrightarrow TW$$

where each $M \in \mathbb{D}$.

Figure 3 shows the three difference preference fields $\{H_i : i = 1, 2, 3)$ on W, as well as the intersections H_M, for $M = \{1, 2\}$ etc.

[Insert Figure 3 here]

Obviously the joint preference field $H_{\mathbb{D}}(u) = \cup H_M(u) : W \twoheadrightarrow TW$ fails the half open property at x since $0 \in \mathrm{Con}[H_{\mathbb{D}}(u)(x)]$. Although $H_{\mathbb{D}}(u)$ is S-continuous, we cannot infer that $C(W, H_{\mathbb{D}}(u)) \neq \Phi$.

Chilchinisky (1992, 1995,1996a,1997a) has obtained similar results for markets, where the condition that the dual is non-empty was termed *market arbitrage,* and defined in terms of global market co-cones associated with each player. Such a dual co-cone, $[H_i(u)]^*$ is precisely the set of prices in the cotangent space that lie in the dual of the prefered cone, $[H_i(u)]$,of the agent. By analogy with the above, she identifies this condition on non-emptiness of the intersection of the family of co-cones as one which is necessary and sufficient to guarantee an equilibrium.

Chichilnisky Theorem. (Chichilnisky ,1997b)

The *limited arbitrage condition* $\cap[H_i(u)]^* \neq \Phi$ is necessary and sufficient for existence of a competitive equilibrium.□

Chichilnisky (1993, 1997c) also defined a topological obstruction to the non-emptiness of this intersection and showed the connection with the existence of a social choice equilibrium.

For a voting rule, $\mathbb{D}$ it is possible to guarantee that $Cycle(W, H_{\mathbb{D}}) = \Phi$ and thus that $C(W, H_{\mathbb{D}}) \neq \Phi$. We can do this by restricting the dimension of W.

Definition 4

(i) Let $\mathbb{D}$ be a family of decisive subsets of the finite society N of size n. If the collegium, $K(\mathbb{D}) = \cap\{M \in \mathbb{D}\}$ is non-empty then $\mathbb{D}$ is called *collegial* and the *Nakamura number* $\kappa(\mathbb{D})$ is defined to be ∞.

(ii) If the collegium $K(\mathbb{D})$ is empty then $\mathbb{D}$ is called *non-collegial.* Define the *Nakamura number* in this case to be $\kappa(\mathbb{D}) = \min\{|\mathbb{D}'|: \mathbb{D}' \subset \mathbb{D}$ and $K(\mathbb{D}') = \Phi\}$.

Nakamura Theorem.

If $u \in U(W)^N$ and $\mathbb{D}$ has Nakamura number $\kappa(\mathbb{D})$ with $dim(W) \leq \kappa(\mathbb{D}) - 2$ then $Cycle(W, H_{\mathbb{D}}(u)) = \Phi$ and $C(W, H_{\mathbb{D}}(u)) \neq \Phi$.

Outline of proof. Consider any subfamily $\mathbb{D}'$ of $\mathbb{D}$ with cardinality $\kappa(\mathbb{D}){-}1$. Then $\cap M \neq \Phi$, so $\cap\{[H_M(u)]^*(x) : M \in \mathbb{D}'\} \neq \Phi$. If $[H_M(u)(x)] \neq \Phi$, we can identify each $[H_M(u)(x)]^*$ with a non-empty convex hull generated by $(du_i(x) : i \in M\}$. These sets can be projected into T_xW where they are convex and compact. Since $dim(W) \leq \kappa(\mathbb{D}) - 2$, then by Helly's Theorem, we see that $\cap\{[H_M(u)]^*(x) : M \in \mathbb{D}\} \neq \Phi$. Thus $Cycle(W, H_{\mathbb{D}}(u)) = \Phi$ and $C(W, H_{\mathbb{D}}(u)) \neq \Phi$.□

See Schofield (1984b), Nakamura (1979) and Strnad (1985).

