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PS 5052
Mathematical Modeling in Political Science

Fall 2011

Monday & Wednesday 10:30-12:00
classroom: Eliot 316

page last revised 08/30/2011

Website under construction
In the meantime, further approximate details can be found here, my website from the last time I taught this course.

Jump directly to current topics & assignments

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Help Sessions: TBA

This course is an introduction to mathematical techniques used to model phenomena studied in political science, with special attention to the analysis of individual action. Mathematical topics covered include: sets, functions, and graphs; matrix algebra; differential calculus and optimization; probability and risk; integral calculus; and sequences, series, and limits. All these topics are useful in many settings in political science, including game theory, dynamic modeling, and statistics.

This course website will be updated to reflect any changes in schedule, topics covered, or assignments, as well as to provide relevant links to materials associated with the course.

Textbooks

Because it's useful to have a textbook for reference and contemplation, we will try to make maximum use of

- Malcolm Pemberton and Nicholas Rau, *Mathematics For Economists: An Introductory Textbook,* 2nd ed. (Manchester University Press, Nov. 2006).

This is available in the Campus Bookstore as well as directly from MacMillan press for about $55-$60. The economics applications don't get in the way, the presentation seems very clear, and the coverage tracks what I want to cover in this course, so I think this will serve us well.

Answers to "exercises" and to "problems" in Pemberton and Rau are available online as (pdfs, about 370K and 540K in size, respectively).

Also recommended is a textbook by our own Professor Gill,

- Jeff Gill, *Essential Mathematics for Political and Social Research* (Cambridge Univ. Press 2006),

The header material and Chapter 1 are available online here. This text doesn't follow my intended coverage so closely, but you may find it a useful reference now and in the future; I will occasionally appeal to it for exercises or additional explanation. Answers to all exercises in the Gill book are available from me on request.

Course requirements

Exams 75%. Grades for the course will be compiled primarily from your performance on three examinations, given at the points indicated in the course outline. These will be untimed, take-home exams, on which you are to work without collaboration of any kind. I will give you at least two weeks warning of any changes in exam dates.

Homework and occasional short quizzes 25%. I will provide frequent problem sets along the way, whose solutions we can discuss in class. I encourage you to collaborate with one another in working on exercises. Grading of these will be based primarily on (1) completion and (2) effort, and may be based on a random sampling of the problems assigned. From time to time, I may also administer a short quiz on the easier aspects of the problems and other material being covered; these would be graded more for accuracy than are the homeworks.

Course Outline and Approximate Schedule

0. Summer Review

Wednesday 8/31 and Wednesday 9/7

I am distributing a collection of questions on elementary topics designed to simultaneously give you a review, reveal something about your background, and prompt your questions about topics on which you may be shaky. These problems, especially any questions arising from them, will be our focus in the first two class meetings, on Wednesday Aug. 31 and Wednesday Sept. 7. **You should try to do all the problems in the first three sections before you come to the first class meeting**, and complete the fourth section by the second class.

Click here for another complete copy of summer review notes and problems. Solutions are posted below, section-by-section.

- Solutions for Section 1, Algebra
- Solutions for Section 2, Sets
- Solutions for Section 3, Functions and Graphs
- Solutions for Section 4, Statements and Proofs

As you can see I am posting answers as well as questions. Take my advice: try to do the problems before you first peek at the answers. Then, try to resolve any differences. Remember, this isn't a test!

Also for the review sessions: you should try to read the following sections in the Pemberton and Rau text. We aren't covering these things in P&R's order, however. The goal is to be finished dong all problems and reading all these sections by the second meeting so we can cover all questions and wrap up.

