

## Formal Models of Elections and Political Bargaining

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Received 16 June 2007; Accepted 27 July 2008

**Abstract** The key theoretical idea in this paper is that activist groups contribute resources to their favored parties in response to policy concessions from the parties. These resources are then used by a party to enhance the leader's valence — the electoral perception of the quality of the party leader. The equilibrium result is that parties, in order to maximize vote share, will balance a centripetal electoral force against a centrifugal activist effect. Under *proportional electoral rule*, there need be no pressure for activist groups to coalesce, leading to multiple political parties. Under *plurality rule*, however, small parties face the possibility of extinction. An activist group linked to a small party in such a polity has little expectation of influencing government policy. The paper illustrates these ideas by considering recent elections in Turkey, Britain and the United States, as well as a number of European polities.

**Keywords** Election, plurality rule, proportional representation, activist groups

**JEL classification** H10, H11

### 1. A spatial model of politics

There are two fundamentally different sets of models of the polity. One class of models has grown out of the attempt to model political competition when the electoral system is based on a majoritarian or plurality method (sometimes called “first past the post”). Early versions of such models assumed that there were at most two parties (Downs 1957; Riker and Ordeshook 1973) and that the policy space was restricted to one dimension. Under the further assumption that parties or candidates adopted positions in order to win, it was inferred that parties would converge to the electoral median, under deterministic voting. This result could not be extended to higher dimensions because of results on the generic non-existence of a core, or voting equilibrium (McKelvey and Schofield 1987; Schofield 1985; Saari 1997). Later work modeled multi-party elections using stochastic methods (Lin et al. 1999; Banks and Duggan 2005; McKelvey and Patty 2006) and suggested that vote maximizing agents would converge to a Nash equilibrium at the electoral mean. In the empirical component of this paper, clear evidence is presented that convergence is very unlikely.

It is not obvious that these electoral models are relevant in a political system with multiple parties. Riker (1962), for example, ignored the question of elections, and fo-

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cused on the nature of post-election bargaining over coalition formation. Riker's work on coalition has led to the second class of models, appropriate for studying politics based on proportional electoral methods (Baron and Ferejohn 1989; Banks and Duggan 2000).

Recent work (Bawn and Rosenbluth 2006; Persson and Tabellini 2000, 2003) has argued that institutional characteristics of political systems, such as presidentialism versus parliamentarianism, and majoritarianism versus proportionality, will have significant effects on the size of government and the extent of redistributive politics. However, these arguments have been based on cross country empirical analyses and relatively simple one dimensional spatial models.

To gain an understanding of the differences between plurality and proportional electoral systems it is necessary to combine models of elections and models of coalition bargaining. Thus a formal theory of politics must connect the nature of the electoral system, the motivations of parties concerning policy and perquisites, and the process of government formation, in a way which is consistent with the empirical phenomena.

This paper attempts to relate pre-election and post-election political behavior by focusing on the nature of activist support for the parties. To illustrate the idea, Section 2 briefly presents a stochastic electoral model of two elections in Turkey in 1999 and 2002. The models use sociodemographic characteristics of voters in the sample to estimate the electoral responses. The equilibria so obtained were quite distinct from the electoral mean, and were found to be similar to the actual estimated positions of the parties. The discrepancy between the vote maximizing positions of the parties and their estimated positions was taken as evidence that the positions were influenced by party activists. The underlying model is one in which activists provide crucial support to party leaders in return for the adoption of policies that the activists prefer.

In the post election context, we can assume that these activists influence the parties in the policies that are to be adopted. This model is very different from the Downsian case, where it was assumed that parties adopted policy positions simply to gain votes. In the model presented here, party leaders can be assumed to have policy preferences induced from those of their supporting activists. In principle, this model is applicable both to plurality and proportional electoral systems.

This model provides a rationale for policy preferences by party leaders in the context of coalition formation, and we use this assumption in Section 3 to propose a spatial model of bargaining.

In multi-party political systems, there is generally no guarantee that the party gaining the most seats will become a member of the governing coalition. This paper will use the idea of the *core*, presented earlier in Laver and Schofield (1990), to argue that a dominant party, located at the center of the policy space, can control the formation of government. Instead of assuming that the 'political game' is constant sum or based on a one-dimensional policy space, we shall consider situations where the policy space may have two or more dimensions, and government results from bargaining between three or more parties. In this post-election phase, the 'positions' of the parties are assumed to be given, as is the distribution of seats. This distribution defines a set of winning coalitions. Given the set of winning coalitions, and party positions, we use the

theory of the “political heart” (Schofield 1999) to discuss coalition bargaining. Under some circumstances, the heart will consist of a single policy point, the “core”. If the “core” is stable under small perturbations in the positions of the parties then it is said to be “structurally stable”. If a party’s position is at the “structurally stable core”, then we shall call this party the “core party”. Under these circumstances, it is argued that the “core party” may form a minority government. If the heart is not given by a point, then it will comprise a domain in the policy space, “the cycle set”. This “cycle set” will be “bounded” by the preferred positions of a particular set of parties. These bounding “proto-coalitions” form the basis for coalitional bargaining. This model of the heart can then be used to describe, heuristically, the general pattern of coalition formation.

Although scholars are in fair agreement concerning the positions of parties in a one-dimensional (left-right) policy space, party positions in two dimensions are much more difficult to ascertain. Empirical models can be constructed on the basis of multi-dimensional data on party policy positions that have been derived from the content analysis of party manifestos in European polities.<sup>1</sup> An alternative method is to use survey data and estimate party positions in a policy space obtained from factor analysis of these data (Schofield and Sened 2006; Schofield et al. 2009a,b). It is generally possible to reduce these data to two dimensions giving a tractable description of the main political issues in these countries. This paper uses survey data in Section 2 and the expert estimates of party position presented in Benoit and Laver (2006) in Section 3.

Using these estimates of party position, we can then determine whether the core is empty, and if it is, deduce the location of the “cycle set” or heart. In two dimensions, it is possible for a core to occur in a structurally stable fashion, but it will generally be necessary that the core party is dominant in terms of its seat strength. Since a core party will be able to veto any coalitional proposal, we expect this party to belong to the government. On the other hand if the core is empty then no party can have a veto of this kind, and it is natural to expect greater uncertainty in coalition outcomes. In such a situation, for any incumbent coalition and policy point, there is always an alternative coalition that can win with a new policy point. This it can do by seducing some members of the incumbent coalition away, by offering them a higher policy payoff than they can expect if they remain loyal to the original coalition. However, because the heart will be bounded by a small number of median arcs, we can identify these arcs with a set of minimal winning coalitions. It is suggested that bargaining between the parties will result in one or other of these coalition governments. In Section 3 of this paper, we shall use the estimated positions and relative sizes of the parties, together with the concepts of the core and heart to suggest a categorization of different types of bargaining environments, distinguishing between *unipolar*, *bipolar* and *triadic* political systems.

In *left unipolar* systems such as Norway, Sweden and Denmark there is typically one larger party and three or four smaller parties. The larger party may be able to dominate coalition politics, and form a minority government with or without the tacit support of one of the other parties. In *triadic* systems, such as Austria and Germany

<sup>1</sup> The original manifesto group used a 54-category policy coding scheme to represent party policy in nineteen democracies. The more recent work (Budge, Klingemann et al. 2001) covers twenty five countries. See also Benoit and Laver (2006) who use expert estimates.

(where typically there are two large and one or two small parties) most coalition cabinets are both minimal winning and minimal connected winning. In *bipolar systems*, such as the Netherlands and Finland, there are typically two large and a number of smaller parties (discussion of these two polities can be found in Schofield 2008). *Center unipolar* systems, such as Belgium, Luxembourg and Ireland typically have two large and at least two other small parties. Minority or surplus coalitions are infrequent and governments are usually minimal winning coalitions. Finally, Italy (until the election of 1994) had a strongly dominant party, the Christian Democrats. This party was in every coalition government, and relatively short-lived governments were very common (Mershon 2002). By 1994, the dominance of the Christian Democrat party had evaporated (Giannetti and Sened 1994).

As we discuss the various polities in Section 3, it is quite clear that under proportional representation, the number of parties and their relative strengths can change in radical ways, inducing complex changes in the possibility of a core and in the configuration of the heart.

Section 4 returns to the activist formal model and considers models of the plurality electoral systems of Britain and the United States. Section 5 concludes by arguing that the motivations of activists under proportional representation and plurality rule are fundamentally different.

## 2. Modelling the election and the legislature

We assume in this section that each party chooses a preferred position (or *bliss point*) in a *policy space*  $X$ . We shall denote the parties as  $P = \{1, \dots, j, \dots, p\}$ , and the vector of party ideal points as  $\mathbf{z} = (z_1, \dots, z_p)$ . After the election we denote the number of seats controlled by party,  $j$ , by  $s_j$  and let  $\mathbf{s} = (s_1, \dots, s_p)$  be the vector of parliamentary seats. We shall suppose that any coalition with more than half the seats is winning, and denote the set of winning coalitions by  $\mathbb{D}$ . This assumption can be modified without any theoretical difficulty. For each winning coalition  $M \in \mathbb{D}$  there is a set of points in  $X$  such that, for any point outside the set there is some point inside the set that is preferred to the former by all members of the coalition. Furthermore, no point in the set is unanimously preferred by all coalition members to any other point in the set. This set is the Pareto set of the coalition. If the conventional assumption is made that the preferences of the actors can be represented in terms of Euclidean distances, then this Pareto set for a coalition is simply the convex hull of the preferred positions of the member parties. (In two dimensions, we can draw this as the area bounded by straight lines joining the bliss points of the parties and including all coalition members.) Since preferences are described by the vector,  $\mathbf{z}$ , we can denote this as  $Pareto(M, \mathbf{z})$ . Now consider the intersection of these Pareto sets for all winning coalitions. If this intersection is non-empty, then it is a set called the *Core* of  $\mathbb{D}$  at  $\mathbf{z}$ , written  $Core(\mathbb{D}, \mathbf{z})$ . At a point in  $Core(\mathbb{D}, \mathbf{z})$  no coalition can propose an alternative policy point that is unanimously preferred by every member of some winning coalition. An alternative way to characterize the core in the case of Euclidean preferences is to define a *median line* in  $X$  (in the two-dimensional case) to be a line joining the positions of two parties,

with the property that the set of parties on either side of the line controls a majority of the seats. In higher dimensions a median hyperplane can be defined analogously. The core will exist if all median lines intersect. When the core is empty then the heart,  $\mathcal{H}(\mathbb{D}, \mathbf{z})$  is defined to be the star shaped figure bounded by these median lines (or hyperplanes in higher dimension). An attractive feature of the heart, regarded as a correspondence is that if  $Core(\mathbb{D}, \mathbf{z})$  is nonempty, and  $\mathbf{z}'$  converges to  $\mathbf{z}$  then  $\mathcal{H}(\mathbb{D}, \mathbf{z}')$  converges to  $Core(\mathbb{D}, \mathbf{z})$ .