For social choice defined by voting games, the Nakamura number for majority rule is 3, except when $n = 4$, in which case $\kappa(\mathbb{D})$ =4, so the Nakamura Theorem can generally only be used to prove a "median voter" theorem in one dimension. However, the result can be combined with the Fan Theorem to prove existence of equilibrium for a political economy with voting rule $\mathbb{D}$, when the dimension of the public good space is no more than $\kappa(\mathbb{D}) - 2$ (Konishi, 1996). Recent work in political economy often only considers a public good space of one dimension (Acemoglu and Robinson,2006). Note however, that if $\mathbb{D}$ is collegial, then $Cycle(W, H_{\mathbb{D}}(u)) = \Phi$.and $C(W, H_{\mathbb{D}}(u)) \neq \Phi$. Such a rule can be called oligarchic, and this inference provides a theoretical basis for comparing democracy and oligarchy (Acemoglu, 2008). Figure 3 showed the preference cones in a majority voting game with 3 agents and Nakamura number 3, so half openess fails in two dimensions.

Extending the equilibrium result of the Nakamura Theorem to higher dimension for a voting rule faces a difficulty caused by Saari's Theorem. We first define a *fine* topology on smooth utility functions.(Hirsch, 1976; Schofield, 1999c, 2003).

Definition 5

Let $(U(W)^N, T_1)$ be the topological space of smooth utility profiles enowed with the the C^1-topology

In economic theory, the existence of isolated price equilibria can be shown to be "generic" in this topological space (Debreu, 1970, 1976; Smale, 1974a,b). In social choice no such equilibrium theorem holds. The difference is essentially because of the coalitional nature of social choice.

Saari Theorem.

For any non-collegial $\mathbb{D}$, there exists an integer $w(\mathbb{D}) \geq \kappa(\mathbb{D}){-}1$ such that $dim(W) > w(\mathbb{D})$ implies that $C(W, H_{\mathbb{D}}(u)) = \Phi$ for all u in a dense subspace of $(U(W)^N, T_1)$ so $Cycle(W, H_{\mathbb{D}}(u)) \neq \Phi$ generically.□

This result was essentially proved by Saari (1997), building on earlier results by Plott (1967), McKelvey (1979), Schofield (1983), McKelvey and Schofield (1987) and Banks (1995). See Saari (1985a,b, 2001a,b, 2008) for related analyses. Indeed, it can be shown that if $dim(W) > w(\mathbb{D})+ 1$ then $Cycle(W, H_{\mathbb{D}}(u))$ is generically dense (Schofield (1984c). The integer $w(\mathbb{D})$ can usully be computed explicitly from $\mathbb{D}$. For majority rule with n odd it is known that $w(\mathbb{D}) = 2$ while for n even, $w(\mathbb{D}) = 3$. Saari showed, for a $q - rule$ where any coalition of size at least $q(< n)$ belongs to the set of decisive colitions, $\mathbb{D}_q$, that

$$w(\mathbb{D}_q) = 2q - n + 1 + max\{\frac{4q - 3n - 1}{2(n - q}, 0\}$$

Although the Saari Theorem formally applies only to voting rules, Schofield (2010) argues that it is applicable to any non-collegial social mechanism, say $H(u)$ and can be interpreted to imply that

$$Cycle(W, H(u)) \neq \Phi \text{ and } C(W, H(u)) = \Phi$$

is a generic phenomenon in coalitional systems. Because preference curves can wander anywhere in any open component of $Cycle(W, H(u))$, Schofield (1979) called this *chaos.* It is not so much the sensitive dependence on initial conditions, but the aspect of indeterminacy that is emphasized. To illustrate, Figure 4 shows the preference cones in an prisoners' dilemma with three players. In this example the preference fields, $\{T_x, T_y, T_z,\}$ of the non-cooperative individuals belong to a different half space from the two person cooperative fields $\{T_{yx}, T_{yz}, T_{zx},\}$.Again half openess fails, so $Cycle(W, H(u)) \neq \Phi$, and we lack any result that would allow us to infer existence of a cooperative equilibrium. Indeed, cooperative and non-cooperative behavior are both possible. On the other hand, existence of a hegemon, as discussed in Section 2, is similar to existence of a collegium, suggesting that $Cycle(W, H(u))$ would be constrained in this case.

Richards (1993) has examined data on the distribution of power in the international system over the long run and and presents evidence that it can be interpreted in terms of a chaotic trajectory. This suggests that the metaphor of the nPD in international affairs does characterise the ebb and flow of the system and the rise and decline of hegemony.

