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Competition for popular support: a valence model of elections in Turkey

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Abstract Models of elections tend to predict that parties will maximize votes by converging to an electoral center. There is no empirical support for this prediction. In order to account for the phenomenon of political divergence, this paper offers a stochastic electoral model where party leaders or candidates are differentiated by differing valences—the electoral perception of the quality of the party leader. If valence is simply intrinsic, then it can be shown that there is a "convergence coefficient", defined in terms of the empirical parameters, that must be bounded above by the dimension of the space, in order for the electoral mean to be a Nash equilibrium. This model is applied to elections in Turkey in 1999 and 2002. The idea of valence is then extended to include the possibility that activist groups contribute resources to their favored parties in response to policy concessions from the parties. The equilibrium result is that parties, in order to maximize vote share, must balance a centripetal electoral force against a centrifugal activist effect. We estimate pure spatial models and models with sociodemographic valences, and use simulations to compare the equilibrium predictions with the estimated party positions.

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1 Introduction: modeling popular support

The early work in modeling elections focused on two-party competition, and assumed a one-dimensional policy space, X, and "deterministic" voter choice. The models showed the existence of a "core" point, unbeaten under majority rule vote, at the median of the electoral distribution. Such models implied that there would be strong centripetal political forces causing parties to converge to the electoral center (Hotelling 1929; Downs 1957). In higher dimensions, such two-party "pure strategy Nash equilibria" (PNE) generally do not exist, so the theory did not cover empirical situations where two or more policy dimensions were relevant.[1] It has been shown, however, that there would exist *mixed* strategy Nash equilibria whose support lies within a subset of the policy space known as the "uncovered set."[2] "Attractors" of the political process, such as the "core", the "uncovered set" or the "heart" (Schofield 1999) are centrally located with respect to the distribution of voters' ideal points. The theoretical prediction that political candidates converge to the center is very much at odds with empirical evidence from U.S. Presidential elections that political candidates do not locate themselves close to the electoral center.[3]

The deterministic electoral model is also ill-suited to deal with the multiparty case. (Here multiparty refers to the situation where the number of candidates or parties, p, is at least three.) As a result, recent work has focused on "stochastic" models which are, in principle, compatible with empirical models of voter choice.[4] In such models, the behavior of each voter is modeled by a vector of choice probabilities. Various theoretical results for this class of models suggested that vote maximizing parties would converge to the mean of the electoral distribution of voter ideal points.[5]

Empirical estimates of party positions in European multiparty polities can be constructed on the basis of various techniques of content analysis of party manifestos.[6] More recent analyses have been based on factor analysis of electoral survey data to obtain a multidimensional description of the main political issues in various countries. All these empirical analyses have obtained policy spaces that are two dimensional. These techniques allow for the estimation of the positions of the parties in the empirically inferred policy space. These estimates have found no general tendency for parties to converge to the center.[7]

The various empirical electoral models can be combined with simulation techniques to determine how parties should respond to electoral incentives to maximize their vote shares. Schofield and Sened (2006), in their simulation of elections in Israel in the period 1988–1996, found that vote maximizing parties did not converge to the electoral origin. It may be objected that factor analysis of survey data gives

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1 See Saari (1997) and the survey in Austen-Smith and Banks (1999).

2 Banks et al. (2006).

3 Poole and Rosenthal (1984), Schofield et al. (2003). See also the empirical work in Schofield et al. (2011).

4 Schofield and Sened (2006).

5 Hinich (1977), Lin et al. (1999), Banks and Duggan (2005), McKelvey and Patty (2006).

6 See Laver and Hunt (1992). Benoit and Laver (2006) use expert estimates.

7 See Adams and Merrill (1999), for example.

only a crude estimate of the variation in voter preferences, while vote maximization disregards the complex incentives that parties face. Nonetheless, as a modeling exercise, the stochastic model for Israel seemed to provide a plausible account of the nature of individual choice[8] as well as the party positioning decision. Although the simulated equilibrium positions of the parties in Israel were not identical to the estimated positions, the positions were generally far from the origin, and for some of the parties very close to their estimated positions. The purpose of this paper is to attempt to extend the stochastic empirical model so as to close the apparent disparity between the simulated equilibrium positions of the parties, and the estimated positions.

The key to the contradiction between the non-convergence result of Schofield and Sened, and the convergence result in other work on the formal stochastic model was the incorporation of an asymmetry in the perception of the quality of the party leaders, expressed in terms of *valence* (Stokes 1963, 1992).

In the model presented here, the average weight, in the voter calculus, given to the perceived quality of the leader of the jth party is called the party's *intrinsic or exogenous valence.* In empirical models this valence is assumed to be exogenous, so it is independent of the party's position. The valence coefficients for each party are generated by the estimation of the stochastic model, based on the "multinomial logit" (MNL) assumption that the stochastic errors have a "Type I extreme value or Gumbel distribution" (Dow and Endersby 2004). These valence terms add to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future.[9]

The Appendix considers a pure spatial stochastic vote model,with party specific exogenous valences, based on the same distribution assumption, and on the assumption that each party leader attempts to locally maximize the party's vote share. Results from Schofield (2007a,b) give the necessary and sufficient conditions under which there is a "local pure strategy Nash equilibrium" (LNE) of this model at the *joint electoral mean* (that is, where each party adopts the same position, z_0, at the mean of the electoral distribution).[10] Theorem 2 in the Appendix shows that a "convergence coefficient", c, incorporating all the parameters of the model, can be defined. This coefficient, c, involves the differences in the valences of the party leaders, and the "spatial coefficient" β. When the policy space, X, is assumed to be of dimension w, then the necessary condition for existence of an LNE at the electoral center[11] is that the coefficient, c, is bounded above by w.

8 Over 60% of the individual votes were correctly modeled.

9 See Penn (2009). Notice, however, that valence refers to the *perception* by voters of the quality of political leaders. Recent work by Westen (2007), for the United States, suggests that voters' perceptions of the characteristics of political candidates are very important. Moreover, Schofield et al. (2011) shows that voter perception of character traits has a strong effect on candidate positions in the United States.

10 A local Nash equilibrium under vote maximization is just a vector of positions such that no small unilateral move by a party can increase its vote. The usual notion of a pure stratgy Nash equilibrium (PNE) cannot be used because in the games we study there may exist no PNE.

11 Again, the electoral center, or origin, is defined to be the mean of the distribution of voter ideal points.

When the necessary condition fails, then parties, in equilibrium, will adopt divergent positions. Because a pure strategy Nash equilibrium must be an LNE, the failure of existence of LNE when all parties are at the electoral mean implies non existence of such a centrist PNE. In this case, a party whose leader has the lowest valence will have the greatest electoral incentive to move away from the electoral mean. As the party moves away from the electoral mean, it increases the probability that voters on the electoral periphery will vote for it.[12] Other low valence parties will follow suit, and the local equilibrium will be one where parties are distributed along a "principal electoral axis."[13] The general conclusion is that, with all other parameters fixed, then a convergent LNE can be guaranteed only when β is "sufficiently" small. Thus, divergence away from the electoral mean becomes more likely the greater are β, the valence differences and the variance of the electoral distribution.[14]

The innovation of this paper is that in additional to exogenous valence, we also incorporate "sociodemographic valence." These party specific valence terms are associated with different groups in the society, and are defined by dichotomous or continuous characteristics of different subgroups in the population. This model is shown to be statistically superior to the spatial model with exogenous valence. This is the case because the exogenous valence model assumes that all voters have the same perception of the quality of the party leaders, whereas with the sociodemographic variables, these perceptions are allowed to vary across different subgroups.

We apply this valence model by considering in some detail a sequence of elections in Turkey from 1999 to 2007. The election results are given in Tables 1, 2, and 3, which also provide the acronyms for the various parties.

As in other related work, the empirical models were based on factor analyses of voter surveys.[15] Figures 1 and 2 show the electoral distributions (based on sample surveys of sizes 635 and 483, respectively) and estimates of party positions for 1999 and 2002.[16] The two dimensions in both years were a "left–right" religion axis and a "north–south" Nationalism axis, with secularism or "Kemalism" on the left and Turkish nationalism to the north. (See also Carkoğlu and Hinich (2006) for a spatial model of the 1999 election.)

Minor differences between these two figures include the disappearance of the Virtue Party (FP) which was banned by the Constitutional Court in 2001, and the change

[12] This follows for theoretical reasons as shown in Schofield (2007a). When $c > w$, at least one of the eigenvalues of the Hessian of the vote share function of a low valence party will be large and positive at the origin. As it moves from the origin, it will lose votes from centrist voters, but gain votes from more radical voters. Simulation of empirical models for Israel (Schofield and Sened 2006) has shown this to be the case.

[13] The principal electoral axis is defined to be the one dimensional subspace along which the variance of the distribution of voter ideal points is maximum.

[14] These results are presented for the reader's convenience in the context of the more general model described in the Appendix.

[15] The estimations presented below are based on factor analyses of sample surveys conducted by Veri Arastima for TUSES.

[16] The party positions were estimated using expert analyses, in the same way as the work by Benoit and Laver (2006).