To construct an electoral model of the choice of the vector  $\mathbf{z} = (z_1, \dots, z_p)$  of party positions as well as the set,  $\mathbb{D}$  of winning coalitions, we shall first adopt a simple stochastic model in which parties attempt to maximize their vote share. We show that the model typically gives heterogenous, non-centrist positions. Using this model, we can then estimate the heart of the legislature, and determine whether there is a core, or a possibly a majority party.

## 2.1 The core and the heart of the legislature: Turkey 1999–2007

We use a stochastic vote model, denoted  $E(\lambda, \theta, \beta; \Psi)$ , to estimate voter utility,  $u_{ij}$ . The model assumes that the errors,  $\varepsilon = \{\varepsilon_1, \dots, \varepsilon_j, \dots, \varepsilon_p\}$  are distributed by the Gumbel distribution,  $\Psi$ , as required for multinomial conditional logit (MNL) estimation (Dow and Endersby 2004). We also assume that the set of voters,  $N$ , are equally weighted. For this model we assume that voter  $i$  utility is given by the expression:

$$u_{ij}(x_i, z_j) = \lambda_j + (\theta_j \cdot \eta_i) - \beta \|x_i - z_j\|^2 + \varepsilon_j$$

Here  $\theta$  is a set of  $m$ -vectors  $\{\theta_j\}$  representing the effect of the  $k$  different sociodemographic parameters (class, domicile, education, income, religious orientation, etc.) on voting for party  $j$  while  $\eta_i$  is an  $m$ -vector denoting the  $i^{\text{th}}$  individual's relevant "sociodemographic" characteristics. The compositions  $\{(\theta_j \cdot \eta_i)\}$  are scalar products. The spatial coefficient is denoted  $\beta$  and  $\lambda = \{\lambda_j : j \in P\}$  are the intrinsic valences for the parties in  $P$ . The vector  $\mathbf{z} = (z_1, \dots, z_p) \in X^p$  is the set of party positions, while  $\mathbf{x} = (x_1, \dots, x_n) \in X^n$  is the set of ideal points of the voters in  $N$ . When  $\beta$  is assumed zero then the model is called pure *sociodemographic* (SD), and denoted  $E(\lambda, \theta; \Psi)$ . When  $\{\theta_j\}$  are all assumed zero then the model is called *pure spatial*, and denoted  $E(\lambda, \beta; \Psi)$ . When all parameters are included then the model is called *joint*, denoted  $E(\lambda, \theta, \beta; \Psi)$ . The differences in log marginal likelihoods for two models then gives the Log Bayes' factor for the pairwise comparison.<sup>2</sup> We use the stochastic model to discuss the Turkish election results in 1999 and 2002, given in Tables 1 and 2. Details of the MNL estimation are available in Schofield, Gallego, Ozdemir and Zakharov (2009).<sup>3</sup> The estimates presented there show that the joint model was statistically superior to the other possible models.<sup>4</sup> We can infer that, though the sociodemographic

<sup>2</sup> Since the Bayes' factor for a comparison of two models is simply the ratio of marginal likelihoods, the log of the Bayes factor is the difference in log likelihoods. See Schofield and Sened (2006).

<sup>3</sup> The estimation is based on a factor analysis of a sample survey conducted by Veri Arastima for TUSES.

<sup>4</sup> The log Bayes factor for the joint model over the sociodemographic model in 1999 was highly significant 31.3 in 1999. Similarly, the log Bayes factor for the joint model over the sociodemographic model was 58.7 in 2002. The Bayes' factors for the joint over the spatial models were slightly significant at 6.13 and 5.17 in

variables are useful in predicting voter choice, it is necessary to use a joint model based on both sociodemographic and spatial variables.

**Table 1.** Turkish election results 1999

Party Name		% Vote	Seats	% Seats
Democratic Left Party	DSP	22.19	136	25
Nationalist Action Party	MHP	17.98	129	23
Virtue Party	FP	15.41	111	20
Motherland Party	ANAP	13.22	86	16
True Path Party	DYP	12.01	85	15
Republican People's Party	CHP	8.71		
People's Democracy Party	HADEP	4.75		
Others		4.86		
Independents		0.87	3	1
Total			550	

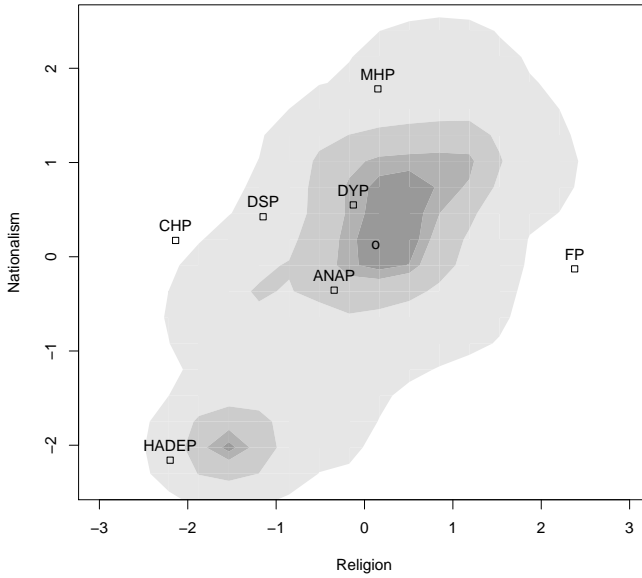
**Table 2.** Turkish election results 2002

Party Name		% Vote	Seats	% Seats
Justice and Development Party	AKP	34.28	363	66
Republican People's Party	CHP	19.39	178	32
True Path Party	DYP	9.54		
Nationalist Action Party	MHP	8.36		
Young Party	GP	7.25		
People's Democracy Party	HADEP	6.22		
Motherland Party	ANAP	5.13		
Felicity Party	SP	2.49		
Democratic Left Party	DSP	1.22		
Others		5.12		
Independents		1.00	9	2
Total			550	

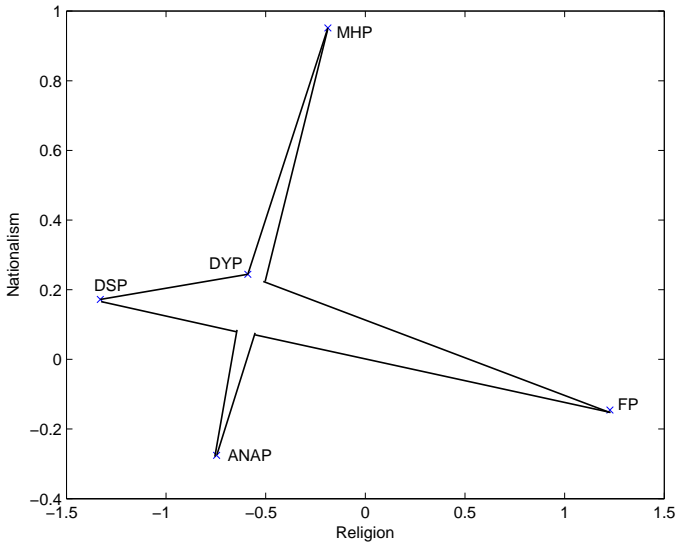
Figures 1 and 3 show the electoral distributions (based on a sample surveys of sizes 635 and 483, respectively) and estimates of party positions for 1999 and 2002.

Minor differences between these two figures include the change of the name of the Kurdish party from HADEP to DEHAP (we retain the name HADEP in Figure 3) and the disappearance of the Virtue Party (FP) which was banned by the Constitutional Court in 2001. In 1999, a DSP minority government formed, supported by ANAP

1999 and 2002, respectively.

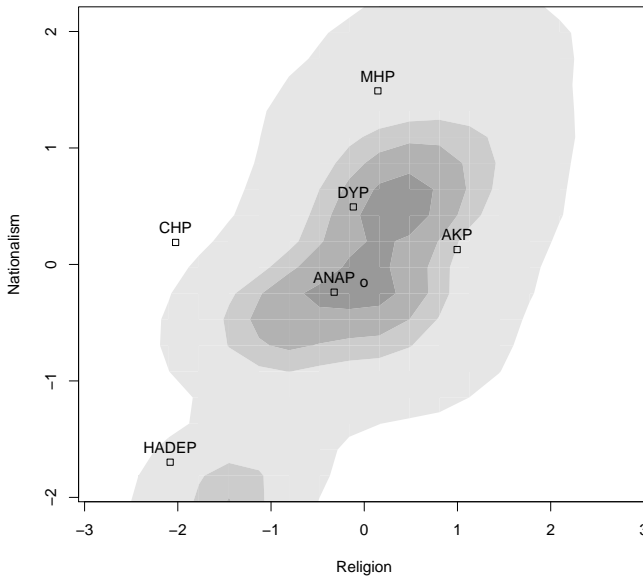


**Figure 1.** Party positions and voter distribution in Turkey in 1999



**Figure 2.** The heart in Turkey in 1999

and DYP. This only lasted about 4 months, and was replaced by a DSP-ANAP-MHP coalition. During the period 1999–2002, Turkey experienced two severe economic



**Figure 3.** Party positions and voter distribution in Turkey in 2002

crises. As Tables 1 and 2 show, the vote shares of the parties in the governing coalition went from about 53% in 1999 to 15% in 2002. The most important change in 2002 was the appearance of the new Justice and Development Party (AKP), which can be regarded as a replacement for the banned FP. The AKP obtained about 35% of the vote and 363 seats out of 550 seats (or 66%), in 2002, indicating that the electoral system had become a much more majoritarian. In 2007, the AKP gained 46.6% of the vote and a majority of 340 seats (or 62%), reflecting the continuing high valence of Recep Tayyip Erdogan, leader of the AKP.<sup>5</sup> We can compute the heart for 1999 by estimating all median lines in Figure 1. The heart for 1999 is the set bounded by these median lines, as shown in Figure 2. If all medians intersect then the core is non-empty. Clearly the AKP had a majority in 2002 and 2007, and so was at the core position.

The estimated valences of the ANAP and MHP, under the pure spatial model dropped between 1999 and 2002. In 1999, the estimated  $\lambda_{ANAP}$  was 0.336, whereas in 2002 it was  $-0.31$ , while  $\lambda_{MHP}$  fell from 0.666 to  $-0.12$ . The estimated valence,  $\lambda_{AKP}$ , of the new Justice and Development Party (AKP) in 2002 was 0.78, in comparison to the valence of the FP of  $-0.159$  in 1999. This we can ascribe to the disillusion of most voters with the other parties, as well as the charisma of Erdogan.

For the pure spatial model, the  $\beta$  coefficient was 0.375 in 1999, and 1.52 in 2002, suggesting that electoral preferences over policy had become more intense.

<sup>5</sup> Although Erdogan was the party leader, Abdullah Gul became Prime Minister after the November 2002 election because Erdogan was banned from holding office. Erdogan took over as Prime Minister after winning a by-election in March 2003.