[Insert Figure 4 here]

In the following sections I shall consider more general social processes in order to examine how $Cycle(W, H)$ may be subject to catastrophic change. The next section considers models of belief aggregation or truth-seeking in a society.

5 Beliefs and Condorcet's Jury Theorem

The Jury theorem formally only refers to a situation where there are just two alternatives $\{1, 0\}$, and alternative 1 is the "true" option. Further, for every individual, i, it is the case that the probability that i picks the truth is ρ_{i1}, which exceeds the probability ,ρ_{i0}, that i does not pick the truth..We can assume that $\rho_{i1}+\rho_{i0}=1$, so obviously $\rho_{i1}>\frac{1}{2}$. To simplify the proof, we can assume that ρ_{i1} is the same for every individual, thus $\rho_{i1}=\alpha>\frac{1}{2}$ for all i. We use χ_i ($=0$ or 1) to refer to the choice of individual i, and let $\chi=\Sigma_{i=1}^{n}\chi_i$ be the number of individuals who select the true option 1. We use Pr for the probability operator, and E for the expectation operator. In the case that the electoral size, n, is odd, then a majority, m, is defined to be $m=\frac{n+1}{2}$. In the case n is even, the majority is $m=\frac{n}{2}+1$. The probability that a majority chooses the true option is then

$$\alpha_{maj}^{n}=\Pr[\chi\geq m].$$

The theorem assumes that voter choice is *pairwise independent*, so that $\Pr(\chi=j)$ is simply given by the binomial expression $\binom{n}{j}\alpha^{j}(1-\alpha)^{n-j}$.

A version of the theorem can be proved in the case that the probabilities $\{\rho_{i1}=\alpha_i\}$ differ but satisfy the requirement that $\frac{1}{n}\Sigma_{i=1}^{n}\alpha_i>\frac{1}{2}$. Versions of the theorem are valid when voter choices are not pairwise independent (Ladha and Miller 1996).

The Jury Theorem. If $1>\alpha>\frac{1}{2}$, then $\alpha_{maj}^{n}\geq\alpha$, and $\alpha_{maj}^{n}\longrightarrow 1$ as $n\longrightarrow\infty$.

Proof. Consider the case with n odd. Now

$$\Pr(\chi=j)=\binom{n}{j}\alpha^{j}(1-\alpha)^{n-j}=\binom{n}{n-j}\alpha^{j}(1-\alpha)^{n-j} \qquad (8)$$

Since $\alpha>\frac{1}{2}$ we see that $\alpha^{n-j}(1-\alpha)^{j}>\alpha^{j}(1-\alpha)^{n-j}$. Thus,

$$\Sigma_{j=0}^{m-1}j\Pr(\chi=n-j)>\Sigma_{j=0}^{m-1}j\Pr(\chi=j)$$

or

$$\Sigma_{k=m}^{n}(n-k)\Pr(\chi=k)>\Sigma_{k=0}^{m-1}k\Pr(\chi=k).$$

Thus

$$n\Sigma_{k=m}^{n}\Pr(\chi=k)>\Sigma_{k=0}^{m-1}k\Pr(\chi=k)+\Sigma_{k=m}^{n}k\Pr(\chi=k).$$

But

$$n\alpha_{maj}=n\Sigma_{k=m}^{n}\Pr(\chi=k)$$

and

$$E(\chi)=\Sigma_{k=0}^{n}k\Pr(\chi=k)=n\alpha.$$

Thus, $\alpha_{maj}>\alpha$ when n is odd.