Table 1 Turkish election results 1999

Party name		% Vote	Seats	% Seats
Democratic Left Party	DSP	22.19	136	25
Nationalist Action Party	MHP	17.98	129	23
Virtue Party	FP	15.41	111	20
Motherland Party	ANAP	13.22	86	16
True Path Party	DYP	12.01	85	15
Republican People's Party	CHP	8.71	–	–
People's Democracy Party	HADEP	4.75	–	–
Others	–	4.86	–	–
Independents	–	0.87	3	1
Total			550	

Table 2 Turkish election results 2002

Party name		% Vote	Seats	% Seats
Justice and Development Party	AKP	34.28	363	66
Republican People's Party	CHP	19.39	178	32
True Path Party	DYP	9.54	–	–
Nationalist Action Party	MHP	8.36	–	–
Young Party	GP	7.25	–	–
People's Democracy Party	HADEP	6.22	–	–
Motherland Party	ANAP	5.13	–	–
Felicity Party	SP	2.49	–	–
Democratic Left Party	DSP	1.22	–	–
Others and Independents	–	6.12	9	2
Total			550	

Table 3 Turkish election results 2007

Party name		% Vote	Seats	% Seats
Justice and Development Party	AKP	46.6	340	61.8
Republican People's Party	CHP	20.9	112	20.3
Nationalist Movement Party	MHP	14.3	71	12.9
Democrat Party	DP	5.4	–	–
Young Party	GP	3.0	–	–
Felicity Party	SP	2.3	–	–
Independents	–	5.2	27[a]	4.9
Others	–	2.3	–	–
Total		100	550	100

[a] Twenty-four of these "independents" were in fact members of the DTP—the Kurdish Freedom and Solidarity Party

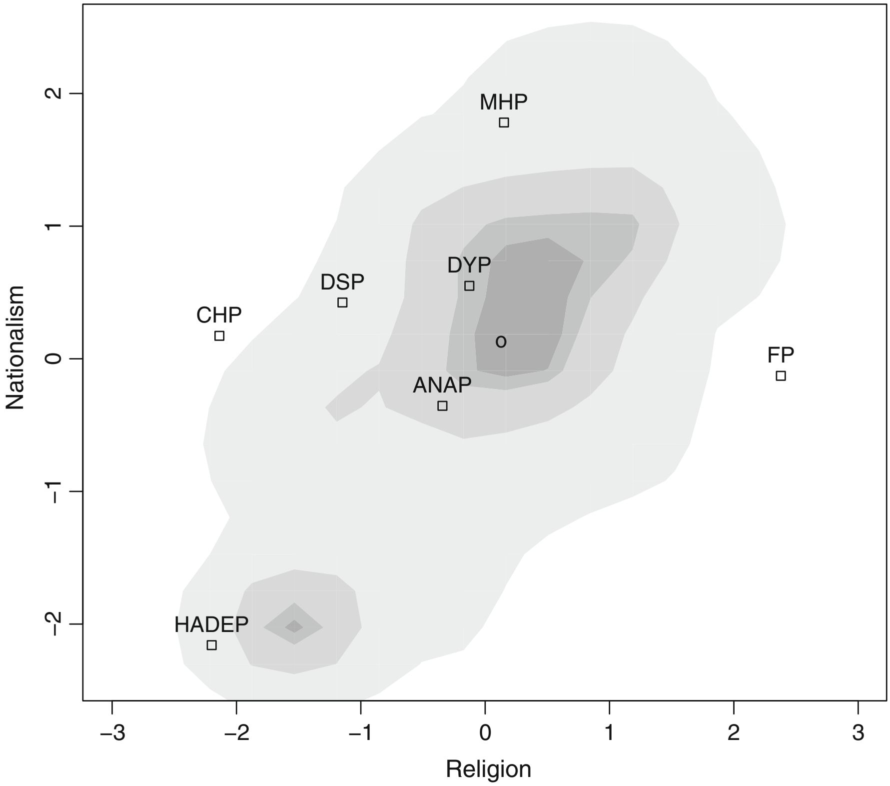

Fig. 1 Party positions and voter distribution in Turkey in 1999

of the name of the pro-Kurdish party from HADEP to DEHAP.[17] The most important change is the appearance of the new Justice and Development Party (AKP) in 2002, essentially substituting for the outlawed Virtue Party.

In 1999, a DSP minority government formed, supported by ANAP and DYP. This only lasted about 4 months, and was replaced by a DSP–ANAP–MHP coalition, indicating the difficulty of negotiating a coalition compromise across the disparate policy positions of the coalition members. During the period 1999–2002, Turkey experienced two severe economic crises. As Tables 1 and 2 show, the vote shares of the parties in the governing coalition went from about 53% in 1999 to less than 15% in 2002. In 2002, a 10% cut-off rule was instituted. As Table 2 makes clear, seven parties obtained less than 10% of the vote in 2002, and won no seats. The AKP won 34% of the vote, but because of the cut-off rule, it obtained a majority of the seats (363 out of 550). In 2007, the AKP did even better, taking about 46% of the vote, against 21% for the CHP. The Kurdish Freedom and Solidarity Party avoided the 10% cut-off rule, by contesting

[17] For simplicity, the pro-Kurdish party is denoted HADEP in the various Figures and Tables. Notice that the HADEP position in Figs 1 and 2 is interpreted as secular and non-nationalistic.

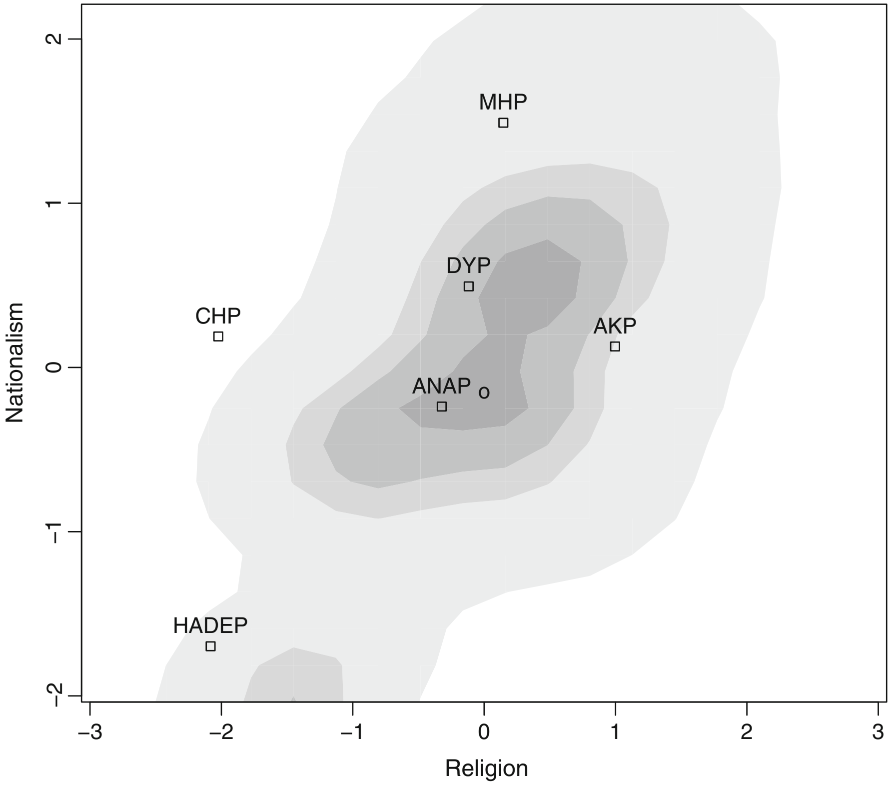

Fig. 2 Party positions and voter distribution in Turkey in 2002

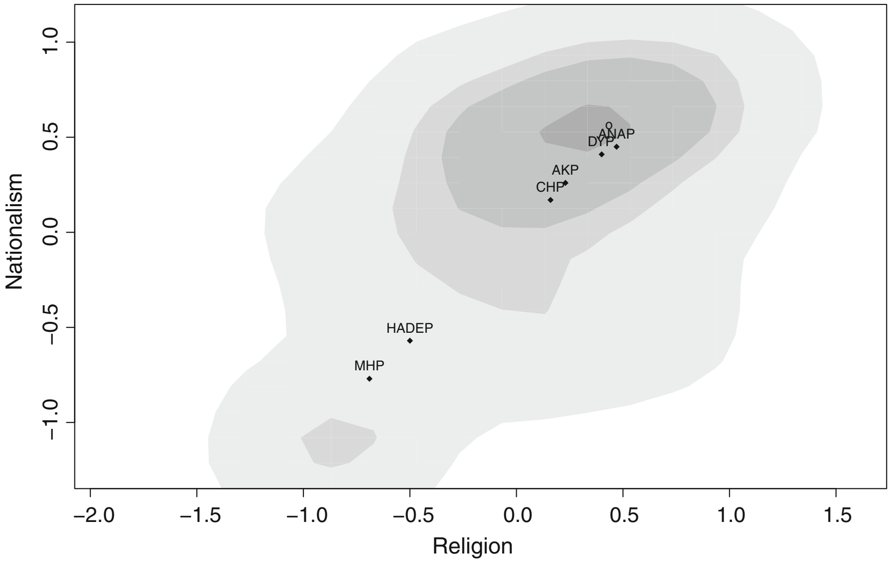

Fig. 3 A Local Nash Equilibrium for the pure spatial model in 2002

the elections as independent non-party candidates, winning 24 seats with less that 5% of the vote.

The point of this example is that a comparison of Figs 1 and 2 suggest that there was very little change in policy positions of the parties between 1999 and 2002. The basis of support for the AKP may be regarded as a similar to that of the banned FP, which suggests that the leader of this party changed the party's policy position on the religion axis, adopting a much less radical position.

In sum, the standard spatial model is unable to explain the change in the electoral outcome, taken together with the relative unchanged positioning of the parties between 1999 and 2002.

Section 2 of this paper considers the details of the MNL model for Turkey for 1999 and 2002. In particular, this section shows that the pure spatial model with exogenous valence predicts that the parties diverge away from the origin. To illustrate, Table 5 shows that the lowest valence party in 2002 was the Motherland Party (ANAP) while the Republican People's Party (CHP) had the highest valence. The convergence coefficient was computed to be 5.94, far greater than the upper bound of 2. Figure 3 presents an estimate of one of the LNE obtained from simulation of vote maximizing behavior of the parties, under the assumption of the pure spatial model with exogenous valence. As expected from the theoretical result, the LNE is non-centrist. Note, however, the the LNE positions for the pure spatial model given in Fig. 3 are quite different from the estimated positions in Fig. 2.

To improve the prediction of the model, we incorporated the sociodemographic variables. Estimating the LNE for this sociodemographic model gave a better prediction. To explain the difference between the estimated positions of the parties, and the LNE from the sociodemographic model, we then added the the influence of party activists to the model. Since sociodemographic variables can be interpreted as specific valences associated with different subgroups of the electorate, we can use these sociodemographic valences to estimate the influence of group-specific activists on party positions.

Theorem 1 in the Appendix[18] gives the first-order *balance* condition for local equilibrium in the stochastic electoral model involving sociodemographic valences and activists. The condition requires the balancing of a *centrifugal marginal activist pull (or gradient)* against a *marginal electoral pull*. In general, if the exogenous valence of a party leader falls, then the marginal electoral pull also falls, so balance requires that the leader adopt a position closer to the preferred position of the party activists.

The pure spatial model, with exogenous valences, and a joint model, with sociodemographic valences, but without activists, are compared using simulation to determine the LNE in these models. This allows us to determine which model better explains the party positions. For example, Fig. 5 shows the LNE based on a joint sociodemographic model for 2002. In this figure, the LNE position for the Kurdish party, HADEP, is a consequence of the high electoral pull by Kurdish voters located in the lower left of the figure. Similarly, the position of the CHP on the left of the figure is estimated to be due to the electoral pull by Alevi voters who are Shia, rather than Sunni and can be

[18] The results in the Appendix extend the version of the activist model originally proposed by Aldrich (1983) and developed in Schofield (2006).