- Sections 1.1, 1.2, and 2.1 on linear equations and inequalities
- Chapter 3 on sets and functions
- Chapter 4 on quadratic expressions, exponents (which P&R call "indices"), and logarithms
- Sections 29.1 on sets, statments, and proofs

Further relevant material you may find helpful in the Gill book:

- on sets: Gill 7.3 (pages 291-306)
- on functions and graphs: Gill 1.5 (pages 18-34) and 2.1 and 2.2 (pages 51-54)
- on logarithms and exponents Gill 1.7

1. Sets, relations, functions, sequences, and series

Monday 9/12 and Wednesday 9/14

- sets and relations [Lecture notes and exercises due Monday 9/19 posted here.]
- **application:** preferences; discrete rational choice
- utility representations; invariance to monotone transformation

Wednesday 9/14 and Monday 9/19

- real numbers, sequences, limits (Pemberton and Rao 5.1, 29.2, 29.3)
- continuous functions (P&R 29.4)
- sums and series (P&R 5.2; see also Gill, pp. 259-266)
- **application:** choices over time (P&R 5.3; chapter 9, except the calculus bits)
- **Exercises** due Monday 9/26: TBA
    - 5.1.3
    - 5.2.1-5.2.3
    - 5.3.2, 5.3.4
    - AND:
        - Consulting the website for the Powerball Lottery for September 24, 2011, assuming that

you are playing in Missouri, what amount can you win after taxes if you take the lump-sum cash prize? What total amount will you receive if you take the annual payments over 30 years?
    - What personal discount factor would make you exactly indifferent between the lump-sum payment and the 30 annual payments? What would you choose if your personal discount factor were higher than that?

2. Differential calculus (P&R, chapters 6-8)

- Wednesday 9/21: The derivative
    - P&R sections 6.1-6.3 Also recommended: Appendix to Chapter 6 (pp. 104-105).
    - do all **exercises**, to turn in Monday 9/26.
- Monday 9/26: Methods of differentiation
    - P&R sections 7.1-7.3.
    - Turn in the following exercises by Monday 10/3 (note there are more exercises due 10/3 below)
        - 7.1.1 and 7.1.2
        - 7.2.1 through 7.2.3
        - 7.3.1 and 7.3.2
- Wed. 9/28: Finding maxima and minima.
    - P&R sections 8.1-8.3
    - Turn in the following exercises by Monday 10/3
        - 8.1.1 through 8.1.3
        - 8.2.1 through 8.2.3
        - 8.3.1, 8.3.2, 8.3.4
- Mon. 10/3: Convexity and concavity
    - P&R section 8.4
    - Do the following exercises and be prepared to discuss in class on Wednesday 10/5:
        - 8.4.1 through 8.4.6
- Wed. 10/5: Review

First exam out Wed. 10/5, due Mon. 10/10

3. Vector and Matrix Algebra (P&R, chapters 11-13, 25, 26)

recommended readings from Gill:

- Chapter 3: 3.1-3.5 (omit examples 3.21, 3.22)
- Chapter 4: 4.3, 4.4, 4.6, 4.7, 4.9 (except example)

4. Multivariate calculus (P&R, chapters 14-17)

Second exam out approximately Wed., Nov. 9; due Mon., Nov. 14

5. Probability

Day 1

- probability

Day 2

- conditional probability and Bayes's rule

Day 3

- **application:** expected utility and choice under risk
- lotteries and risk aversion

Day 4

- probability with continuous outcomes: PDF, CDF

Exercises due TBA

- Click here to download exercises as a pdf file.

6. Integral calculus (P&R, chapters 19-20)

Possible exercises

- **Exercise 1.** Use a limit of sums of rectangular areas to calculate the area under the function f(x) = 3x from x = 1 to x = 3. Hint: you may assume that the following formula holds: for any positive integer n,

  $1 + 2 + ... + n = n(n+1) / 2.$

- Silverman p. 121 #2, 4, 5
- Silverman p. 151 #2, 4
- **Exercise 6.** Use integration to find the CDF, mean, and variance of a random variable X whose PDF is f(x) = 1 for x in [0, 1] and f(x) = 0 everywhere else.
- **Exercise 7.** Find the mean and variance of a "triangular distribution" on [0, 2] defined as follows:
  - f(x) = x for x in [0, 1] ;
  - f(x) = 2 - x for x in [1, 2] ; and

    - $f(x) = 0$ everywhere else.
- **Exercise 8.** Find the CDF, mean and variance for a random variable X having the uniform distribution on [a, b], where $a < b$.

Final exam out Tuesday 12/12; due Monday 12/17

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