The case with n even follows in similar fashion, taking $m=\frac{n}{2}+1$, and using an equi-probable tie-breaking rule when $k=\frac{n}{2}$.This gives

$$\alpha_{maj}=\Sigma_{k=m}^{n}\Pr(\chi=k)+\frac{1}{2}\Pr(\chi=\frac{n}{2}).$$

For both n being even or odd, as $n\longrightarrow\infty$, the fraction of voters choosing option 1 approaches $\frac{1}{n}E(\chi)=\alpha>\frac{1}{2}$. Thus, in the limit, more than half the voters choose the true option. Hence the probability $\alpha_{maj}^{n}\longrightarrow 1$ as $n\longrightarrow\infty$.$\square$

Laplace also wrote on the topic of the probability of an error in the judgement of a tribunal. He was concerned with the degree to which jurors would make just decisions in a situation of asymmetric costs, where finding an innocent party guilty was to be more feared than letting the guilty party go free. As he commented on the appropriate rule for a jury of twelve, "I think that in order to give a sufficient guarantee to innocence, one ought to demand at least a plurality of nine votes in twelve" (Laplace 1951[1814]:139). Schofield (1972a,b) considered a model derived from the jury theorem where uncertain citizens were concerned to choose an ethical rule which would minimize their disappointment over the the likely outcomes, and showed that majority rule was indeed optimal in this sense.

Models of belief aggregation extend the Jury theorem by considering a situation where individuals receive signals, update their beliefs and make an aggregate choice on the basis of their posterior beliefs (Austen-Smith and Banks, 1996). Models of this kind can be used as the basis for analysing correlated beliefs.[28] and the creation of belief cascades (Easley and Kleinberg, 2010).

Schofield (2002, 2006) has argued that Condorcet's Jury theorem provided the basis for Madison's argument in Federalist X (Madison, 1999.[1787]) that the judgments of citizens in the extended Republic would enhance the "probability of a fit choice." However, Schofield's discussion suggests that belief cascades can also fracture the society in two opposed factions, as in the lead up to the Civil War in 1860.[29]

There has been a very extensive literature recently on cascades[30] but it is unclear from this literature whether cascades will be equilibriating or very volatile. In their formal analysis of cascades on a network of social connections, Golub and Jackson (2010) use the term *wise* if the process can attain the truth. In particular they note that if one agent in the network is highly connected, then untrue beliefs of this agent can steer the crowd away from the truth. The recent economic disaster has led to research on market behavior to see if the notion of cascades can be used to explain why markets can become volatile or even irrational in some sense. (Acemoglu et al. 2010; Schweitzer et al. 2009). Indeed the literature that has developed in the last few years has dealt with the nature of herd instinct, the way markets respond to speculative behavior and the power law that characterizes market price movements.[31] The general idea is that the market can no longer be regarded as efficient. Indeed, as suggested by Ormerod (2001) the market may be fundamentally chaotic.

"Empirical" chaos was probably first discovered by Lorenz (1962, 1963) in his efforts to numerically solve a system of equations representative of the behavior of weather. A very simple version is the non-linear vector equation

[28] Schofield 1972 a,b; Ladha 1992, 1993, 1995, 1996; Ladha and Miller 1996.

[29] Sunstein (2006, 2011) also notes that belief aggregation can lead to a situation where subgroups in the society come to hold very disparate opinions. See also Ladha (1992,1993) and Ladha and Miller (1996) for a Jury Theorem with correlated beliefs

[30] Gleick (1987), Buchanan, 2001, 2003; Gladwell, 2002; Johnson, 2002; Barabasi 2003, 2010; ; Strogatz, 2004; Watts, 2002, 2003; Surowiecki , 2005; Ball, 2004; Christakis and Fowler, 2011.

[31] See, for example, Mandelbrot and Hudson (2004), Shiller (2003, 2005), Taleb (2007), Barbera (2009), Cassidy (2009), Fox (2009).

$$\frac{dx}{dt} = \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} = \begin{bmatrix} -a_1(x_1 - x_2) \\ -x_1x_3 \ + \ a_2x_1 - x_2 \\ x_1x_2 - a_3x_3 \end{bmatrix}$$

which is chaotic for certain ranges of the three constants, a_1, a_2, a_3.

The resulting "butterfly" portrait winds a number of times about the left hole (as in Figure 5),then about the right hole,then the left,*etc.* Thus the "phase prortrait" of this dynamical system can be described by a sequence of winding numbers $(w_l^1, w_k^1, w_l^2, w_k^2,\ etc.)$. Changing the constants a_1, a_2, a_3 slightly changes the winding numbers. Note that the picture in Figure 5 is in three dimensions, The butterfly wings on left and right consist of infinitely many closed loops. The whole thing is called the Lorentz "strange attractor." A slight perturbation of this dynamic system changes the winding numbers and thus the qualitative nature of the process. Clearly this dynamic system is not structurally stable, in the sense used by Kaufmann (1993). The metaphor of the butterfly gives us pause, since all dynamic systems whether models of climate, markets, voting processes or cascades may be indeterminate or chaotic.