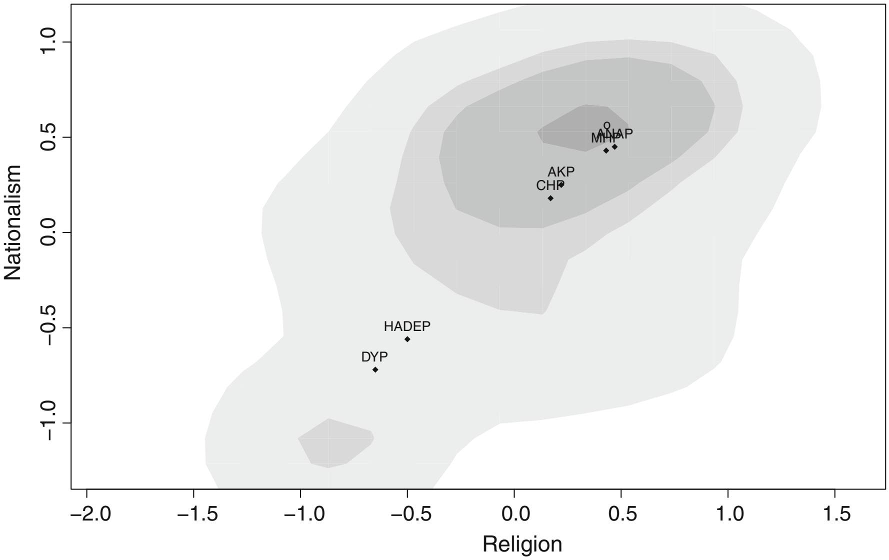

Fig. 4 A second local Nash Equilibrium for the pure spatial model in 2002

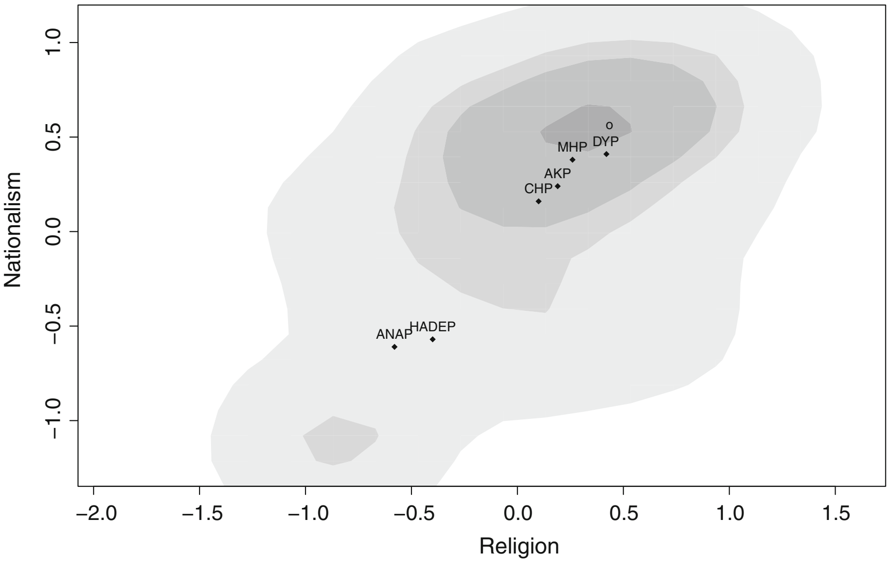

Fig. 5 A local Nash Equilibrium for the joint model in 2002

regarder as supporters of the secular state. Although Fig. 5 gives a superior prediction of the party positions than Fig. 3, there is still a discrepancy between the estimated positions of Fig. 2 and the LNE in Fig. 5. We argue that the difference between these two vectors of party positions, as presented in Figs. 2 and 5, can be used to provide an estimation of the marginal activist pulls influencing the parties.

More generally, we suggest that the combined model, with sociodemographic variables and activists, can be used as a tool with which to study the political configuration of such a complex society. In the conclusion we suggest that the full model involving activists may be applicable to the study of what Epstein et al. (2006) call "partial democracies", where a political leader must maintain popular support, not just by winning elections, but by maintaining the allegiance of powerful activist groups in the society.

2 Elections in Turkey 1999–2007

The Appendix defines an empirical electoral model, denoted $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta; \mathbf{V})$ which utilizes sociodemographic variables, denoted $\boldsymbol{\theta}$.

The symbol, $\mathbf{V}$, denotes a family of egalitarian vote functions, one for each party, and under which all voters are counted equally. The formal model of the Appendix considers a more general class of vote functions where the voters vary in their weights, thus allowing for complex electoral rules. In the Appendix, the egalitarian family is denoted $\mathbf{V}_{\mathrm{e}}$. The symbol, Ψ, denotes the Gumbel stochastic distribution on the errors. To simplify notation in the applications that follow we delete reference to $\mathbf{V}$ and Ψ.

This empirical model assumes that the utility function of voter i is given by the expression

$$u_{ij}(x_i, z_j) = \Lambda_j + (\theta_j \cdot \eta_i) - \beta \|x_i - z_j\|^2 + \varepsilon_j.$$

Here, the *spatial coefficient* is denoted β and $\boldsymbol{\Lambda} = \{\Lambda_j : j \in P\}$ are the *exogenous valences* (relative to a baseline party, k^*).[19] The relative *exogenous* valence, Λ_j, gives the average belief of the voters in the electorate concerning the quality of the leader of party j in comparison to the leader of the baseline party, k^*. The symbol, $\boldsymbol{\theta}$, denotes a set of m-vectors $\{\theta_j\}$ representing the effect of the m different sociodemographic parameters (class, domicile, education, income, religious orientation, etc.) on the beliefs of the various subgroups in the polity on the competence of party j. The symbol η_i is an m-vector denoting the ith individual's relevant "sociodemographic" characteristics. The composition $(\theta_j \cdot \eta_i)$ is the scalar product and can be interpreted as the group-specific valence ascribed to party j as a consequence of the various sociodemographic characteristics of voter i. Again, these sociodemographic variables will be normalized with respect to the baseline party k^*, essentially by estimating $((\theta_j - \theta_{k^*}) \cdot \eta_i)$. This scalar term is called the *total sociodemographic valence* of voter i for party j. The tth term in this scalar is called the *sociodemographic valence* of i as a result of membership by i of the tth group, or, more briefly, the tth group specific sociodemographic valence for the leader of party j.[20]

[19] Note that in the empirical models discussed below, these are specified relative to the baseline party, the DYP.

[20] For example, in Tables 6 and 7 there are six sociodemographic variables, so $m = 6$. An individual who is Alevi has $\eta_{i,\mathrm{Alevi}} = 1$. The coefficient for the CHP party for an Alevi is 3.089 in 1999, and this is the group-specific valence that a voter who is a member of the group of Alevi voters has for this party. Note again that this is specified relative to the baseline party, the DYP. These valences may be the result of the

The vector $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ is the set of party positions, while $\mathbf{x} = (x_1, \ldots, x_n) \in X^n$ is the set of ideal points of the voters in N. When β is assumed zero then the model is called pure *sociodemographic* (SD), and denoted $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta})$. When $\{\theta_j\}$ are all assumed zero then the model is called *pure spatial*, and denoted $\mathbb{M}(\boldsymbol{\Lambda}, \beta)$. The pure spatial model implicitly assumes that the ranking over valence is identical among voters.The empirical model, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta)$, including the sociodemographic terms is called *joint*. These sociodemographic variables allow us to incorporate characteristics common to specific groups of supporters of any party, and this permits the valence ranking to vary among voters in a way which depends on sociodemographics. Not accounting for these characteristics in the analysis will bias the estimates of the exogenous valences of the parties. Tables 4 and 5 give the details of the pure spatial MNL models for the elections of 1999 and 2002 in Turkey, while Tables 6 and 7 give the details of the joint MNL models. The differences in log marginal likelihoods for the three different models then gives the log Bayes' factor for the pairwise comparisons.[21] The log Bayes' factors show that the joint and pure spatial MNL models were clearly superior to the SD models. In addition, the joint models were superior to the pure spatial models.[22] We can infer that, though the sociodemographic variables are useful, by themselves they do not give an accurate model of voter choice.[23] It is necessary to combine the pure spatial model, including the valence terms, with the sociodemographic valences to obtain a superior estimation of voter choice.

Comparing Tables 4 and 5, it is clear that the relative valences of the ANAP and MHP, under the pure spatial model, dropped between 1999 and 2002. In 1999, the estimated Λ_{ANAP} was $+0.336$, while the confidence interval on Λ_{ANAP} for 1999 in Table 4 shows that the hypothesis that $\Lambda_{\text{ANAP}} = 0$ should be rejected. In contrast, the estimated value of Λ_{ANAP} for 2002 was -0.31, and the confidence interval on Λ_{ANAP} does not allow us to reject the hypothesis that $\Lambda_{\text{ANAP}} = 0$.[24] Similarly, Λ_{MHP} fell from a significant value of +0.666 in 1999 to -0.12 in 2002. The estimated relative valence, Λ_{AKP}, of the new Justice and Development Party (AKP) in 2002 was $+0.78$, in comparison to the valence of the FP of -0.159 in 1999. Since the AKP can be regarded as a transformed FP, under the leadership of Recep Tayyip Erdogan, we can infer from the confidence intervals on these two relative valences that this was a significant change due to Erdogan's leadership.[25]

Footnote 20 continued
perception of the leader's ability, as displayed in the past, or of the particular partiality of these voters to choose the party, independently of the party's policy position.

21 Since the Bayes' factor (Kass and Raftery 1995) for a comparison of two models is simply the ratio of marginal likelihoods, the log Bayes' factor is the difference in log likelihoods.

22 The log Bayes factors for the joint models over the sociodemographic models were highly significant at +31 in 1999 and +58 in 2002. The Bayes' factors for the joint over the spatial models were also significant, and estimated to be +6 and +5 in 1999 and 2002, respectively.

23 Sociodemographic models are standard in the empirical voting literature.

24 These tables show the standard errors of the coefficients, as well as the *t-values*, the ratios of the estimated coefficient to the standard error.

25 Although Erdogan was the party leader, Abdullah Gul became Prime Minister after the November 2002 election because Erdogan was banned from holding office. Erdogan took over as Prime Minister after winning a by-election in March 2003.