The formal model presented in Schofield (2007) obtains necessary and sufficient conditions for the joint origin  $\mathbf{z}_0 = (0, \dots, 0)$  to be a Nash Equilibrium in the vote maximizing game. In 1999, the FP had the lowest valence, and the Hessian of the FP vote share function at  $\mathbf{z}_0$  can be computed to be

$$C_{FP} = \begin{bmatrix} -0.24 & 0.45 \\ 0.45 & -0.27 \end{bmatrix}.$$

The eigenvalues of the Hessian can be shown to be  $-0.74$ , with eigenvector  $(1, -1.12)$  and  $0.23$ , with the orthogonal eigenvector or principal axis given by the vector  $(1, 0.89)$ . This principal axis is aligned at approximately 45 degrees to the religion axis in Figure 1. Obviously, the origin is a saddlepoint for the FP vote function, and theory requires that this party move up or down the principal axis, away from the origin. Other parties should follow suit. Clearly this theoretical prediction catches the gross configuration of party positions in Figure 1.

In 2002, the lowest valence party is ANAP, and in precisely the same way, the Hessian of the vote share of ANAP at  $\mathbf{z}_0 = (0, \dots, 0)$  can be computed to be

$$C_{ANAP} = \begin{bmatrix} 2.01 & 1.88 \\ 1.88 & 1.93 \end{bmatrix}.$$

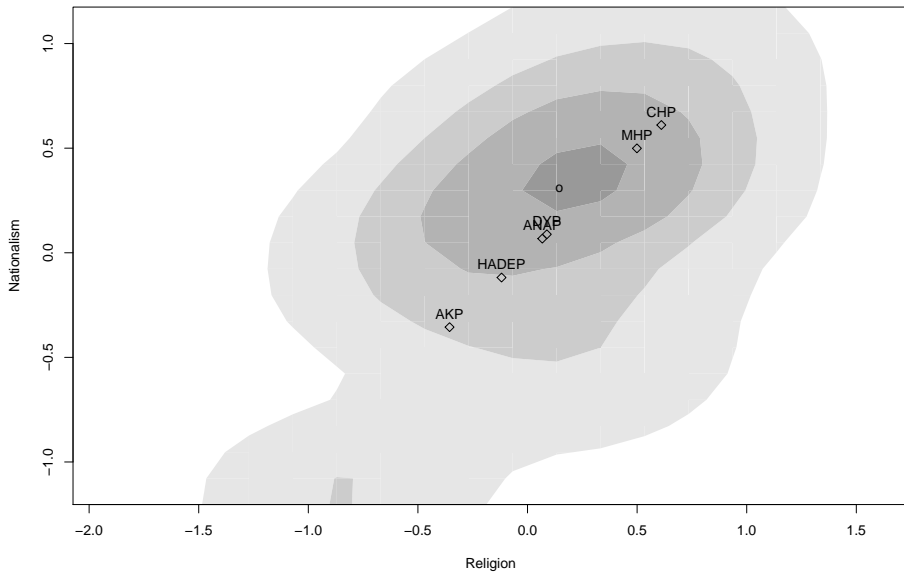
The major eigenvalue for ANAP is 3.85, with eigenvector  $(1.02, 1.0)$  and minor eigenvalue 0.09, with orthogonal eigenvector  $(-1.0, 1.02)$ . In this case  $\mathbf{z}_0$  is a minimum for the ANAP vote function, and we expect all parties to scatter away from the origin. Figure 4 presents an LNE obtained from simulation of the pure spatial model for 2002.

Notice that the estimated position of the CHP (the Republican People's Party) is much further to the left on the Religion axis in both 1999 and 2002, than obtained in the simulated LNE for the pure model. Supporters of the CHP tend to be Alevi, a non-Sunni religious community, who are adherents of Shia Islam rather than Sunni, and may be viewed as activists for "Kemalism" or the secular state. Indeed, in the joint model,  $E(\lambda, \theta, \beta; \Psi)$ , voters who are Alevi have a very high additional valence for the CHP. The FP is also far from the principal axis, to the right on the religion axis in 1999, as expected for a party whose adherents are Sunni. We now introduce the joint model to obtain a better estimate of LNE. We also introduce a more general model based on activists.

## 2.2 Extension of the model for Turkey

In this section we present a model of party activists, and then use the joint sociodemographic model to obtain information about activists. These activist functions  $\{\mu_j : j \in P\}$  are functions of party position, rather than exogenous constants. Schofield (2006a) shows that the first order condition for a local equilibrium in this model is given by the set of gradient balance conditions:

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j) + \frac{1}{2\beta} \frac{d\mu_j}{dz_j}(z_j) = 0$$



**Figure 4.** A Local Nash equilibrium for the pure spatial model for Turkey in 2002

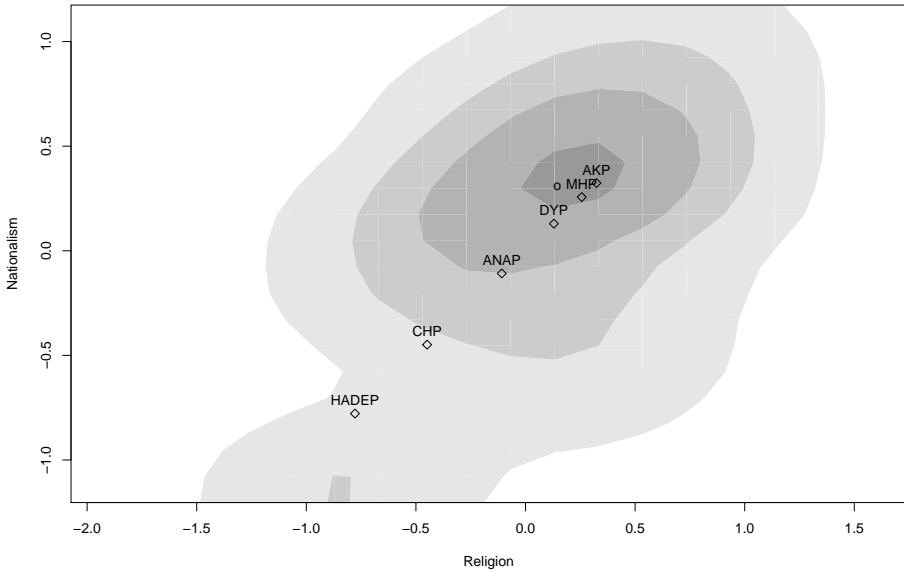
The terms  $\{\frac{d\mu_j}{dz_j}\}$  are the *marginal activist pulls (or gradients)* (giving the marginal activist effects on each party  $j$ ), while the gradient terms  $\{\frac{d\phi_j^*}{dz_j}(z_j) = z_j^{el} - z_j\}$  are the *electoral pulls on the parties*, each one pointing towards the weighted electoral mean,  $z_j^{el}$ , of the party. The weighted electoral mean essentially weights voter policy preferences by the degree to which the group-specific valences influence the choice of the voter. The joint model,  $E(\lambda, \theta, \beta; \Psi)$ , allows us to draw some inferences about equilibrium positions. First we note that the sociodemographic variables imply that we must use the weighted electoral mean, as defined for party  $j$  in Section 4:

$$z_j^{el} \equiv \sum_{i=1}^n \alpha_{ij} x_i,$$

$$\text{where } \alpha_{ij} = \frac{\rho_{ij} - \rho_{ij}^2}{\sum_{k \in N} (\rho_{kj} - \rho_{kj}^2)}.$$

Figure 5 gives one of the local Nash equilibrium, obtained by simulation of the joint model with group specific valences. In this model the activist functions are not included, so  $\frac{d\mu_j}{dz_j}(z_j) = 0$ . This equilibrium vector gives estimates of  $\{z_j^{el}\}$  and the simulation allows us to infer that:

$$\mathbf{z}^{el} = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x\text{-axis} & -0.5 & 0.2 & 0.1 & -0.7 & -0.1 & 0.4 \\ y\text{-axis} & -0.5 & 0.2 & 0.1 & -0.7 & -0.1 & 0.4 \end{bmatrix}$$



**Figure 5.** A LNE for the joint model for Turkey in 2002

Notice that the party positions in this LNE are much closer to the estimated positions of the parties. Moreover, they also lie on the principal component given by an eigenvector (1.0, 1.0), which is almost identical to the eigenvector for the LNE obtained for the pure spatial model. The estimated positions of the parties in Figure 3 are:

$$\mathbf{z}^* = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x\text{-axis} & -2.0 & 0.0 & 0.0 & -2.0 & -0.2 & 1.0 \\ y\text{-axis} & 0.1 & 1.5 & 0.5 & -1.5 & -0.1 & 0.1 \end{bmatrix}$$

That is, assuming that this vector is in equilibrium with respect to the full model involving activists, then we can identify this vector as  $\mathbf{z}^*$ . Then, as in Section 4,

$$\mathbf{z}^* - \mathbf{z}^{el} = \frac{1}{2\beta} \left[ \frac{d\mu_1}{dz_1}, \dots, \frac{d\mu_p}{dz_p} \right].$$

Assuming that the joint model with activists is valid, then the difference between these two vectors gives us an estimate of the vector of marginal pulls on the parties:

$$\mathbf{z}^* - \mathbf{z}^{el} = \begin{bmatrix} \text{Party} & \text{CHP} & \text{MHP} & \text{DYP} & \text{HADEP} & \text{ANAP} & \text{AKP} \\ x\text{-axis} & -1.5 & -0.2 & -0.1 & -1.3 & -0.1 & 0.6 \\ y\text{-axis} & 1.5 & 1.3 & 0.4 & -0.8 & -0.1 & -0.3 \end{bmatrix}$$

From the joint model, the group specific valences for HADEP (or DEHAP) by Kurdish voters were very high:

$$(\theta_{\text{HADEP}} \cdot \eta_{\text{Kurd}}) = 5.9 \text{ in 1999}$$

$$(\theta_{\text{DEHAP}} \cdot \eta_{\text{Kurd}}) = 6.0 \text{ in 2002}$$

The joint electoral model would predict that the party would move close to Kurdish voters. Kurdish voters will tend to be located on the left of the religion axis, and are also anti-nationalistic. We can assert that, with very high probability, the group valence effects will be significant for HADEP in the two elections. We can further infer that the estimated activist pull on HADEP is very high, pulling the party to the left on the religion axis, and in an anti-nationalist direction on the  $y$  axis.

Similarly, the estimated activist pull on the CHP is extremely high, and we can infer that this is due to the influence of Alevi voters. The Alevis are a non-Sunni religious community, who are adherents of Shia Islam rather than Sunni, and may be viewed as supporters of “Kemalism” or the secular state. Another way of expressing this is that Alevi voters have very high group specific valence for the CHP, with

$$\begin{aligned}(\theta_{CHP} \cdot \eta_{Alevi}) &= 3.1 \text{ in } 1999, \\(\theta_{CHP} \cdot \eta_{Alevi}) &= 2.6 \text{ in } 2002.\end{aligned}$$

As a consequence, the CHP will move to a vote maximizing position, on the left of the religious axis, as in Figure 5, which allows it to take advantage of this support. This asymmetry will cause Alevi activists to provide further differential support for the CHP. It is thus plausible that secular voters (on the left of the religious axis in Figures 1 and 3) would offer further support to the CHP, located close to them. This would affect the party’s marginal activist pull, and induce the CHP leader to move, in equilibrium, even further left.