[Insert Figure 5 here]..

A chaotic market model is provided by Coros et al. (2002) who consider a formal model of the market, based on the reasoning behind Keynes's "beauty contest" (Keynes 1936). There are two coalitions of "bulls" and "bears". Individuals randomly sample opinion from the coalitions and use a *critical* cutoff-rule. For example if the individual is bullish and the sampled ratio of bears exceeds some proportion then the individual flips to bearish. The model is very like that of the Jury Theorem but instead of guaranteeing a good choice the model can generate chaotic flips between bullish and bearish markets, as well as fixed points or cyclic behavior, depending on the cut-off parameters. Taleb's argument (Taleb, 1997) about black swan events can be applied to the recent transformation in societies in the Middle East and North Africa that resemble such a cascade. (Taleb and Blyth, 2011). As in the earlier episodes in Eastern Europe, it would seem plausible that the sudden onset of a cascade is due to a tipping switch in a critical coalition.

The notion of "criticality" has spawned in enormous literture particularly in fields involving evolution, in biology, language and culture.[32] Bak and Sneppen (1993) refer to the self organized critical state as the

> "edge of chaos" since it separates a frozen inactive state from a "hot" disordered state.

Flyvbjerg et al (1993) go on to say

> species sit at local fitness maxima..and occasionally a species jumps to another maximum [in doing so it] may change the fitness landscapes of other species which depend on it. ..Consequently they immediately jump to new maxima. This may affect yet another species in a chain reaction, a *burst* of evolutionary activity.

[32] See for example Cavallli-Sforza and Feldman (1981), Bowles et al (2003)..

This work was triggered by the earlier ideas on "punctuated equilibrium" by Eldredge and Gould (1972).[33] As Gould (2002:782) writes

> [T]hus, the pattern of punctuated equilibrium establishes species as effective individuals and potential Darwinian agents in the mechanisms of macroevolution.

There are a number of points to be made about these remarks. First of all, the fitness model can be viewed as essentially a game, based on a search for a critical (or local) Nash equilibrium.[34].When Dawkins (1976) wrote of the "selfish gene" he seemed to imply that evolution could be structured in the form of a non-cooperative game But Jablonka and Lamb (2005:38) observe

> For Gould, the central focus of evolutionary studies has to be organisms, groups, and species. For Dawkins, it has to be the gene, the unit of hereditary.

This suggests that "cooperation" is at the center of evolution. Following Gould, the critical Nash equilibria at the level of species can be destroyed by a "catastrophe" (Zeeman, 1977). Finding the new evolutionary trajectory may require joint changes in a *coalition* of the species. Indeed there may be numerous different co-adaptive species.

Second, evolutionary transformations at all levels are largely the result of new configurations of coalitions of genes. For Margulis and Sagan (2002:12, 20),

> the major source of inherited variation is not random mutation. Rather the important transmitted variation that leads to evolutionary novelty comes from the acquisition of genomes. Entire sets of genes.. are acquired and incorporated by others.
>
> We must begin to think of organisms as communities, as collectives. And communities are ecological entities.

At the level of the gene, evolutionary change may requires new genomic structures that again are coalitions of genes.

The point to be emphasized is that the evolution of a species involves bifurcation, a splitting of the pathway. We can refer to the bifurcation as a *catastrophe* or a *singularity*. The portal or door to the singularity may well be characterized by chaos or uncertainty, since the path can veer off in many possible directions, as suggested by the bifurcating cones in Figures 3 and 4. At every level that we consider, the bifurcations of the evolutionary trajectory seem to be locally characterized by chaotic domains. I suggest that these domains are the result of different coalitional possibilities. The fact that the trajectories can become indeterminate suggests that this may enhance the exploration of the fitness landscape.

[33] See also Eldredge (1976), Gould (1976).

[34] See Maynard Smith (1982) for the game theoretic notion of evolutionary stable strategy.