Table 4 Pure Spatial Model of the Turkish election 1999

Party name		Λ_k	Std. error	|t-Value|
Democratic Left Party	DSP	0.724*	0.153	4.73
Nationalist Action Party	MHP	0.666*	0.147	4.53
Virtue Party	FP	−0.159	0.175	0.9
Motherland Party	ANAP	0.336	0.153	2.19
True Path Party	DYP	–	–	–
Republican People's Party	CHP	0.734*	0.178	4.12
People's Democracy Party	HADEP	−0.071	0.232	0.3
Normalized with respect to DYP				
Spatial coefficient β	0.375*		0.088	4.26
Convergence coefficient c	1.49*		0.22	6.77

$n = 635$; Log marginal likelihood (LML) $= -1183$
* Significant with probability <0.001

Table 5 Pure Spatial Model of the Turkish election 2002

Party Name		Λ_k	Std. error	|t-Value|
Justice and Development Party	AKP	0.78*	0.15	5.2
Republican People's Party	CHP	1.33*	0.18	7.4
True Path Party	DYP	–	–	–
Nationalist Action Party	MHP	−0.12	0.18	0.66
Young Party	GP	–	–	–
People's Democracy Party	HADEP	0.43	0.21	2.0
Motherland Party	ANAP	−0.31	0.19	1.63
Normalized with respect to DYP				
Spatial coefficient β	1.52*		0.12	12.66
Convergence coefficient c	5.94*		0.27	22.0

$n = 483$; Log marginal likelihood (LML) $= -737$
* Significant with probability <0.001

It should be noted that the β coefficients for the pure spatial models were 0.375 in 1999, and 1.52 in 2002. Both of these are estimated to be non-zero at the 0.001 level. Indeed, they are significantly different from each other,[26] suggesting that electoral preferences over policy had become more intense.

We first use the results of the formal pure spatial model given in the Appendix to compute estimates of the convergence coefficients. These computations suggest that convergence to an electoral center is not to be expected in these elections. We then use simulation to determine the LNE of the empirical joint models, again showing non-convergence. This allows us to obtain information about activist support for the parties.

[26] The 95% confidence interval for β_{1999} is [0.2,0.55] and for β_{2002} it is [1.28,1.76]

Table 6 Joint Model of the 1999 Election in Turkey (normalized with respect to DYP)

Variable	Party	Coefficient		95% Confidence interval	
		Est	Std Err	Lower bound	Upper bound
Spatial coeff. β		0.456*	0.104	0.243	0.648
Relative valence Λ_k	ANAP	−0.114	0.727	−1.513	1.227
	CHP	−0.673	0.770	−2.166	0.786
	DSP	0.463	0.720	−0.930	1.825
	FP	1.015	0.878	−0.709	2.755
	HADEP	−0.610	1.230	−3.004	1.803
	MHP	2.447*	0.669	1.167	3.664
Age	ANAP	0.001	0.012	−0.021	0.023
	CHP	−0.009	0.013	−0.033	0.016
	DSP	−0.008	0.012	−0.031	0.014
	FP	−0.023	0.014	−0.050	0.003
	HADEP	−0.053*	0.023	−0.103	−0.014
	MHP	−0.044*	0.012	−0.067	−0.022
Education	ANAP	0.006	0.065	−0.115	0.130
	CHP	0.106	0.063	−0.012	0.232
	DSP	0.077	0.058	−0.024	0.197
	FP	−0.129	0.081	−0.285	0.018
	HADEP	0.144	0.097	−0.038	0.335
	MHP	−0.060	0.061	−0.175	0.070
Urban	ANAP	0.531	0.367	−0.156	1.279
	CHP	0.354	0.395	−0.374	1.078
	DSP	0.582	0.359	−0.147	1.271
	FP	0.417	0.416	−0.418	1.183
	HADEP	0.264	0.634	−0.918	1.497
	MHP	−0.201	0.378	−0.922	0.593
Kurd	ANAP	1.132	0.924	−0.410	3.138
	CHP	1.715*	0.911	0.194	3.637
	DSP	−0.102	1.083	−2.650	2.098
	FP	1.116	0.972	−0.733	3.024
	HADEP	5.898*	0.926	4.290	7.904
	MHP	0.063	0.933	−1.751	2.148
Soc. Econ. Status	ANAP	0.080	0.165	−0.302	0.394
	CHP	0.163	0.176	−0.195	0.499
	DSP	−0.010	0.158	−0.322	0.333
	FP	0.120	0.179	−0.230	0.458
	HADEP	−0.119	0.264	−0.598	0.384
	MHP	0.168	0.159	−0.147	0.469
Alevi	ANAP	−0.697	0.972	−2.687	1.168

Table 6 continued

Variable	Party	Coefficient		95% Confidence interval	
		Est	Std Err	Lower bound	Upper bound
	CHP	3.089	0.693	1.965	4.715
	DSP	0.934	0.729	−0.383	2.423
	FP	0.346	0.939	−1.374	2.007
	HADEP	1.355	0.972	−0.332	3.605
	MHP	−0.873	0.925	−3.225	0.676

$n = 635$; Log marginal likelihood = −1178
* Prob <0.001

2.1 The 2002 election

Figure 3 shows the smoothed estimate of the voter ideal points in 2002. This distribution gives the 2×2 voter covariance matrix, with an electoral variance on the first axis (religion) estimated to be 1.18 while the electoral variance on the second axis (nationalism) was 1.15. The total electoral variance was $\sigma^2 = 2.33$, with an electoral standard deviation of $\sigma = 1.52$ The covariance between the two axes was equal to 0.74.

Thus, the voter covariance matrix is

$$\nabla_0 = \begin{bmatrix} 1.18 & 0.74 \\ 0.74 & 1.15 \end{bmatrix}$$

with $\text{trace}(\nabla_0) = 2.33$.

The eigenvalues of this matrix are 1.9, with major eigenvector $(+1.0, +0.97)$ and 0.43, with minor eigenvector $(-0.97, +1.0)$. The major eigenvector corresponds to the *principal electoral axis*, aligned at approximately 45° to the religion axis.

For the pure spatial model $\mathbb{M}(\boldsymbol{\Lambda}, \beta)$, the β coefficient was 1.52, The valence terms are estimated in contrast with the valence of the DYP, and the the party with the lowest relative valence is ANAP with $\Lambda_{\text{ANAP}} = -0.31$. By definition, $\Lambda_{\text{DYP}} = 0$. The vector of relative valences is then

$$(\Lambda_{\text{ANAP}}, \Lambda_{\text{MHP}}, \Lambda_{\text{DYP}}, \Lambda_{\text{HADEP}}, \Lambda_{\text{AKP}}, \Lambda_{\text{CHP}}) \\ = (-0.31, -0.12, 0.0, 0.43, 0.78, 1.33).$$

When all parties are at the origin, the probability, ρ_{ANAP}, that a voter chooses ANAP, in the model $\mathbb{M}(\boldsymbol{\Lambda}, \beta)$, is independent of the voter. The Appendix, Eq. 7, shows that this is given by

$$\frac{\exp(-0.31)}{\exp(-0.31) + \exp(-0.12) + \exp(0.0) + \exp(0.43) + \exp(0.78) + \exp(1.33)} \\ = [1 + \exp(0.19) + \exp(0.31) + \exp(0.74) + \exp(1.09) + \exp(1.164)]^{-1}$$

Table 7 Joint Model of the 2002 Election in Turkey (normalized with respect to DYP)

Variable	Party	Coefficient		95% Confidence interval	
		Est	Std Dev	Lower bound	Upper bound
Spatial coeff β		1.445*	0.143	1.180	1.723
Valence Λ_k	AKP	1.968*	0.667	0.708	3.432
	CHP	1.103	0.797	−0.579	2.615
	HADEP	2.596	1.246	−0.254	5.049
	MHP	1.714	0.889	−0.021	3.426
	ANAP	−0.567	0.880	−2.487	1.133
Age	AKP	−0.031	0.011	−0.052	−0.010
	CHP	−0.019	0.013	−0.045	0.005
	HADEP	−0.060	0.024	−0.110	−0.014
	MHP	−0.067	0.017	−0.103	−0.034
	ANAP	−0.004	0.014	−0.031	0.022
Education	AKP	−0.070	0.062	−0.185	0.045
	CHP	−0.007	0.068	−0.136	0.115
	HADEP	−0.142	0.108	−0.365	0.079
	MHP	−0.048	0.079	−0.202	0.106
	ANAP	−0.078	0.076	−0.237	0.064
Urban	DYP	0.050	0.406	−0.770	0.844
	CHP	0.121	0.443	−0.744	1.001
	HADEP	−1.138	0.688	−2.426	0.236
	MHP	−0.570	0.536	−1.649	0.504
	ANAP	0.661	0.479	−0.228	1.628
Kurd	AKP	2.086	1.105	0.203	4.596
	CHP	1.251	1.171	−0.891	3.839
	HADEP	5.996*	1.208	3.960	8.945
	MHP	1.595	1.312	−0.960	4.258
	ANAP	1.603	1.199	−0.535	4.358
Soc. Econ. Status	AKP	0.142	0.160	−0.160	0.457
	CHP	0.198	0.191	−0.196	0.560
	DEHAP	−0.217	0.281	−0.755	0.301
	MHP	0.317	0.204	−0.083	0.703
	ANAP	0.214	0.209	−0.182	0.613
Alevi	AKP	−0.249	0.983	−2.125	1.743
	CHP	2.567*	0.817	1.111	4.489
	DEHAP	0.377	1.045	−1.519	2.540
	MHP	−0.529	1.410	−3.565	2.292
	ANAP	1.392	0.931	−0.323	3.560

$n = 483$; Log marginal likelihood = -732
* Prob <0.001

$$
\begin{aligned}
&= [1 + 1.2 + 1.36 + 2.09 + 2.97 + 3.2]^{-1}\\
&= 0.08.
\end{aligned}
$$

Below, we show that the 95% confidence interval on ρ_{ANAP} is [0, 05, 0.11], which includes the actual vote share (5.13%) in 2002.