A different argument holds for the AKP in 2002. The relative valence  $\lambda_{AKP} = 1.97$ , for the joint model is large and significant, so the weighted electoral mean,  $z_{AKP}^{el}$  lies on the principal eigenvector, while activist support for the AKP would move it to the right on the religion axis, as well as in an anti-nationalism direction.

In contrast, if the military provides activist support for the MHP on the nationalism axis, then this party will move left in a secular direction, and north on the nationalism axis. Overall, we note that we can expect activist valence to strongly influence party positioning, and we can proxy this support to some degree using the sociodemographic variables.

In the 2007 election, the members of Kurdish Party (now called the Freedom and Solidarity Party, DTP) contested the election as independents, and thus were not subject to the 10 percent cut-off, and so were able to win 24 seats. The AKP took 46.6 percent of the vote, reflecting the continuing high valence of Erdogan. On August 29, 2007, Abdullah Gul, Erdogan’s ally in the AKP, was elected president of Turkey. The tensions between the DTP and the authorities increased on 18 December 2007 when the Turkish military arrested Nurettin Demirtas, the leader of the DTP, alleging that he had forged documents to avoid military service.<sup>6</sup>

<sup>6</sup> It was also alleged that there were links between Demirtas and the outlawed Kurdish Workers’ Party, the PKK.

**Table 3.** Turkish election results 2007

Party Name		% Vote	Seats	% Seats
Justice and Development Party	AKP	46.6	340	61.8
Republican People's Party	CHP	20.9	112	20.3
Nationalist Movement Party	MHP	14.3	71	12.9
Democrat Party	DP	5.4		
Young Party	GP	3.0		
Felicity Party	SP	2.3		
Independents		5.2	27 <sup>7</sup>	4.9
Others		2.3		
Total		100	550	100

### 3. Typologies of “multiparty” polities

The previous examples suggest that parties do not appear to adopt Nash equilibrium positions based on a simple vote maximizing game. The next section considers a more general electoral model, where each party is dependent on activist support. In this model parties gain support from activists, as long as the party position is chosen in response to activist demands. We can interpret this to mean that the party implicitly has policy preferences. However, since there may well be many potential activist groups in a polity, we may expect a number of parties to respond to activist demands. Below, we shall discuss the simpler case of plurality rule, as in the US, where there will tend to be two major parties. In polities using electoral systems based on proportional representation (PR) there appears to be no rationale forcing activist groups to coalesce. In the following discussion of legislative politics we shall use estimates of party positions, and examine the nature of the core, or heart, under the assumption that the party positions are chosen to maximise vote share, as in the above example from Turkey. If the reasoning presented in the previous section is valid, then we should expect minority governments in situations where there is a core party.

#### 3.1 Left unipolar systems — Denmark, Sweden and Norway

The empirical analysis of Laver and Schofield (1990) suggests that the frequent minority governments in the period 1945–1987 in Denmark, Sweden and Norway were based on core parties on the left of the policy space.

*Denmark.* The political system has a high degree of fragmentation (the effective number<sup>8</sup> increased from about 3.8 in the late 1940s to 7.0 in 1970). The largest party is the Social Democrat Party (SD) with 30–40 percent of the seats, and the Liberals (or Venstre, V) with 20 to 30 percent. The SD is the only dominant party. The SD was

<sup>7</sup> Twenty-four of these “independents” were in fact members of the DTP — the Kurdish Freedom and Solidarity Party.

<sup>8</sup> The effective number is the inverse of the Herfindahl measure of concentration. This measure is obtained by summing the squares of the seat proportions.

in 13 out of 21 governments in the period 1945–1987, while Venstre was a member of the remaining governments.

Governments without a clear majority are typical in Denmark, though tacit support is often provided by small parties. The pattern that emerges is one of SD minority governments with support of the radical liberals (RV), Socialist People's Party (SF) or Communist Party (DKP) alternating with governments consisting of the Venstre and the Conservative People's Party (KF).

For example, Table 4 gives the election results for 1957 and 1964. Because the parties on the right controlled more than a majority of the seats in 1957, we can infer that the core is empty. In 1964, the right coalition gained only 84 seats, and the core SD formed a minority government.

**Table 4.** Elections in Denmark, 1957 and 1964

Party	Seats	
	1957	1964
Communists	DKP	6
Socialist People's Party	SF	10
Social Democrats	SD	70
Radical Venstre	RV	14
Venstre or Liberals	V	45
Conservative Party	KF	30
Justice Party	RF	9
Others		1
Total		175
Actual governments:	1957 to 1960: {SD, RV, RF}	
	1960 to 1964: {SD, RV}	
	1964 to 1968: SD minority	

Note however that the Danish system became more fragmented, so that the possibility of a core declined. Figure 6 gives the estimates of positions in 2001, including those of new parties: the Center Democrats (CD), located very close to the SD position, but not marked, the Christian Peoples Party (KrF), Danish People's Party (DF) and the Red-Greens or Enhedslisten (Enh). The figure shows the median lines. The heart is the star shaped set given in the figure, generated by the SD, DF, KF and V positions.

In the election 2001, the effective number was over 6.5, and a coalition of {V, KF} formed, controlling 72 seats, out of 179. This coalition gained 70 seats in 2005, and stayed in power. It would seem that the major party positions may have changed very little over time, but there is a clear indication of an increase in fragmentation.

*Sweden.* The dominance of the Social Democratic Party (SAP) in Sweden was quite pronounced, since it typically obtained just less than 50 percent of the vote, until 1970. This implied that the only coalition excluding the SAP was a counter coalition of four other parties on the right, making the SAP a natural core party.

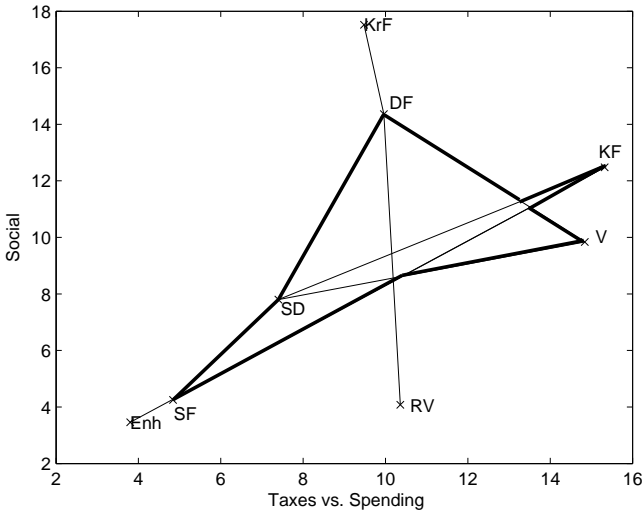


Figure 6. The heart in Denmark in 2001

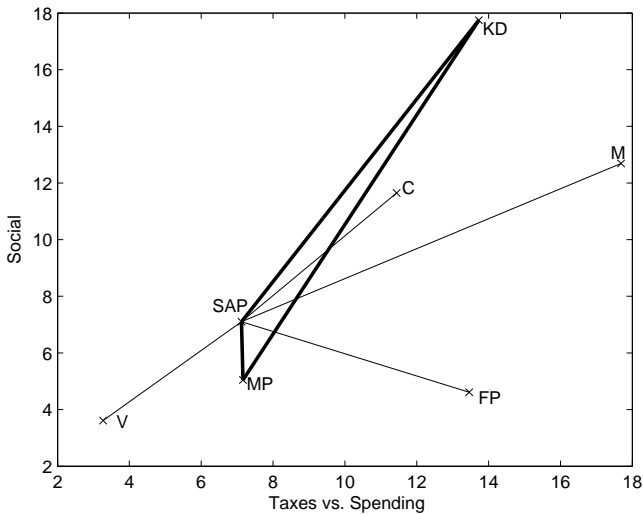


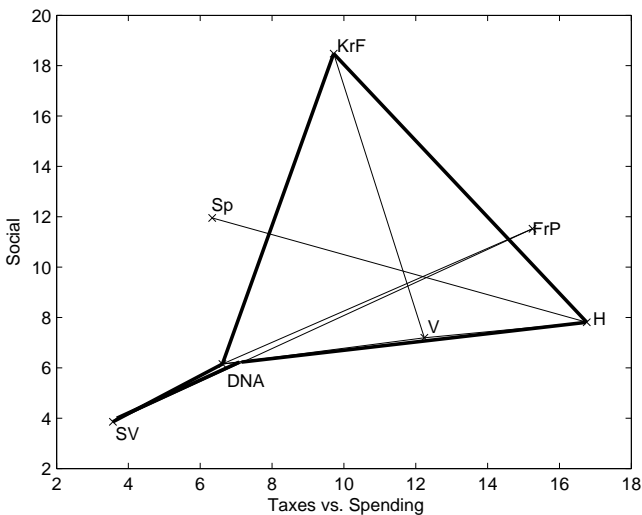
Figure 7. The heart in Sweden in 2002

In contrast, Figure 7 shows the political configuration in 2002. The heart is a triangle bounded by the positions of the Christian Democrats (KD, with 33 seats, out of 349), the SAP (with 144) and the Green Party (MP, with 17 seats). The parties outside the heart are the Center Party (C, with 22 seats), the Moderate Party (M, with 55 seats), the Liberal People's Party (FP, with 48 seats) and the Left Party (V for Vansterpartiet,

with 30 seats). Thus, in 2002, the SAP, the Left Party and the Greens together took 53% of the vote and 191 seats out of 349. In the 2006 election, the four parties of the right (KD, M, FP and C) formed a pre-election coalition, gained 48% of the vote and 178 seats, and were able to form the government.

*Norway.* The Labor Party (Det Norske Arbeiderpartie or DNA) occupies a position similar to that of the SAP in Sweden. Indeed the DNA has often been the strongly dominant party. Until 1961 it controlled a majority of the seats. The Socialist Left Party (SV) took only 2 seats (out of 155) in 1977 but jumped to 17 seats in 1989, and in the recent election in September 2005 took 15 out of 169. After the election of 1981, the three parties on the right (Center Party, Sp; Christian People’s Party, KrP; and Conservatives, H) controlled a majority. From 1989, the radical right wing populist Progress Party (FrP), founded by Anders Lange, grew rapidly, gaining 38 seats in 2005. After the 1989 and 1993 elections the DNA was essentially at the core position with a plurality of the vote and was able to form a minority government.

In 1997 however, the DNA lost a couple of seats, and the DNA leader, Jagland, stepped down, leading to the formation of a minority right wing coalition, led by Bondevik of the KrP, together with the Center Party. The unwillingness of the three right wing parties to form a coalition with the FrP led to the minority right wing coalition from 1997 to 2005. In the 2005 election, the Center Party switched, forming a Red-Green coalition with the DNA and the SV. This alliance took 87 seats out of 169, and was able to form the first majority coalition in Norway since 1985. (See Strom 1991 for an earlier discussion of minority coalitions in Norway.) Note however, that if the parties on the right could agree to form a coalition with the Progress Party, then the heart is the set bounded by the positions of the DNA, the Sp and the Liberals (V), making the Sp a pivot party between coalitions of the left and right. See Figure 8.