A more general remark concerns the role of climate change. Climate has exhibited chaotic or catastrophic behavior in the past.[35] There is good reason to believe that human evolution over the last million years can only be understood in terms of "bursts" of sudden transformations (Nowak, 2011) and that language and culture co-evolve through group or coalition selection (Cavallli-Sforza and Feldman 1981, Wilson, 2012). Calvin (2003) suggests that our braininess was cause and effect of the rapid exploration of the fitness landscape in response to climatic forcing. Stringer (2012) calls this theory of rapid transformation the *Social Brain Hypothesis* or *Machiavellian intelligence. The* cave art of Chauvet,in France, and elsewhere in Europe, dating back about 36,000 years hints at belief in magic and the supernatural.Indeed Stringer speculates that religion played an important role in human evolution. providing a key mechanism for the retention of important social technology. Indeed he notes that evolution is still occurring, as a consequence of the coevolution of humans with their diseases (Diamond,1997). We might speculate that our minds exhibit evolutionary competition between mathematical ability and and religious belief.

6 A Moral Compass

The results in topological social choice,discussed above provide some justification for the assertion by Popper (1959) that prediction is impossible in the social sciences. Barabasi (2011) argues, in contrast, that our ability to process data will allow us in the future to model society in a deterministic fashion. However, even attempts to analyze climate change itself has shown how difficult it is to model inter-related processes with positive feedback mechanisms (Edwards (2010).

If my interpretation of the models presented here is correct, then uncertainty is a fundamental fact of coalitional forces in society. In particular the uncertainty involves the critical transition from what appears to be an equilibrium situation to a situation where the social trajectory becomes highly volatile. This seems to be the lesson to be drawn from Arrow's Impossibility theorem. Moreover the Radical Enlightenment project of constructing a rational society may be impossible for the same reason. Indeed Haidt (2012) argues that people do have strong moral bases for their perceptions and preferences. Perhaps our society will have to consider the necessity of constructing a moral compass that will help in deciding how to deal with climate change. Damasio (2003) suggests going back to Spinoza's emphasis on the importance of feelings as a basis for the human sciences These comments imply a challenge for social choice theory that will give greater weight to moral attitudes as proposed by Baigent (2007).

[35] Indeed as I understand the dynamical models, the chaotic episodes are due to the complex interactions of dynamical processes in the oceans, on the land, in weather, and in the heavens. These are very like interlinked *coalitions* of non-gradient vector fields.

7 Conclusion.

Parfit's remarks on climate chnge are worth quotng here:

> What matters most is that we rich people give up some of our luxeries, ceasing to overheat the Earth's atmosphere, and taking care of this planet in other ways, so that it continues to support intelligent life. If we are the only rational animals in the Universe, it matters even more whether we shall have descendants during the billions of years in which that would be possible. Some of our descendants might live lives and create worlds that, though failing to justify past suffering, would give us all, including those who suffered, reason to be glad that the Universe exists.(Parfit, 2011: 419)

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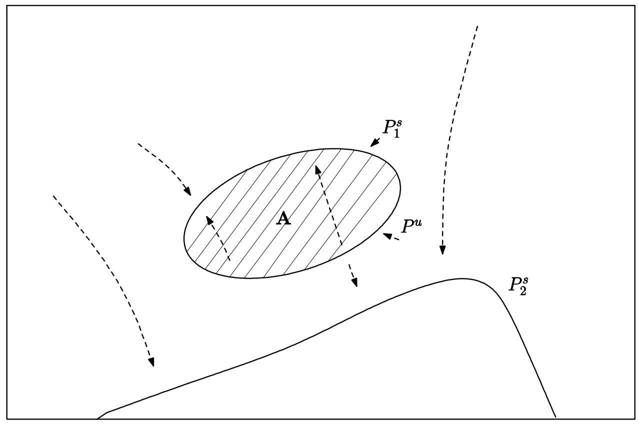