Appendix shows that the Hessian of the vote share function of ANAP, when all parties are at the origin, is given by the characteristic matrix of ANAP:

$$
\begin{aligned}
C_{\text{ANAP}} &= 2\beta(1 - 2\rho_{\text{ANAP}})\nabla_0 - I\\
&= 2 \times (1.52) \times [(1 - (2 \times 0.08)]\nabla_0 - I\\
&= (2.55)\begin{bmatrix} 1.18 & 0.74\\ 0.74 & 1.15 \end{bmatrix} - I\\
&= \begin{bmatrix} 2.01 & 1.88\\ 1.88 & 1.93 \end{bmatrix}.
\end{aligned}
$$

Moreover, the convergence coefficient,

$$c = 2\beta(1 - 2\rho_{\text{ANAP}})\text{trace}(\nabla_0) = 2.55 \times 2.33 = 5.94.$$

This greatly exceeds the upper bound of +2.0 for convergence to the electoral origin. The major eigenvalue for the ANAP characteristic matrix is +3.85, with eigenvector (+1.0, +0.98), while the minor eigenvalue is +0.09, with orthogonal, minor eigenvector (−0.98, +1.0). The eigenvectors of this Hessian are almost perfectly aligned with the principal and minor components, or axes, of the electoral distribution.

Although the electoral origin satisfies the first order condition for local equilibrium, it follows from a standard result that the electoral origin is a *minimum* of the vote share function of ANAP, when the other parties are at the same position. On both principal and minor axes, the vote share of ANAP increases as it moves away from the electoral origin, but because the major eigenvalue is much larger than the minor one, we can expect that the AKP (as well the other parties) in equilibrium to adopt positions along a single eigenvector. Figures 3 and 4 present two LNE obtained from simulation of the pure spatial model. These are:

$$
\mathbf{z}_1 = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP}\\ x: \text{rel} & 0.16 & -0.69 & 0.40 & -0.50 & 0.47 & 0.23\\ y: \text{nat} & 0.17 & -0.77 & 0.41 & -0.57 & 0.45 & 0.26 \end{bmatrix}.
$$

$$
\mathbf{z}_2 = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP}\\ x: \text{rel} & 0.17 & 0.43 & -0.65 & -0.51 & 0.47 & 0.22\\ y: \text{nat} & 0.18 & 0.43 & -0.72 & -0.56 & 0.45 & 0.25 \end{bmatrix}.
$$

Note that all the positions in these two LNE lie close to the principal axis given by the eigenvector (1.0, 1.0). The higher valence parties, the AKP and CHP lie closer to the origin, while the lower valence parties tend to be further from the origin.

In contrast, the estimated positions of the parties for 2002 in Fig. 2 are:

$$\mathbf{z}^* = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & -2.0 & 0.0 & 0.0 & -2.0 & -0.2 & 1.0 \\ y:\text{nat} & +0.1 & 1.5 & 0.5 & -1.5 & -0.1 & 0.1 \end{bmatrix}.$$

The equilibrium positions of the CHP and MHP, particularly, are very far from their estimated positions.

2.1.1 Errors in the models

The Appendix shows that the standard error on Λ_{ANAP} is $h = 0.19$, so

$$\begin{aligned} \rho_{\text{ANAP}}(\Lambda_{\text{ANAP}} + h) &= \rho_{\text{ANAP}}(\Lambda_{\text{ANAP}}) + h\frac{d\rho_{\text{ANAP}}}{d\Lambda} \\ &= \rho_{\text{ANAP}}(\Lambda_{\text{ANAP}}) + h\rho_{\text{ANAP}}(1 - \rho_{\text{ANAP}}). \end{aligned}$$

This gives a standard error of 0.014 and a 95% confidence interval on ρ_{ANAP} of [0.05, 0.11]. Since the standard error on β is 0.12, giving a confidence interval on β of approximately [1.28, 1.76], the standard error on c is 0.27. Using the lower bound on β and upper bound on ρ_{ANAP} gives an estimate for the 95% confidence interval on c of [4.65, 7.38], so we can assert that, with very high probability, the convergence coefficient exceeds 4.0. Another way of interpreting this observation is that even if we use the upper estimate of the relative valence for ANAP, and the lower bound on β, then the joint origin will still be a minimum of the vote share function for ANAP.

We now repeat the analysis for the election of 1999.

2.2 The 1999 election

The empirical model presented in Table 4 estimated the electoral variance on the first axis (religion) to be 1.20 while on the second axis (nationalism) the electoral variance, σ^2, was 1.14, giving a total electoral variance, σ^2, of 2.34, with the covariance between the two axes equal to +0.78.

The electoral covariance matrix is the 2×2 matrix

$$\nabla_0 = \begin{bmatrix} 1.20 & 0.78 \\ 0.78 & 1.14 \end{bmatrix}.$$

For the model ,the β coefficient was 0.375, while the party with the lowest valence was FP with $\Lambda_{\text{FP}} = -0.16$. The vector of valences is:

$$\begin{aligned} &(\Lambda_{\text{FP}}, \Lambda_{\text{MHP}}, \Lambda_{\text{DYP}}, \Lambda_{\text{HADEP}}, \Lambda_{\text{ANAP}}, \Lambda_{\text{CHP}}, \Lambda_{\text{DSP}}) \\ &\quad = (-0.16, +0.66, 0.0, -0.071, +0.34, +0.73, +0.72). \end{aligned}$$

When all parties are located at the origin, the probability, ρ_{FP}, that a voter chooses FP under $\mathbb{M}(\boldsymbol{\Lambda},\beta)$ is equal to

$$\frac{1}{[1+\exp(0.82)+\exp(0.16)+\exp(0.09)+\exp(0.5)+\exp(0.89)+\exp(0.88)]}$$
$$=[11.27]^{-1}=0.08.$$

The standard error on Λ_{FP} is 0.175, so the 95% confidence interval can be estimated to be $[0.01, 0.15]$. The FP vote share in 1999 was 15.41%, suggesting that the pure spatial model should be extended to include sociodemographic valences.

Now $2\beta(1-2\rho_{\mathrm{FP}})=2\beta\times(1-2\times(0.08))=2\times 0.38\times 0.84=0.64$, so the characteristic matrix of the FP is

$$\begin{aligned} C_{\mathrm{FP}} &= (0.64)\begin{bmatrix} 1.20 & 0.78 \\ 0.78 & 1.14 \end{bmatrix} - I \\ &= \begin{bmatrix} -0.24 & 0.448 \\ 0.448 & -0.27 \end{bmatrix}. \\ \text{and} \quad c &= 0.64\times 2.34 = 1.49. \end{aligned}$$

Although $c<2.0$, we can compute the eigenvalues of C_{FP} to be -0.74 with minor eigenvector $(+1,-1.116)$ and $+0.23$, with major eigenvector $(+1,+0.896)$, giving a saddlepoint for the FP Hessian at the joint origin. As with the 2002 election, on the basis of the pure spatial model, we again expect all parties to align along the major eigenvector, at approximately 45° to the religion axis. Note, however, that the standard error on c is of order 0.22, so unlike the result for the election of 2002, we cannot assert that there is a high probability that the convergence coefficient exceeds 2. However, there is a probability exceeding 0.95 that one of the eigenvalues is positive.

In comparing the pure spatial models of the elections of 1999 and 2002, we note there is very little difference between the model predictions.

2.3 Extension of the model for Turkey

We now use the empirical joint model, $\mathbb{M}(\boldsymbol{\Lambda},\boldsymbol{\theta},\beta)$, in order to better model party positioning. We use this model to estimate the influence of party activists in a more general activist model, denoted $\mathbb{M}(\boldsymbol{\Lambda},\boldsymbol{\mu},\beta)$. In the activist model,the activist functions $\boldsymbol{\mu}=\{\mu_j : j\in P\}$ are presumed to be functions of party position, rather than exogenous constants. The idea behind this model is that activists provide campaign contributions to specific parties, and these contributions can be used by the parties to affect valence. For the game theoretic foundations of this model see Grossman and Helpman (1994, 2001). Grossman and Helpman (1996, p. 265) also define two distinct motives for these activists:

> Contributors with an *electoral* motive intend to promote the electoral prospects of preferred candidates. Those with an *influence motive* aim to influence the politicians' policy pronouncements.

Here we use a reduced form of the activist functions, based on Schofield (2006), since we only need the fact that the activist contribution to party j is a differentiable function of the party's position, and positively affects the parties valence. Galiani et al. (2010) provide a partial game theoretic model of this activist game that is consistent with Schofield (2006).

Theorem 1 of the Appendix shows that the first-order condition for a local equilibrium, $\mathbf{z}^* = (z_1^*, \ldots z_p^*)$, in the activist model is given by the set of *gradient balance conditions*:

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j^*) + \frac{1}{2\beta}\frac{d\mu_j}{dz_j}(z_j^*) = 0. \tag{1}$$

Each term, $\frac{d\mu_j}{dz_j}(z_j)$ is the *the marginal activist pull (or gradient) at* z_j, giving the marginal activist effects on party j, while the gradient term $\frac{d\mathcal{E}_j^*}{dz_j}(z_j) = \left[z_j^{\mathrm{el}} - z_j\right]$ is the *gradient electoral pull on the party*, at z_j, pointing towards its weighted electoral mean, z_j^{el}, as defined for party j in (5) in the Appendix:

$$z_j^{\mathrm{el}} \equiv \sum_{i=1}^{n} \varpi_{ij} x_i, \quad \text{where } [\varpi_{ij}] = \left[\frac{[\rho_{ij} - \rho_{ij}^2]}{\sum_{k \in N}[\rho_{kj} - \rho_{kj}^2]}\right]. \tag{2}$$

The weighted electoral mean essentially weights voter policy preferences by the degree to which the sociodemographic valences influence the choice of the voter.

Note in particular that (2) gives the first-order condition for any of the various models considered here. In particular, if the sociodemographic and activist terms are zero, then (2) reduces to $[\varpi_{ij}] = \frac{1}{n}$, and, by the obvious coordinate transformation, we obtain $z_j = 0$, for all j, as the first-order condition.

The joint model, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta)$, allows us to draw some inferences about equilibrium positions. First we note that in the joint model, the sociodemographic valences are substitutes for the relative valences. Table 7 shows that the only relative valence that is significantly non zero in 2002 is $\boldsymbol{\Lambda}_{\mathrm{AKP}}$. A number of the sociodemographic valences are, however, very significant.[27]

Figures 5 gives an LNE, $\mathbf{z}_3$, obtained by simulation of the joint model, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta)$:

$$\mathbf{z}_3 = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & 0.12 & 0.26 & 0.40 & -0.50 & -0.58 & 0.19 \\ y:\text{nat} & 0.16 & 0.38 & 0.41 & -0.51 & -0.61 & 0.24 \end{bmatrix}.$$

[27] The Bayes factors, or differences between the log marginal likelihoods of the joint models over the pure spatial models were +5 in both years.