**Figure 8.** The heart in Norway in 2001



### 3.2 Center unipolar systems — Belgium, Luxembourg and Ireland

*Belgium.* Belgium is an interesting example with respect to the theoretical prediction about the core. In the period up to the late 1960s, the political configuration based on three parties meant that the core was empty and minimal winning coalition governments the rule. However after 1970, increasing political fragmentation resulting from conflicts over language and regional autonomy led to the replacement of the three party system with a multiparty system generated by the federalist-unitary dimension. The entrance of new parties, including the nationalist *Voksunie* (VU) in 1954, the *Rassemblement Wallon* (RW) and the *Francophone Democratic Front* (FDF), increased the effective seat number (to 6.0 by 1971). The centrist *Christelijke Volkspartij* (CV) was almost continually in power until the election of 1999, when it lost its plurality status, gaining only 22 seats out of 150, in comparison to the 23 seats of the *Flemish Liberal and Democrat Party* (VLD). In 1999, a coalition of six parties with 94 seats formed the government: VLD, the two wings of the *Socialist Party* (*Parti Socialiste*, or PS and the *Socialistische Partij*, SP, with 33 seats between them), the *Free Democrat Party* (FDF) with 18 seats, and 20 seats from two other small, green parties (*Ecolo*, EC, and *Agalev*, AG). Figure 9 shows the party positions, on the assumption that the two socialist parties (PS and SP) were at the same position. The heart illustrates the various coalition possibilities. The *Volksunie* had split into a nationalist wing (VU&ID) and a more federalist component, the *Flemish Block* (VB). These parties, together with the *National Front* (FN) are shown to be positioned outside the heart.

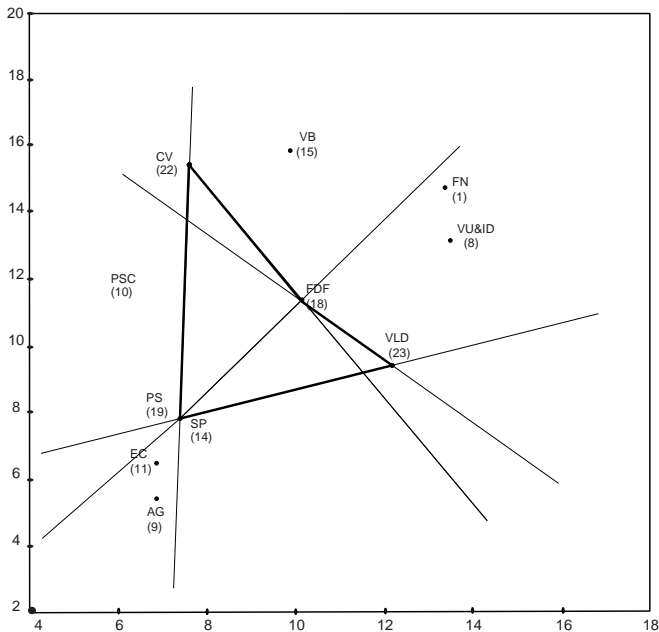
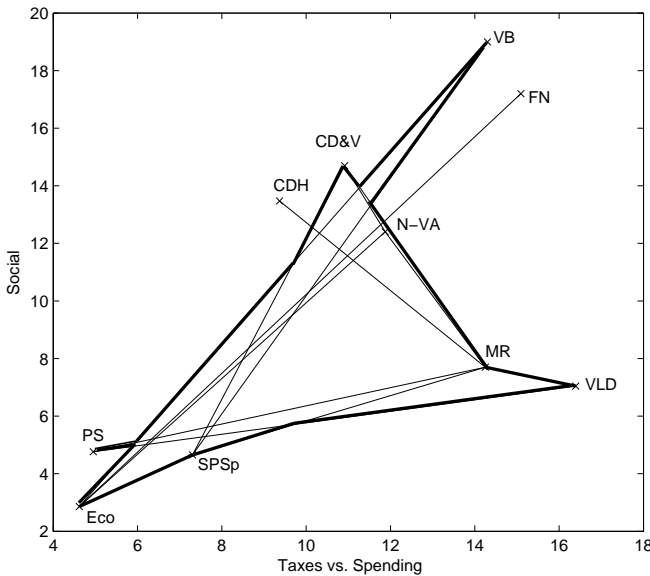


Figure 9. The heart in Belgium in 1999

In 2003, the CV renamed itself the Christian Democratic and Flemish Party (CD&V) and won 21 seats while the FDF was renamed the Reformist Movement (MR) and won 24 seats. The green parties only won four seats. The other small parties were the New Flemish Alliance (N-VA) with 1 seat and the Humanistic Democratic Center (CDH) with 8. The Francophone Socialist Party (PS) won 25 seats while the Flemish Socialist Party (SP) formed an alliance with Spirit (Sp), a small offshoot of the VU, and this cartel won 23 seats. Assuming that the two parties, PS and SPSp, were at distinct positions gives the heart as shown in Figure 10. This illustrates the more complex coalition possibilities as a result of the increasing fragmentation that occurred between 1999 and 2003. The effective number increased from 7.0 to 8.0 between these elections. Guy Verhofstadt of the VLD became prime minister in 2003.



**Figure 10.** The heart in Belgium in 2003

In the election of 10 June 2007, the CD&V went from 21 seats to 30 (out of 150), becoming the largest party in the Chamber of Representatives. After a month of negotiation, King Albert II asked the leader of the CD&V, Yves Leterme, to be *formateur* of a coalition government. Leterme found this impossible, and resigned from the task on 23 August. Belgium was without an effective government for a record six months. On December 23, 2007, the VLD under Guy Verhofstadt formed an “interim government” and won a vote of confidence in parliament, with 97 votes in favor, 46 opposed, and one abstention, thus assuring it legitimacy for three months. Finally, on March 20, 2008, Yves Leterme was sworn in as prime minister, backed by a five party coalition.

*Luxembourg.* The largest party is the Christian Social Party (CSV) with about one third of the seats, followed by the Luxembourg Socialist Workers’ Party (LSAP) with

between one quarter and one third of the seats. The smaller Democratic Party (DP) generally gains just less than one fifth of the seats. The heart is clearly based on the triad of the positions {LSAP, CVP, DP}, and governments tend to be associated with pairwise minimal winning coalitions: {LSAP, DP} in 1974–1979, {CVP, DP} in 1979–1984 and 1999–2004, and {LSAP,CVP} after the election of 2004.

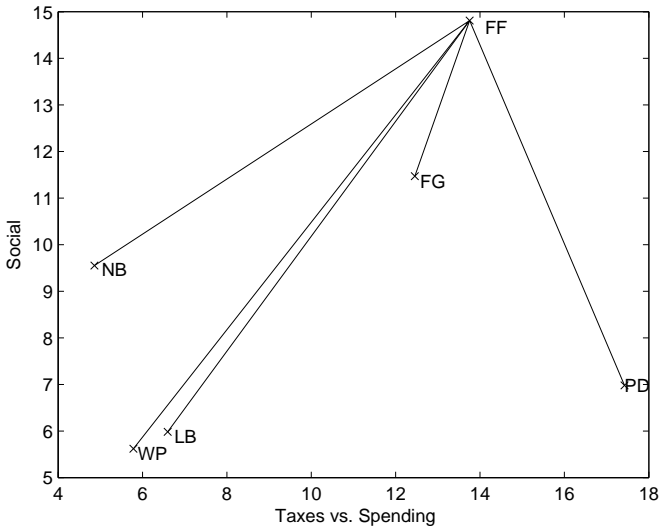
*Ireland.* Ireland is especially interesting because it has a dominant center party (Fianna Fail) and unlike Belgium or Luxembourg, there have been a number of minority (Fianna Fail) governments. To see the complexity of the bargaining possibilities, consider Table 5 which lists the seat strengths after February 1987 in the Dail Eireann.

**Table 5.** Party and faction strengths in the Dáil Eireann, 1987

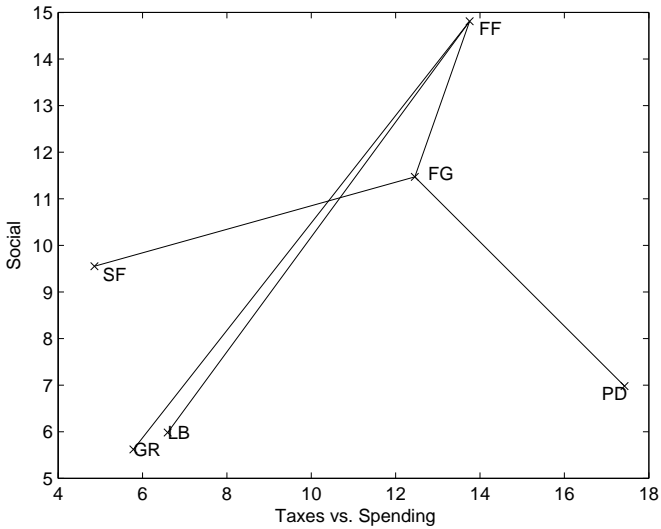
<i>Left</i>	
Workers' Party (WP)	4
Democratic Socialist Party	1
Labor (LB)	12
Tony Gregory (Left wing Independent)	1
Sean Treacy (Ex-Labor Independent)	1
Ceann Comhairle: Neil Blaney (Independent, NB)	1
<i>Center</i>	
Fine Gael (FG)	51
Fianna Fáil (FF)	81
Progressive Democrats (PD)	14
Total	166

A coalition of Fine Gael and Labor had collapsed in January 1987, and Garret Fitzgerald remained Toaiseach, leading a caretaker minority Fine Gael government. Clearly the natural minimal winning coalitions were {Fianna Fáil, Progressive Democrats} with 94 seats, {Fianna Fáil, Fine Gael}, {Fianna Fáil, Labor} with 93, and an unlikely coalition of Fianna Fáil with the far left parties. Figure 11 indicates the median lines based on estimates of the party positions and that of the independent, Neil Blaney at NB. We may infer that Fianna Fáil was indeed a core party, suggesting a minority government. This is precisely what occurred. Sean Treacy became Ceann Comhairle (Chairman) of the Dail. Tony Gregory abstained and Haughey (with Neil Blaney) had 82 votes out of 164, with Treacy casting the deciding vote for the government.