Figure 1: Stable and unstable components of the global Pareto Set

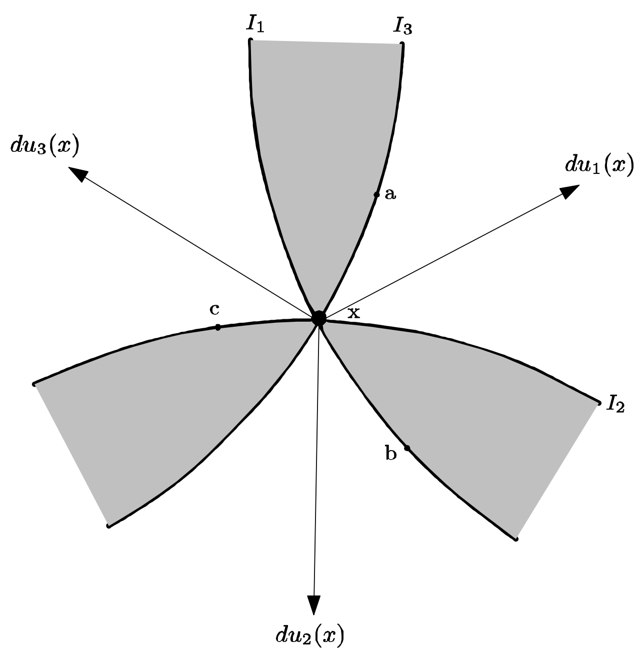

Figure 2: Cycles in a neighborhood of x.

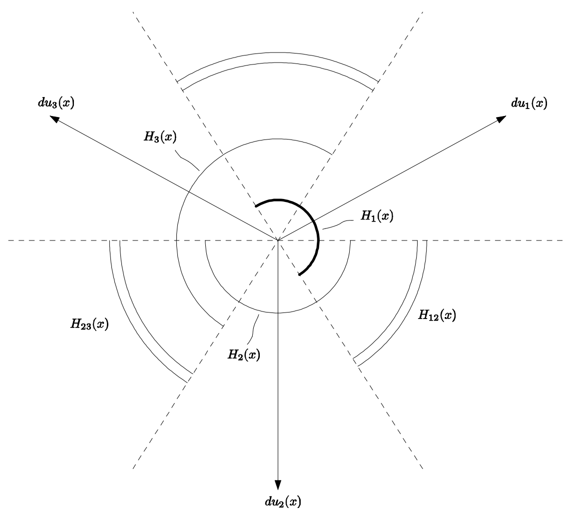

Figure 3: The failure of half-openess of a preference field

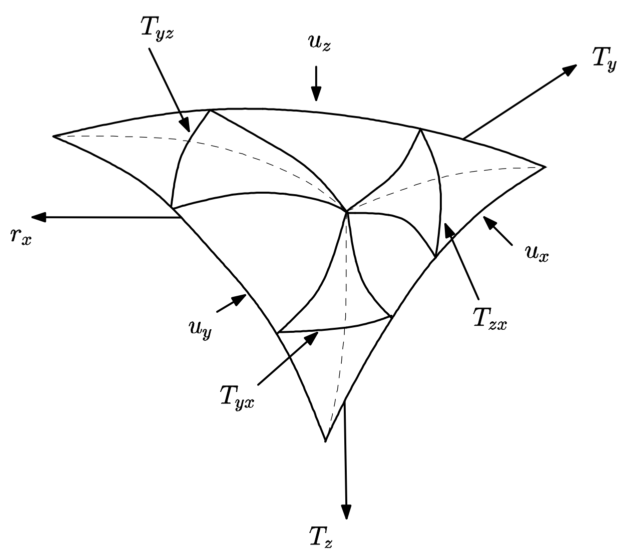

Figure 4: Failure of half-openess in the three person prisoners' dilemma

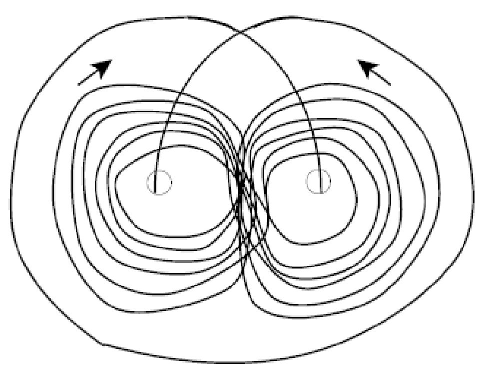

Figure 5: The butterfly