Again the estimated positions are:

$$\mathbf{z}^* = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & -2.0 & 0.0 & 0.0 & -2.0 & -0.2 & 1.0 \\ y:\text{nat} & +0.1 & 1.5 & 0.5 & -1.5 & -0.1 & 0.1 \end{bmatrix}.$$

Comparing the joint model with the pure spatial model, we see that the equilibrium positions are slightly better predictors for HADEP, MHP, and ANAP

For this joint model, Tables 6 and 7 show that the sociodemographic valences for HADEP (or DEHAP) by Kurdish voters were very high:

$$(\theta_{\text{HADEP}} \cdot \eta_{\text{Kurd}}) = 5.9 \text{ in } 1999$$
$$(\theta_{\text{HADEP}} \cdot \eta_{\text{Kurd}}) = 6.0 \text{ in } 2002.$$

Keeping the other variables at their means in 2002, then changing η_{Kurd} from non-Kurd to Kurd increases the probability of voting for HADEP from 0.013 to 0.45. The high significance level of the sociodemographic variables indicates that the joint electoral model would predict that HADEP would move close to Kurdish voters who tend to be located on the left of the religion axis, and are also anti-nationalistic. The position marked HADEP in Fig. 2 is consistent with this inference.

The joint model also shows that Alevi voters have very high sociodemographic valences for the CHP, with

$$(\theta_{\text{CHP}} \cdot \eta_{\text{Alevi}}) = 3.1 \quad \text{in } 1999$$
$$(\theta_{\text{CHP}} \cdot \eta_{\text{Alevi}}) = 2.6 \quad \text{in } 2002.$$

The Alevis are a non-Sunni religious community, who are adherents of Shia Islam rather than Sunni, and may be viewed as supporters of "Kemalism" or the secular state. Again, with other variables at their means, changing η_{Alevi} from non-Alevi to Alevi increases the probability of voting for CHP in 2002 from 0.16 to 0.63. Thus, the joint model indicates that the CHP will move to a vote maximizing position, on the left of the religious axis, again as indicated in Fig. 2.

Conversely, for Alevi voters $\theta_{\text{AKP}} \cdot \eta_{\text{Alevi}}) = -0.25$ in 2002, and we can infer that the AKP may move right to attract Sunni voters.

In the model $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta)$, we do not consider activist terms, so this is equivalent to setting $\{\frac{d\mu_j}{dz_j}\} = 0$. We can infer from (1) that the first-order balance condition will be satisfied at a vector $\mathbf{z} = (z_1, \ldots z_p)$ when

$$\frac{d\mathcal{E}_j^*}{dz_j} \equiv \left[z_j^{\text{el}} - z_j\right] = 0, \quad \text{for each } j.$$

Thus, we can use $\mathbf{z}_3$ as the estimator for the vector of weighted electoral means. We find that

$$
\begin{aligned}
\mathbf{z}^* - \mathbf{z}_3 &= \\
&\left[\begin{array}{lcccccc} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & -2.0 & 0.0 & 0.0 & -2.0 & -0.2 & 1.0 \\ y:\text{nat} & +0.1 & 1.5 & 0.5 & -1.5 & -0.1 & 0.1 \end{array}\right] \\
&-\left[\begin{array}{lcccccc} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & 0.12 & 0.26 & 0.40 & -0.50 & -0.58 & 0.19 \\ y:\text{nat} & 0.16 & 0.38 & 0.41 & -0.51 & -0.61 & 0.24 \end{array}\right]. \\
&=\left[\begin{array}{lcccccc} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x:\text{rel} & -3.2 & -0.26 & -0.40 & -1.50 & +0.38 & 0.81 \\ y:\text{nat} & -0.15 & +1.12 & 0.09 & -0.99 & +0.51 & -0.14 \end{array}\right]
\end{aligned}
$$

Assuming that this vector is an LNE with respect to the full model, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\mu}, \beta)$ involving activists, then by (13) in the Appendix, we can make the identification:

$$\frac{1}{2\beta}\left[\frac{\mathrm{d}\mu_1}{\mathrm{d}z_1}, \ldots, \frac{\mathrm{d}\mu_p}{\mathrm{d}z_p}\right] = \mathbf{z}^* - \mathbf{z}_3$$

Here, $\{\frac{\mathrm{d}\mu_1}{\mathrm{d}z_1}, \ldots, \frac{\mathrm{d}\mu_p}{\mathrm{d}z_p}\}$ are the marginal activist pulls at the equilibrium vector $\mathbf{z}^*$. Under the hypothesis that the joint model with activists is valid, then the difference between these two vectors gives us an estimate of the vector of marginal pulls on the parties:

The estimated activist pull on HADEP is very high, pulling the party to the left on the religion axis, and in an anti-nationalist direction on the y-axis. Similarly,the estimated activist pull on the CHP is even higher on the religious axis, pulling the party in a secular direction, and we can infer that this is due to the influence of Alevi voters.

As a consequence, this asymmetry will cause Alevi activists to provide further differential support for the CHP. It is thus plausible that secular voters (on the left of the religious axis in Figs. 1, 2) would offer further support to the CHP, located close to them. This would affect the party's marginal activist pull, and induce the CHP leader to move even further left, towards its inferred equilibrium position in the full activist model.

We suggest that activist support for the AKP would move it slightly to the right on the religion axis, as well as in an anti-nationalism direction. This would result in its estimated position as in Fig. 2.

In contrast, we might conjecture that the military provides activist support for the MHP on the nationalism axis, and this will move the party to the left in a secular direction, and north on the nationalism axis, resulting in its position in Fig. 2.

Overall, we note that we can expect activist valence to strongly influence party positioning, and we can proxy this support to some degree using the sociodemographic variables. Notice that the sociodemographic variables are estimated at the vector $\mathbf{z}^*$, so the estimated sociodemographic valences have been influenced by activist support. The LNE obtained from the joint model is a hypothetical solution to the vote maximizing game involving the parties, based on some empirical assumptions about the underlying nature of the important sociodemographic groups in the polity.

2.4 General remarks on Turkish elections

Although we have not performed a MNL analysis of the 2007 election, it seems obvious that some of the changes in the nature of party strategies were due to changes in the electoral laws. The election results of 1999 were based on an electoral system that was quite proportional, whereas in 2002 and 2007, the electoral system was highly majoritarian. In 2002, for example, the AKP gained 66% of the seats with only 34% of the vote, while in 2007 it took 46.6% of the vote and 340 seats (or 61.8%), reflecting the continuing high valence of Erdogan. Similarly, the CHP went from about 9% of the vote in 1999 (and no seats) to 19% of the vote in 2002, and 32% of the seats. This is mirrored by the increase in the valence estimates of the joint model from $\Lambda_{\text{CHP}} = -0.673$ in 1999 to $\Lambda_{\text{CHP}} = 1.103$, in 2002. In contrast the MHP went from 18% of the vote in 1999 to 8% in 2002, while Λ_{MHP} for the joint model fell from 2.5 to 1.7. The turn around in the vote share of the MHP between 2002 and 2007 could be a result of increasing support for this party from nationalist activist groups in an attempt to offset the high valence and electoral support for the AKP in 2002. Indeed, the increased concentration of the vote share between 1999 and 2007 may be a consequence of the greater significance of activist influence as the electoral system became more majoritarian due to the nature of the electoral cut-off rule.[28]

In such a non-proportional electoral system there are incentives for members of different sociodemographic groups to to engage in strategic voting. There is some indication from the formal model that the intensity of the political contest between secularist, nationalistic, and religious activist groups had increased prior to 2007, and recent events suggest that this is continuing.

After the 2007 election, Abdullah Gul, Erdogan's ally in the AKP was elected as the country's 11th President, despite strong opposition from the army and many secular interests. In late February 2008, the Turkish military invaded the Kurdish controlled territory in north west Iraq in an attempt to destroy the bases of the P.K.K. (the Kurdistan Workers' Party). The secular Constitutional Court has also considered banning many members of the AKP. In September 2008, Turkey formed a Caucasus Stability and Cooperation Platform with five neighboring countries, in reponse to Russian aggression in Georgia, and President Gul visited Armenia, one of the countries in the Platform. On 30 January 2009, Erdogan returned home from the World Economic Forum in Davos after walking out of a televised debate with Shimon Peres, the Israeli President, over Israel's war on the Gaza Strip. The moderator had refused to allow Erdogan to rebut Peres' justification of the war. Erdogan was welcomed back in Turkey as a hero.

However, more secular voters have begun to worry that Erdogan had become more autocratic, and in the municipal elections in March, 2009, the vote for the AKP dropped

[28] The Herfindahl concentration measure of the vote shares went from 0.11 in 1999 to 0.16 in 2002 to 0.27 in 2007.

from 47 to 39%. It appears that the Turkish electorate had divided geographically into four different political regions: a liberal, secular litoral, a conservative interior, with a nationalistic center, and a Kurdish nationalistic southeast.[29] The conflicts between the secular military and the non-secular government have come to a head over the Ergenekon affair, which has involved the prosecution of more than 200 people, allegedly involved in plotting against the state. In February 2010, the government arrested a further 40 people, including three high-ranking ex-military officers, and in March the government proposed constitutional changes that would limit the power of the Constitutional Court, making it more difficult for the Court to ban parties, as it has in the past. The changes would also make it more difficult to restrict membership of the forces to those who had no allegiance to religious groups, and would also permit trials in civilian rather than military courts for officers who were accused of plotting against the government. These constitutional changes require 367 votes out of the 550 in the Parliament. Both opposition parties, the Nationalist Movement Party (MHP) and Republican's Peoples Party (CHP) oppose these changes in the constitution, while the AKP only has 340 votes.

In his visit to Turkey in April 2009, Barack Obama made it clear that in his view, Turkey should become a member of the European Union. At the same time, he urged Turkey to undertake more democratic reforms. Although Turkey has many of the characteristics of a full democracy, there does appear to be severe conflict between the government and secular activist groups such as the military and judiciary.

Although many business interests favor membership of the European Union, the opposition to this by President Sarkozy of France and Chancellor Merkel of Germany may cause Turkey to turn east. In October 2009, Erdogan visited Tehran and met with President Ahmadinejad of Iran, while Turkey and Russia are also discussing the possibility of having Russian gas supplies transit through Turkey. On May 31, there was an attack by Israeli commandoes against a boat traveling in international waters and carrying humanitarian supplies for Gaza. Nine people in the convey were killed. The convoy was partly organized by a Turkish organization, Insani Yardim Vakfi. The repercussions for Turkish Israel relations were likely to be extreme. On 8 June 2010, Erdogan met with President Ahmadinejad and Prime Minister Vladimir Putin of Russia at a regional security summit in Istanbul. Turkey may be shifting from its pro-western stance and seeking to be an independent power in the region.