After the 2002 election, Fianna Fáil obtained 80 seats, out of 166, while the other party strengths were: Fine Gael (FG,31), Labor (LB, 21), Progressive Democrats (PD, 8), Greens (GR, 6), Sinn Féin (SF, 5), with 15 seats belonging to other independents and factions. Bertie Ahern, leader of Fianna Fáil formed a coalition with the Progressive Democrats, controlling 88 seats, sufficient for a majority. In the May 24, 2007, election, Fianna Fáil won 78 seats, while the Progressive Democrats only won 2 seats, not enough to form a majority coalition. Fine Gael increased its strength to 51, while Labor dropped to 20. Only 5 seats went to independents, while the Greens won 6 seats



**Figure 11.** The heart in Ireland in 1987



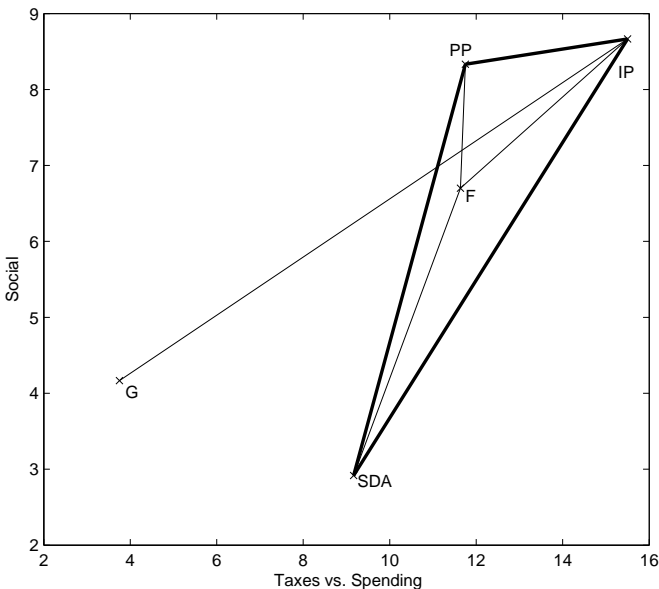
**Figure 12.** The heart in Ireland in 2007

and Sinn Féin won 4. Figure 12 suggests the nature of the heart. The medians through the FG position are based on the assumption that 1 seat will be taken by the Ceann Comhairle and the independents' positions are between FG and LB. Enda Kenny, the

leader of Fine Gael, initially refused to concede defeat, as it was theoretically possible for him to put together a majority anti-Fianna Fáil coalition, but this would require the support of either Sinn Féin or all four independents. Kenny's ambition was squashed by the formation of a coalition government on June 14, led by Ahern, involving Fianna Fáil, the Greens and the Progressive Democrats, controlling 86 seats. The Greens bargained for specific policy objectives and cabinet positions. The coalition, together with four of the independents, elected John O Donoghue as Ceann Comhairle by 90 to 75. A corruption scandal forced Ahern to announce his resignation as from May 6.

### 3.3 A right unipolar system

*Iceland.* To some extent Iceland is a mirror image of the three Scandinavian political systems. In the 2003 election, the largest party was the right-wing Independence Party (IP) which took 22 seats out of 63. At the center are two parties: the Progressive Party (PP) with 12 seats in 2003 and a small Liberal Party (F) with 4 seats. On the left was the Social Democratic Alliance (SDA) with 20 seats, and the Left-Green Movement (G) with 4. The heart is given by the triad of positions {SDA, PP, IP} indicating the likelihood of minimal winning coalitions. David Oddsson, the leader of IP, had served as Prime Minister from 1991 to 1995, in alliance with SDA, and then from 1995 to 2004 in alliance with the PP. Oddsson was succeeded in September 2004 by Halldor Asgrimsson of the PP. A coalition government of the IP, under Geir Haarde, with the PP formed in June 2006. In the election of May 13, 2007, the Independence



**Figure 13.** The heart in Iceland in 2003

Party picked up two additional seats for a total of 24, while the Progressive Party went from 12 seats to 8, leading to an IP-SDA coalition government under Haarde. This collapsed dramatically on January 26, 2009, as a result of the financial collapse threatening the country. Johanna Sigurdardottir became prime minister, leading a caretaker coalition government of the Social Democrats and the Left-Green Movement for the three months until the new election on April 25, 2009. At that election, the SDA won 20 seats to 14 for the Left-Greens, while the IP was reduced to 16 seats, consolidating Sigurdardottir's position as prime minister, and her call for Iceland to enter the European Union.

Figure 13 gives an estimate of the heart in 2003. The increase in the size of the Left-Greens created a new median line between SDA and G, so the heart in 2009 is the triangle [SDA, G, IP].

### 3.4 Triadic systems

*Austria.* In Austria the large parties are the Social Democrat Party (SPO) and People's Party (OVP). Until 1959 the Communists (KPO) had roughly four seats, while the Freedom Party (FPO, but called the League of Independents before 1956), generally won between six and eleven seats up until 1979. The OVP won majorities in 1945 (with 85 seats) and in 1966 (with 84 seats). The SPO, under Bruno Kreisky, gained majorities in the elections of 1971, 1975 and 1975, and between 1983 to 1986 formed a coalition with the FPO. From 1986 until 1999 the grand SPO-OVP coalition governed. From 1995 to 1999, partly under the leadership of Jörg Haider, the FPO increased in strength from 41 to 52 seats, making it an obvious coalition partner for the OVP (also

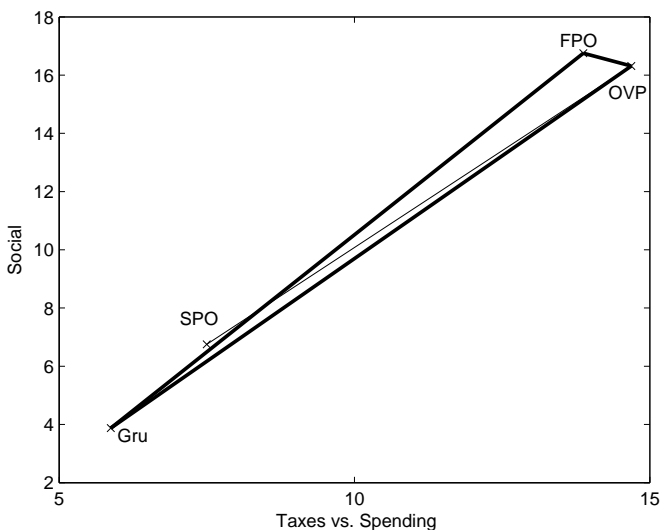


Figure 14. The heart in Austria in 2006

with 52 seats out of 183). Surprisingly, the FPO gained a slightly larger proportion of the vote than the OVP. Various controversies over the FPO leadership lead to a new election in 2002. Haider had resigned the leadership of the FPO in 2000, and, in the 2002 election, the FPO strength fell to 18 seats, while the OVP jumped to 79 seats. For the first time since 1966, the OVP gained a higher proportion of the vote than the SPO (presumably because of the collapse of the FPO). In 2005, Haider formed a new party, the “Alliance for the future of Austria”, BZO), which only gained 7 seats in the 2006 election. The OVP, with 66 seats, then formed a coalition with the FPO (with its 21 seats), against the SPO, with its 68 seats and the Greens (Gru) with its 21 seats. Figure 14 shows the heart for the election of 2006. Assuming that the BZO is located at the FPO position, the heart is based on the triad SPO/Gru, OVP, FPO}.

In the election of September 2008, the far-right parties gained substantially, presumably because of anti-immigration sentiments. The Freedom Party, led by Heinz-Christian Strache, won 18 percent of the vote, a gain of 7 percentage points over 2006, while the BZO, still led by Jörg Haider, got 11 percent, nearly tripling its result of 2006.

*Germany.* Figure 15 shows the heart for the election of 2002 in Germany, where the Christian Democrats (CDU/CSU) gained 248 seats, the Social Democrat Party (SPD) gained 251 seats, and the Free Democrat Party (FDP) gained 47 seats. The Greens (GRU) with 55 seats formed a minimal winning coalition with the SPD until the September 2005 election. As the figure indicates, the {SPD, GRU} median line is one of the boundaries of the heart, and so this coalition is a natural one to form. After the September 2005 election, however, the Greens gained 51 seats against 61 for the

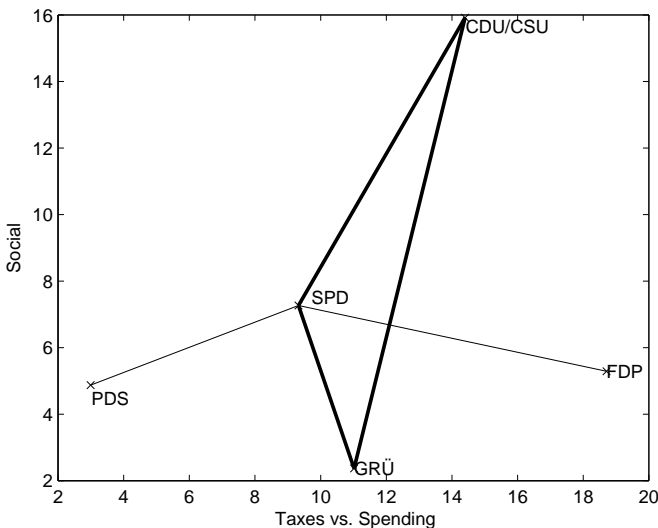


Figure 15. The heart in Germany in 2002

FDP and 54 for the Party of Democratic Socialism (PDS). Since the CDU only gained 225 seats in contrast to 222 for the SPD there was an impasse. The coalition {PDS, GRU, SPD} is now possible, causing a contraction of the heart. Eventually Merkel, of the CDU, became Chancellor, leading the grand CDU/CSU-SPD coalition. In the election of 27 September, 2009, the CDU/CSU won 239 seats in all, and their new coalition allies, the FDP, won 93, giving a majority of 332 out of 622. The SPD fell to 146 seats, with the Left winning 76, and the Greens with 68.

### 3.5 A collapsed core

*Italy.* Italy needs a category of its own, as it was originally a center unipolar system, where the dominant party, the Christian Democrat Party (DC) was in a uniquely powerful position until the 1994 election. The DC went from 206 seats (out of 630) in 1992 to 33 in 1994. Until 1987 the DC controlled about 40 percent of the seats, with the Communist Party (PCI) and Socialists (PSI) controlling less than 30 percent each. The smaller parties include the Social Democrats (PSDI), Republicans (PRI), Liberals (PLI), Monarchists (PDIUM), and Neofascists (MSI). Aside from the first two governments in 1946 and 1947, the Communists never belonged to a coalition government. The DC was strongly dominant, and the only party able to position itself at a structurally stable core in a two-dimensional policy space, as indicated in Figure 16 (based on Giannetti and Sened, 2004).

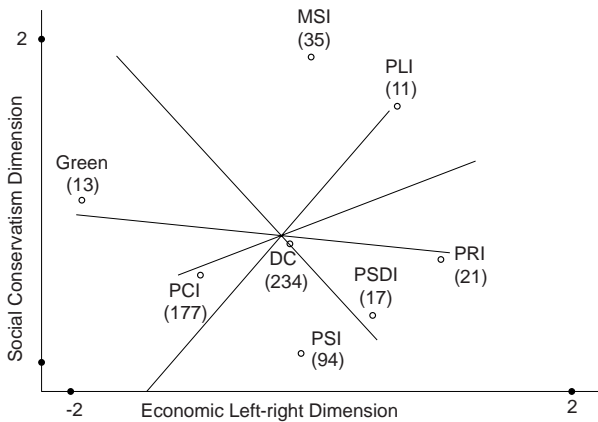
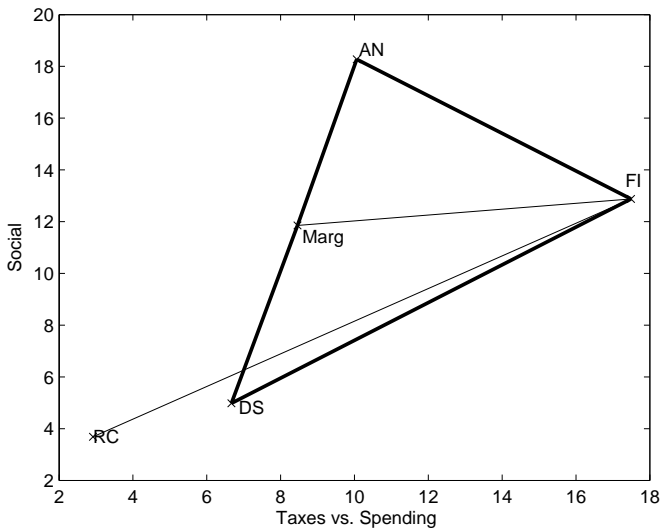


Figure 16. The core in Italy in 1987

The persistence of the Pentapartito coalition (1979–1989) comprising a coalition of DC, PSI, PRI, PLI and the PSDI is further evidence that the core was non-empty. To control the distribution of government perquisites, the DC maintained a grand, anti-PCI coalition. Schofield (1993) suggested that corruption associated with these perquisites eventually led to an anti-DC coalition based on new parties such as the Northern League and the Greens. Mershon (2002), Giannetti and Sened (2004) and



Schofield and Sened (2006) discuss the dramatic changes in Italian politics that occurred in the period 1992–1996. Figure 17 indicates the quite new Italian configuration based on the positions of the parties in 2001: the Alleanza Nazionale (AN, 24 seats), Democratici di Sinistra (DS, 31 seats), Forza Italia (FI, 62 seats), La Margherita (Marg, 27 seats), and Rifondazione Comunista (RC, 11 seats).



**Figure 17.** The heart in Italy in 2001

Since then there have been oscillations between left and right, the most recent being Berlusconi's election success on April 14, 2008.

#### 4. A spatial model of elections

The model of coalition bargaining, discussed in the previous sections, suggest that even when there is no majority party then a large, centrally located party, at a “core” position in the policy space, will be dominant. Such a core party can, if it chooses, form a minority government by itself and control policy outcomes (see Schofield, Grofman and Feld 1989; Laver and Schofield 1990, 1998; Banks and Duggan 2000; Schofield and Sened 2006). If party leaders are aware of the fact that they can control policy from the core, then this centripetal tendency should lead parties to position themselves at the center. Moreover, the “mean voter theorem,” based on a stochastic model of election and on vote maximization, suggests that the electoral origin will be a Nash equilibrium (Adams 1999a,b, 2001; Adams and Merrill 1999; Lin, Enelow and Dorussen 1999; Banks and Duggan 2005; McKelvey and Patty 2006). These two very different models of political strategy suggest that parties will tend to locate themselves at the electoral center.

Yet, contrary to this intuition, there is ample empirical evidence that party leaders do not necessarily adopt centrist positions. The previous sections present evidence to this effect.

Section 2 of this paper briefly considered a formal stochastic model developed in Schofield (2007) that is based on the valence of the party. Valences are party biases, derived from voters' judgements about characteristics of the candidates, or party leaders, which cannot be ascribed to the policy choice of the party. One may conceive of the valence that a voter ascribes to a party leader as a judgement of the leader's quality or competence.

This section considers a more general valence model (Schofield 2006a) based on activist support for the parties (Aldrich 1983a,b; Aldrich and McGinnis 1989; Aldrich 1995; Stokes 1992). This activist valence model presupposes that party activists donate time and other resources to their party. Such resources allow a party to present itself more effectively to the electorate, thus increasing its valence. Since activists tend to be more radical than the average voter, parties are faced with a dilemma. By accommodating the political demands of activists, a party gains resources that it can use to enhance its valence, but by adopting the radical policies demanded by activists, the party may appear too extreme and lose electoral support. The party must therefore balance the electoral effect against the activist valence effect. The result gives this a first order balance condition between electoral and activist support. Since valence in this model is affected by activist support, it may exhibit "decreasing returns to scale" and this may induce concavity in the vote share functions of the parties. Consequently, when the concavity of activists' valence is sufficiently pronounced then a pure strategy Nash equilibrium (PNE) of the vote maximizing game will exist. The result indicates that there is no reason for this equilibrium to be one where all parties adopt centrist positions.

Throughout it is assumed that the stochastic errors have the Type I extreme value (or Gumbel) distribution,  $\Psi$ . The formal model based on  $\Psi$  parallels the empirical models based on multinomial logit (MNL) estimation (Dow and Endersby 2004).

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of party declarations, or manifestos, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that party leaders cannot predict vote response precisely, but can estimate an expected vote share.

**Definition 1.** The Stochastic Vote Model  $E(\lambda, \mu, \beta; \Psi)$  with Activist Valence:

The data of the spatial model is a distribution,  $\{x_i \in X : i \in N\}$ , of voter ideal points for the members of the electorate,  $N$ , of size  $n$ . We assume that  $X$  is an open, convex subset of Euclidean space,  $\mathbb{R}^w$ , with  $w$  finite. Each of the parties in the set  $P = \{1, \dots, j, \dots, p\}$  chooses a policy,  $z_j \in X$ , to declare. Let  $\mathbf{z} = (z_1, \dots, z_p) \in X^p$  be a typical vector of party policy positions.

Given  $\mathbf{z}$ , each voter,  $i$ , is described by a vector

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \dots, u_{ip}(x_i, z_p)),$$

where  $u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta \|x_i - z_j\|^2 + \varepsilon_j = u_{ij}^*(x_i, z_j) + \varepsilon_j$ .

Here  $u_{ij}^*(x_i, z_j)$  is the observable component of utility. The term,  $\lambda_j$ , is the fixed or *intrinsic valence* of agent  $j$ , while the function  $\mu_j(z_j)$  is the component of valence generated by activist contributions to agent  $j$ . The term  $\beta$  is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of the Euclidean norm,  $\|\cdot\|$ , on  $X$ . The vector  $\varepsilon = (\varepsilon_1, \dots, \varepsilon_j, \dots, \varepsilon_p)$  is the stochastic error, whose multivariate cumulative distribution,  $\Psi$ , is the Type I extreme value distribution with the closed form

$$\Psi(x) = \exp[-\exp[-x]].$$

Voter behavior is modeled by a probability vector. The probability that a voter  $i$  chooses party  $j$  at the vector  $\mathbf{z}$  is

$$\begin{aligned} \rho_{ij}(\mathbf{z}) &= \Pr[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l), \text{ for all } l \neq j] \\ &= \Pr[\varepsilon_l - \varepsilon_j < u_{ij}^*(x_i, z_j) - u_{il}^*(x_i, z_j), \text{ for all } l \neq j]. \end{aligned}$$

Here  $\Pr$  stands for the probability operator generated by the distribution assumption on  $\varepsilon$ .

The *expected vote share* of agent  $j$  is

$$V_j(\mathbf{z}) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(\mathbf{z}).$$

The differentiable function  $V : X^P \rightarrow \mathbb{R}^P$  is called the *party profile function*.

A strategy vector  $\mathbf{z}^* = (z_1^*, \dots, z_{j-1}^*, z_j^*, z_{j+1}^*, \dots, z_p^*) \in X^P$  is a *local Nash equilibrium* (LNE) for the profile function  $V : X^P \rightarrow \mathbb{R}^P$  iff, for each party  $j \in P$ ,  $V_j(z_1^*, \dots, z_{j-1}^*, -, z_{j+1}^*, \dots, z_p^*)$  is locally maximized at  $z_j^*$ .

Schofield (2006a) shows that the first order condition for  $\mathbf{z}^*$  to be a LNE is that it be a *balance solution*.

**Definition 2.** The balance solution for the model  $E(\lambda, \mu, \beta; \Psi)$ :

Let  $\rho_{ij}(\mathbf{z}) = \rho_{ij}$  be the matrix of voter probabilities at the vector  $\mathbf{z}$  and let

$$\alpha_{ij} = \frac{\rho_{ij} - \rho_{ij}^2}{\sum_k^n (\rho_{kj} - \rho_{kj}^2)}$$

be the  $p$  by  $n$  matrix of coefficients. The *balance equation* for  $z_j^*$  is given by expression

$$z_j^* = \frac{1}{2\beta} \frac{d\mu_j}{dz_j} + \sum_{i=1}^n \alpha_{ij} x_i.$$

The vector  $\sum_i \alpha_{ij} x_i$  is called the *weighted electoral mean* for party  $j$ , and can be written

$$z_j^{el} = \sum_{i=1}^n \alpha_{ij} x_i.$$

The term

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j) = z_j^{el} - z_j$$

is called the *marginal electoral pull of party j* at  $z_j$  and is a gradient vector pointing towards the weighted electoral mean. This weighted electoral mean is that point where the electoral pull is zero. Then  $z_j^*$  solves the balance equation if

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j^*) + \frac{1}{2\beta} \frac{d\mu_j}{dz_j} = 0,$$

where the vector  $\frac{d\mu_j}{dz_j}$  is called *the marginal activist pull for party j*.

If  $\mathbf{z}^* = (z_1^*, \dots, z_{j-1}^*, z_j^*, z_{j+1}^*, \dots, z_p^*)$  has the property that each  $z_j^*$  satisfies the balance equation then call  $\mathbf{z}^*$  the *balance solution*.

**Theorem 1.** (Schofield 2006a) *Consider the electoral model  $E(\lambda, \mu, \beta; \Psi)$  based on the Type I extreme value distribution, and including both intrinsic and activist valences. The first order condition for  $\mathbf{z}^*$  to be an LNE is that it is a balance solution. If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then the balance solution will be a PNE.*

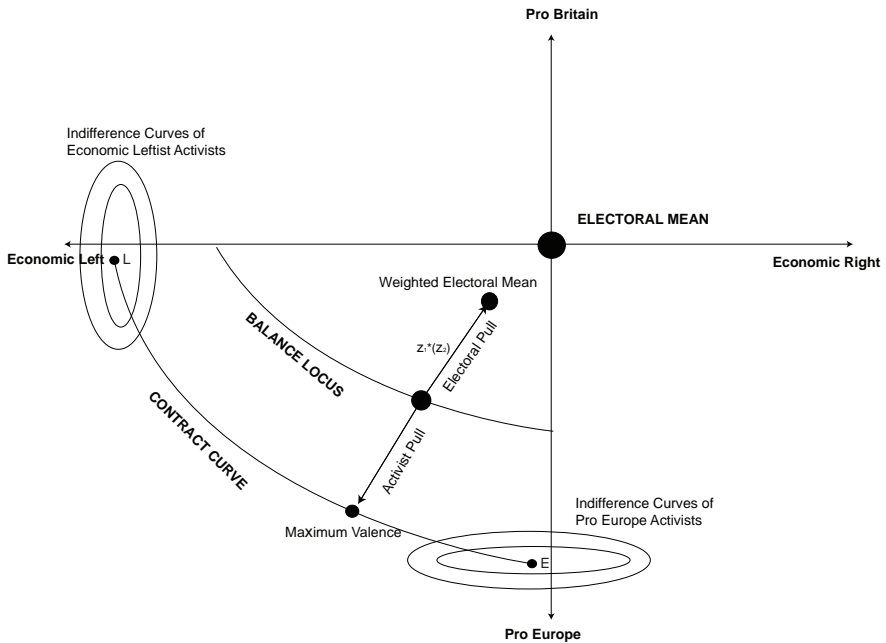
The *marginal electoral pull of party j* is a gradient vector pointing towards the weighted electoral mean of the party. This weighted electoral mean is that point where the electoral pull is zero. This gradient points toward the electoral center and represents the *centripetal pull* to the center. The *marginal activist pull for party j* represents the *centrifugal force* generated by the resources made available by activists.