3 Concluding remarks

Recent works by Acemoglu and Robinson (2006), Boix (2003), and Przeworski et al. (2000) have explored the transition from autocratic regimes to democracy. A recent contribution by Epstein et al. (2006) has emphasized the existence of the category of "partial democracies." These exhibit mixed characteristics of both democratic and autocratic regimes. In fact, Epstein et al. give Turkey as a prime illustration of the possible degree of democratic volatility of a regime. They observe that, in terms of Polity IV scores, Turkey fell from being a full democracy to an autocracy first in the

[29] Asli Aydintasbas in the *New York Times,* 7 April 2009.

mid 1960s and again in the early 1980s, and since then has hovered between partial and full democracy. Epstein et al. (2006, p. 564) also comment, on the basis of their empirical analysis, that "the determinants of the behavior of partial democracies elude our understanding." These models of democratic transitions have tended to consider a single economic axis, and to utilize the notion of a median citizen, or median king-maker as the unique pivotal player.[30] While these models have been illuminating, we believe it necessary to consider policy spaces of higher dimension and to utilize a stochastic model so as to emphasize the aspect of uncertainty.

The analysis of Turkey in this paper indicates that both religion and nationalism define the political space.[31] The military in Turkey can be represented by a pro-nationalist, pro-secular position, far from the AKP, and it is this phenomenon which means that Turkish politics cannot be understood in terms of a median voter. Modeling partial democracies would seem to require a very explicit analysis of the power of activist groups.

This paper has applied a theoretical stochastic model to present an empirical analysis of elections in Turkey, and argues that there is no evidence of a centripetal tendency towards an electoral center. Instead it suggests that activist groups will tend to be located far from the electoral center. Once the sociodemographic valences have caused the parties to move away from the center to gain electoral support, the influence of activists will separate the parties even further, pulling them towards policy positions preferred by the activists. Thus, simulation of the joint model with sociodemographic valence can be used to infer aspects of this activist influence.

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Appendix: Formal and empirical electoral models

The model with activists

The electoral model presented here is an extension of the multiparty stochastic model of McKelvey and Patty (2006), modified by inducing asymmetries in terms of valence. The justification for developing the model in this way is the empirical evidence that valence is a natural way to model the judgements made by voters of party leaders and candidates. There are a number of possible choices for the appropriate model for multiparty competition. The simplest one, which we first present, is that the utility function for leader j is proportional to the popular support, V_j, of the party in the election.[32]

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[30] See Gallego and Pitchik (2004).

[31] Schofield and Sened (2006) found the electoral model for Israel to be very similar to Turkey, with two electoral axes, religion and security. Schofield and Zakharov (2010) found nationalism to be one of the principal axes in Russia, but the second axis was defined by attitudes to capitalism/communism, perhaps comparable to religion.

[32] The popular support may be identical to the vote share in a democratic election, or may be weighted by individual characteristics, such as domicile, income or ownership of land, in non-democratic polities.

With this assumption, we can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE). Because the vote share functions are differentiable, we use calculus techniques to obtain conditions for positions to be locally optimal. Thus, we examine what we call *local pure strategy Nash equilibria* (LNE). From the definitions of these equilibria it follows that a PNE must be a LNE, but not conversely. A necessary condition for an LNE is thus a necessary condition for a PNE. A sufficient condition for an LNE is not a sufficient condition for PNE. Indeed, additional conditions of concavity or quasi-concavity are required to guarantee existence of PNE.

The stochastic model essentially assumes that candidates cannot predict vote response precisely, but that they can estimate the effect of policy proposals on the expected vote share. In the model with valence, the stochastic element is associated with the weight given by each voter, i, to the average perceived quality or valence of each candidate. We also consider a formal model where the perceptions of the leader qualities vary across different sociodemographic groups in the society.

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, N, of size n. We assume that X is a subset of Euclidean space, of dimension w with w finite. Without loss of generality, we adopt coordinate axes so that $\frac{1}{n}\Sigma x_i = 0$. By assumption $0 \in X$, and this point is termed the *electoral mean*, or alternatively, the *electoral origin*. Each of the parties in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare prior to the specific election to be modeled. Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of party policy positions.

Given $\mathbf{z}$, each citizen, i, is described by a utility vector

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$$

where

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta||x_i - z_j||^2 + \varepsilon_{j.} = u^*_{ij}(x_i, z_j) + \varepsilon_j. \quad (3)$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The constant term, λ_j, is the fixed or *exogenous valence* of party j, The function $\mu_j(z_j)$ is the component of valence generated by activist contributions to agent j. We can also refer to this term as *endogenous valence*. The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of a metric induced from the Euclidean norm, $||\cdot||$, on X. The vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_p)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by Ψ. The notation $\lambda_j + \mu_j(z_j)$ is intended to imply that this is the average valence for party j among the electorate, but the realized valence is a distributed by Ψ. The most common assumption in empirical analyses is that Ψ is the *Type I extreme value distribution* (sometimes called Gumbel). This cumulative distribution has the closed form

$$\Psi(x) = \exp\left[-\exp\left[-x\right]\right].$$

The theorems presented in this Appendix are based on this assumption. This distribution assumption is the basis for much empirical work based on MNL estimation (Dow and Endersby 2004). The variance of ε_j is fixed at $\frac{\pi^2}{6}$, so that by definition β has dimension L^{-2}, where L is whatever unit of measurement is used in X.

In empirical models, the exogenous valences are simply real numbers, estimated by the model. Since they are all finite, they can be ranked. We therefore assume that the exogenous valence vector is given by

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_p) \text{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1.$$

This is a strong assumption, in that it assumes that every voter ranks the parties in this fashion. Adding sociodemographic valences, as in the body of the paper, means that this ranking over valences differs among the electorate.

Voter behavior is modeled by a probability vector. The probability that a voter i chooses party j at the vector $\mathbf{z}$ is

$$\rho_{ij}(\mathbf{z}) = \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \quad \text{for all } l \neq j].$$

Here Pr stands for the probability operator generated by the distribution assumption on ε.

With this distribution assumption on Ψ, it follows, for each voter i, and leader j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{p} \exp u^*_{ik}(x_i, z_k)}. \tag{4}$$

For any voting model the *likelihood* of a model is

$$\mathbb{L} = \prod_{i \in N,\ j_i \in P} \rho_{ij_i}(\mathbf{z}),$$

where j_i is the party that i chooses. The log likelihood of the model is $\log_e(\mathbb{L})$. Clearly as $\mathbb{L}$ approaches 0 then $\log_e(\mathbb{L})$ approaches $-\infty$.

To compare two models, $\mathbb{M}_1$ and $\mathbb{M}_2$, the *Bayes Factor* is $\mathbb{L}(\mathbb{M}_1)/\mathbb{L}(\mathbb{M}_2)$ and the *log Bayes factor* of $\mathbb{M}_1$against $\mathbb{M}_2$ is $\log_e(\mathbb{L}(\mathbb{M}_1) - \log_e(\mathbb{L}(\mathbb{M}_2)$. A log Bayes factor over 5.0 for $\mathbb{M}_1$ against $\mathbb{M}_2$ is considered strong support for $\mathbb{M}_1$ (Kass and Raftery 1995).

The *expected popular support* for leader j is

$$V_j(\mathbf{z}) \equiv \sum_{i \in N} s_{ij} \rho_{ij}(\mathbf{z}).$$

Here $\{s_{ij}\}$ are different weights that can be associated with different voters. In the case all weights are equal to $\frac{1}{n}$, we call the model *egalitarian.*

It is useful to have a formal model where voter weights differ. For example, in US Presidential elections, it is not the vote share *per se* but the *share of the electoral college total.* Voter weights in different States will therefor vary.

To present the model we now regard $\mathbf{V} = \{V_j : j \in P\}$ as a set of *vote share functions*, and identify $\mathbf{V}$ as a differentiable *profile function*, $\mathbf{V} : X^p \to \mathbb{R}^p$. We denote the egalitarian profile function as $\mathbf{V}_e$.

In this stochastic electoral model, it is assumed that each party j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p)$.

Thus a vector $\mathbf{z}^*=(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is called a *local strict Nash equilibrium* (LSNE) if each z_j strictly locally maximizes V_j, conditional on $\mathbf{z}_{-j}$, while $\mathbf{z}^*$ is a *local weak Nash equilibrium* (LNE) if each z_j weakly locally maximizes V_j, conditional on $\mathbf{z}_{-j}$. The notion of LSNE is convenient so as to avoid degeneracy problems associated with the Hessians.

In the same way the vector $\mathbf{z}^*$ is a *strict (or weak) pure strategy Nash equilibrium* (PSNE or PNE) if each party j chooses z_j to strictly (or weakly) maximize V_j on X.

Now assume that the vector $\mathbf{z}$ is fixed, and let $\rho_{ij}(\mathbf{z}) = \rho_{ij}$ be the probability that i picks j. Define the p by n matrix array of weights by

$$[\varpi_{ij}] \equiv \left[\frac{s_{ij}[\rho_{ij} - \rho_{ij}^2]}{\sum_{k \in N} s_{kj}[\rho_{kj} - \rho_{kj}^2]} \right] \tag{5}$$

The vector $\sum_i \varpi_{ij} x_i$ is a convex combination of the set of voter ideal points and is called the *weighted electoral mean* for party j. Define

$$z_j^{\mathrm{el}} \equiv \sum_{i=1}^{n} \varpi_{ij} x_i \quad \text{and} \quad \frac{\mathrm{d}\mathcal{E}_j^*}{\mathrm{d}z_j}(z_j) \equiv \left[z_j^{\mathrm{el}} - z_j \right].$$

Then the *balance equation* for z_j^* is given by the expression

$$\frac{\mathrm{d}\mathcal{E}_j^*}{\mathrm{d}z_j}(z_j^*) + \frac{1}{2\beta} \frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}(z_j^*) = 0. \tag{6}$$

The term $\frac{\mathrm{d}\mathcal{E}_j^*}{\mathrm{d}z_j}(z_j)$ is the *marginal electoral pull of party* j at the point z_j and can be regarded as a gradient vector, at z_j, pointing towards the weighted electoral mean of the party. (Note that this electoral pull depends on the positions of all leaders.) When z_j is equal to the weighted electoral mean then the electoral pull is zero. The gradient vector $\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}(z_j)$ is called *the marginal activist pull for party* j at z_j.

When $\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}(z_j) = 0$, then then the balance equation reduces to setting $z_j = z_j^{\mathrm{el}}$.

If $\mathbf{z}^* = (z_1^*, \ldots z_j^*, \ldots z_p^*)$ is such that each z_j^* satisfies the balance equation then call $\mathbf{z}^*$ a *balance solution*. The balance solution requires that the electoral and activist gradients are directly opposed, for every party leader.

The model just presented is denoted $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \mathbf{V})$. Schofield (2006) proves the following theorem for this model.

Theorem 1 *Consider the electoral model* $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \mathbf{V})$ *based on the distribution,* Ψ*, including both exogenous and activist valences, and defined by the family* $\mathbf{V}$ *of vote share functions.*

(i) *The first order condition for* $\mathbf{z}^*$ *to be an LSNE is that it is a balance solution.*
(ii) *If all activist valence functions are sufficiently concave,*[33] *then a balance solution will be a PNE.*

In the full activist model, $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \beta; \mathbf{V})$, with valence functions $\{\mu_j\}$ that are not identically zero or constant, then it is the case that generically $\mathbf{z}_0$ cannot satisfy the first order conditions for LNE even when $\mathbf{V}$ is egalitarian. Instead the vector $\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}$ "points towards" the position at which the activist valence for leader j is maximized. When this marginal or gradient vector, $\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}$, is increased (as activist groups become more willing to contribute to leader j) then the equilibrium position is pulled away from the weighted electoral mean of the leader, and we can say the "activist effect" for the leader is increased. In the case of two opposed leaders, j and k, if the activist valence functions are fixed, but the exogenous valence, λ_j, is increased, or λ_k, is decreased, then the weighted electoral mean, z_j^{el}, approaches the electoral origin. Thus, the local equilibrium of leader j is pulled towards the electoral origin. We can say the "electoral effect" is increased.

The egalitarian model without activists

In the case that the activist valence functions are identically zero, or constant, we denote the model by $\mathbb{M}(\boldsymbol{\lambda}, \beta; \mathbf{V})$. The key consideration for the egalitarian model, $\mathbb{M}(\boldsymbol{\lambda}, \beta; \mathbf{V}_e)$,when all voter weights are identical, is whether the electoral origin is a LSNE. For this model it can be shown that if all parties are at the same position, so $\mathbf{z}^* = (z^*, z^*, \ldots z^*)$ then every $\{\rho_{ij}(\mathbf{z}^*) : i \in N\}$ is independent of i, and can thus be written $\rho_j(\mathbf{z}^*)$. This implies that all α_{ij} in (5) are identical at $\mathbf{z}^*$ and equal to $\frac{1}{n}$. Thus, when there is only exogenous valence, the equation $z_j^* = \frac{1}{n}\Sigma x_i$ satisfies the balance solution for all j. By an appropriate coordinate change, we can assume $\frac{1}{n}\Sigma x_i = 0$. In this case, all marginal electoral pulls are zero at $\mathbf{z}_0 = (0, \ldots 0)$, so $\mathbf{z}_0$ satisfies the first-order conditions. However, to determine whether $\mathbf{z}_0$ is an LNE it is necessary to examine the Hessians of the vote share functions.

We first define the *electoral covariance matrix,* ∇_0, and then use ∇_0 to define the convergence coefficient of the model $\mathbb{M}(\boldsymbol{\lambda}, \beta; \mathbf{V}_e)$. Let $X = \mathbb{R}^w$ be endowed with a system of coordinate axes $r = 1, \ldots, w$. For each coordinate axis let $\xi_r = (x_{1r}, x_{2r}, \ldots, x_{nr})$ be the vector of the rth coordinates of the set of n voter ideal points. The scalar product of ξ_r and ξ_s is denoted $(\xi_r \cdot \xi_s)$. Let $(\sigma_r \cdot \sigma_s) = \frac{1}{n}(\xi_r \cdot \xi_s)$ be the electoral covariance between the r and s axes, and σ_s^2 be the variance on the s axis.

[33] By this we mean that the eigenvalues of the activist functions are negative and of sufficient magnitude everywhere. That is to say, there exists $\alpha < 0$, such that all eigenvalues $<\alpha$ is sufficient to guarantee existence of a PNE.

(i) The symmetric $w \times w$ *electoral covariance matrix* about the origin is denoted ∇_0 and is defined by

$$\nabla_0 \equiv [(\sigma_r \cdot \sigma_s)]_{s=1,\ldots,w}^{r=1,\ldots,w}.$$

(ii) The total electoral variance is $\sigma^2 \equiv \sum_{s=1}^{w} \sigma_s^2 = \text{trace}(\nabla_0)$.

(iii) At the vector $\mathbf{z}_0 = (0, \ldots, 0)$ the probability $\rho_{ij}(\mathbf{z}_0)$ that i votes for party j is independent of i, and is given by

$$\rho_j = \left[1 + \sum_{k \neq j} \exp\left[\lambda_k - \lambda_j\right]\right]^{-1}. \tag{7}$$

(iv) The Hessian of the egalitarian vote share function of party j at $\mathbf{z}_0$ is a positive multiple of the w by w *characteristic matrix*,

$$C_j \equiv 2\beta(1 - 2\rho_j)\nabla_0 - I. \tag{8}$$

(Here I is the identity matrix.)

The *convergence coefficient* of the egalitarian model, $\mathbb{M}(\boldsymbol{\lambda}, \beta; \mathbf{V}_e)$, is defined to be

$$c \equiv c(\boldsymbol{\lambda}, \beta; \mathbf{V}_e) \equiv 2\beta[1 - 2\rho_1]\sigma^2. \tag{9}$$

Note that the β-parameter has dimension L^{-2}, so that c is dimensionless, and therefore independent of the scale used to measure positions. We can therefore use c to compare different models.

Theorem 2 *Consider the electoral model* $\mathbb{M}(\boldsymbol{\lambda}, \beta; \mathbf{V}_e)$ *where all activist valence functions are zero (or constant) and* $\mathbf{V}_e$ *is the egalitarian party profile.*

(i) *The joint origin* $\mathbf{z}_0 = (0, \ldots, 0)$ *satisfies the first order condition to be a LSNE for this model.*

(ii) *In the case that X is w dimensional then the necessary condition for* $\mathbf{z}_0$ *to be a LNE for this model is that* $c(\lambda, \beta; \mathbf{V}_e) \leq w$.

(iii) *In the case that X is 2 dimensional, a sufficient condition for* $\mathbf{z}_0$ *to be a LSNE for this model is that* $c(\boldsymbol{\lambda}, \beta; \mathbf{V}_e) < 1$.

The proof and some applications of Theorem 2 are given in Schofield (2007a,b)

Empirical models

In empirical models with exogenous valence alone it is necessary to estimate the model with respect to the valence of a baseline party, say k^*. We set $\Lambda_j = \lambda_j - \lambda_{k^*}$, and call these the relative valences. We denote this egalitarian model by $\mathbb{M}(\boldsymbol{\Lambda}, \beta; \mathbf{V}_e)$.

At the joint origin,$\mathbf{z}_0$, we see that

$$\rho_{ij}(\mathbf{z}_0) = \frac{\exp(\lambda_j)}{\sum_{k=1}^{p}\exp(\lambda_k)} = \frac{\exp(\lambda_j - \lambda_{k^*})}{\sum_{k=1}^{p}\exp(\lambda_k - \lambda_{k^*})} = \frac{\exp(\Lambda_j)}{\sum_{k=1}^{p}\exp(\Lambda_k)} \tag{10}$$

is again independent of the individual, i, and can be written as ρ_j.

To estimate the standard error on ρ_j, we use Taylor's Theorem, which asserts that

$$\rho_j(\Lambda_j + h) = \rho_j(\Lambda_j) + h\frac{\mathrm{d}\rho_j}{\mathrm{d}\Lambda_j} = \rho_j(\Lambda_j) + h\rho_j(1-\rho_j). \tag{11}$$

Empirical models with sociodemographic valences

As described in the body of the paper, in empirical applications with sociodemographic variables, we typically assume that $\mathbf{V}$ is the egalitarian party profile function,$\mathbf{V}_e$, so the model $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta; \mathbf{V}_e)$ is based on the assumption that voter utility has the form

$$u_{ij}(x_i, z_j) = \Lambda_j + (\theta_j \cdot \eta_i) - \beta\|x_i - z_j\|^2 + \varepsilon_j.$$

The estimate of voter i's valence will then be $\Lambda_j + (\theta_j \cdot \eta_i)$, so this will vary from one voter to another. A consequence of this is that, in the expression (5) for the weighted electoral mean, even when all parties are at the origin, then the denominator term $\{\rho_{kj}(\mathbf{z}_0) : k \in N\}$ will depend on voter k. This implies that voters will be weighted differently, and generically, $\mathbf{z}_0$ will not satisfy the first order condition for LNE. However, the joint empirical model, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta; \mathbf{V}_e)$, assumes that the sociodemographic effects are independent of party positions, and this implies $\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j} = 0$, for all j. Using (6), we infer that the various LNE obtained by simulation of the joint model provides an estimate of a set of vectors of weighted electoral means: $\{\mathbf{z}^{\text{el}} = (z_1^{\text{el}}, \ldots, z_p^{\text{el}})\}$.

Assuming that the estimated party positions are given by the vector $\mathbf{z}^* = (z_1^*, \ldots, z_p^*)$ and that this is in equilibrium with respect to the full activist model, then choosing one joint LNE, $\mathbf{z}^{\text{el}}$, gives an estimate of

$$\left[z_j^{\text{el}} - z_j^*\right] \equiv \frac{\mathrm{d}\mathcal{E}_j^*}{\mathrm{d}z_j}(z_j^*) = -\frac{1}{2\beta}\left[\frac{\mathrm{d}\mu_j}{\mathrm{d}z_j}\right]. \tag{12}$$

Thus,

$$\left[\mathbf{z}^* - \mathbf{z}^{\text{el}}\right] = \frac{1}{2\beta}\left[\frac{\mathrm{d}\mu_1}{\mathrm{d}z_1}, \ldots, \frac{\mathrm{d}\mu_p}{\mathrm{d}z_p}\right]. \tag{13}$$

This observation suggests how the gradients of the activist valence functions may be inferred from a comparison of LNE of the joint empirical model with the estimated political configuration.

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