In principle, this model can be used to examine positions of parties as they respond to activist demands in order to gain resources that can be used to contest elections. This model has been used to consider electoral competition when there are only two dimensions of policy, and a small number of parties competing under plurality rule.

Figure 18 gives an illustration taken from Schofield (2005) based on an empirical model for Britain for recent elections. In the figure there are two dimensions, one labelled the economic left/right and one labelled Pro-Britain/Pro-Europe. The Labor Party (using the U.S. spelling) benefits from resources from two potential activist groups, with preferred policy positions at L and E. The contract curve is the curve connecting these preferred positions of an activist group (L) on the economic left and an activist group (E), supporting membership of a strong European Union. This model, applied to British elections for 1992–2002, did appear to give some insight into the position of the Labor Party under Blair, near to the electoral center, in contrast to the Conservative Party, whose political leaders were estimated to have relatively low valence.

Miller and Schofield (2003, 2008) and Schofield and Miller (2007) have used this model (based on an economic axis and a social axis) to account for conflicts between economic and social conservatives, positioned at E and C respectively in Figure 19, over support for Republican Party candidates. As the figure also indicates, there are potential conflicts between pro-labor activists at L, and social activists at S.

As suggested by the notion of a balance locus, candidates for office in a two party system must balance the centripetal electoral gradient against a centrifugal activist gradient.



**Figure 18.** The electoral and activist pulls for the Labor Party in Britain

Figure 19 illustrates these formal results, by showing the contract curves between E and C. The equilibrium position for a Republican candidate will depend on the Republican intrinsic valence and the position adopted by the opposition candidate. When there is a single economic dimension, then the valence difference between the contenders will separate them on left and right. Potential activist concerns can then bring the second, social dimension into existence. Optimal, or vote maximizing, candidate positions will lie on the two balance loci. In general the optimal position for a low valence candidate like Goldwater will lie on a balance locus farther from the electoral center than that of a candidate like Bush whose valence is relatively higher. Figure 20 illustrates the voter distribution and candidate positions in the 2000 election in the United States obtained by Schofield, Claasen, Ozdemir and Zakharov (2009).

As these figures suggest, the changing configuration of centripetal and centrifugal forces appears to lead to a slow rotation in the configuration of the parties. Schofield, Miller and Martin (2003) and Schofield (2006b) suggest that a political realignment (Sundquist 1973) occurs when the two party configuration is changed suddenly (as the result of a constitutional quandary). Indeed, the recent election of Barack Obama may be seen as marking the resolution of such a political and economic quandary. The historical analysis offered by Schofield (2006b) suggests that this process of realigning transformation has tended to occur in a “clockwise” direction since the election of Lincoln in 1860.

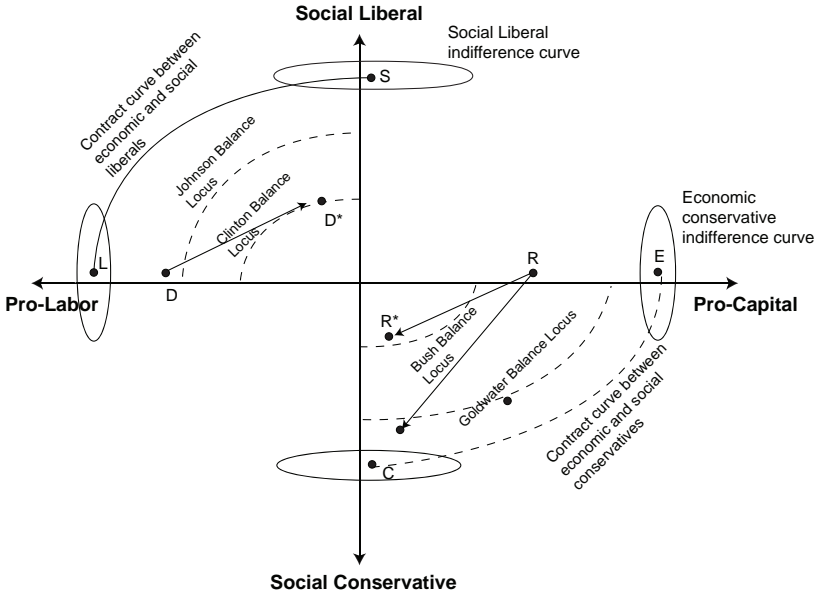


Figure 19. The Balance Loci in the US

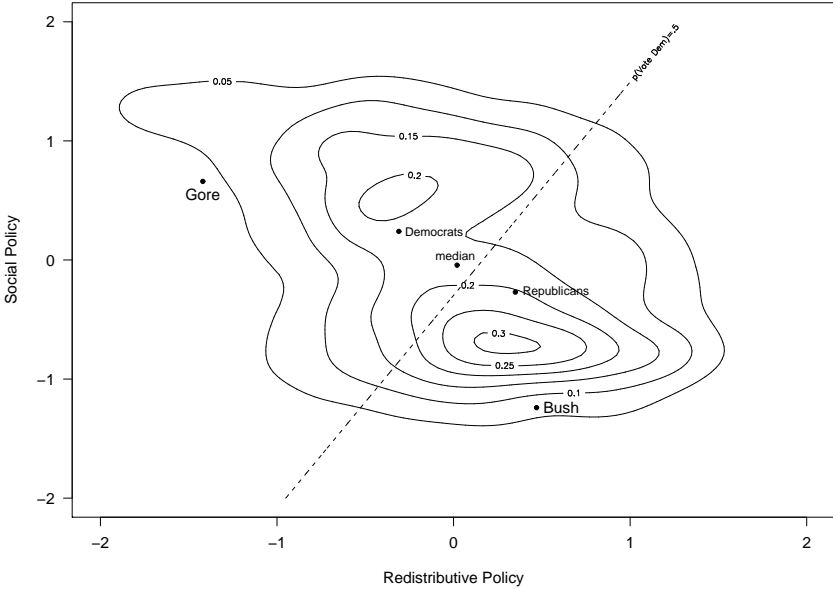


Figure 20. The electoral distribution and candidate positions in the 2000 election in the US

## 5. Concluding remarks

This paper has discussed a number of European multiparty polities with electoral systems based on proportional representation, as well as Turkey, the United States and Britain. It is evident that they all display complex and distinct characteristic features.

Although this paper has suggested a typology of the multiparty polities based on the qualitative features of the core and the heart, it is evident that the suggested typology does not give a full account of the complexities of coalitional bargaining. The key features of this typology is the degree of fragmentation, and the extent of centrality (i.e. whether a dominant party occupies the core position). What is remarkable, however, is the degree to which each country exhibits a pattern of coalition government that is consistent, in some sense, over time. It is hardly surprising that comparative scholars have found these patterns to be of such great theoretical interest. Estimating party positions, and attempting to model coalition bargaining between the parties is a major challenge for comparative research. Recent work by Benoit and Laver (2006) on estimating party position for a large number of political configurations is a significant advance, and their estimates have proved invaluable as a means to estimate the legislative heart in these polities.

The purpose of the spatial analysis presented here is to give some insight into the complexities of multi-party bargaining. The typology presented here has used the theory based on the existence of core parties and on the heart as an indication of the bargaining domain when the core is empty. Some countries are characterized by the existence of a dominant party, able to attain enough seats to be strongly dominant and command the core position. In the bipolar polities there are two potentially dominant parties, each one of which may be able to gain enough seats on occasion to control the core. Increasing fragmentation may make it less likely that a core party can exist. As the configuration of the heart becomes more complex, then bargaining over government will also become more complex. It is hardly surprising that fragmentation will be associated with less durable government (see King et al. 1990).

The main theoretical point that emerges is that the configuration of the heart in these polities suggests that there is hardly any centripetal tendency towards an electoral center (as suggested by the “mean voter theorem” of Lin et al. 1999). It is consistent with this analysis presented here that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and Riker (1953) and note that under proportional electoral methods, there is very little motivation for interest groups to coalesce. Consequently, the fragmentation of interest groups will lead to a degree of fragmentation in the polity. Fragmentation may be mitigated by the electoral system (especially if there is a relatively high electoral requirement which determines whether a party will obtain some legislative representation). However, even when there is a degree of majoritarianism in the electoral system (as in Italy in recent years) this may have little effect on reducing fragmentation. Clearly if one party dominates coalition policy for a long period of time then there will be a much higher degree of stability than indicated purely by government duration. However, as the situation in Italy circa 1994 suggests, if there is a core party facing little in terms of real political opposition, then corruption may become persistent. For democratic polities, there may be an element

of a quandary associated with the choice of an electoral system. If it is based on proportional representation then there may be the possibility of dominance by a centrally located party. Alternatively, there may be coalitional instability resulting from a fragmented polity and a complex configuration of parties. Another way of expressing, in simplified form, the difference between proportional representation and plurality rule is this: under proportional electoral methods, bargaining to create winning coalitions occurs *after* the election. Under plurality rule, if interest groups do not form a coalition *before* the election, then they can be obliterated, creating a pressure to coalesce.

The spatial maps together with the formal results based on the spatial model of elections suggest the following set of conclusions:

- (i) The pure spatial model of direct democracy indicates that the occurrence of a core, or unbeaten alternative, is very unlikely in a direct democracy using majority rule, when the dimension of the policy is at least two (McKelvey and Schofield 1987; Saari 1997). However, a social choice concept known as the heart, a generalization of the core, will exist, and converges to the core when the core is non-empty (Schofield 1999).
- (ii) A legislative body, made up of democratically elected representatives, can be modeled in social choice terms. Because party strengths will be disparate, a large, centrally located party may be located at a core position. Such a party, in a situation with no majority party, may be able to form a minority government. Instances from Scandinavia, Italy, and possibly Ireland are discussed here.
- (iii) A more typical situation is one with no core party. In such a case, the legislative heart can give an indication of the nature of bargaining between parties as they attempt to form a winning coalition government.
- (iv) This theory of legislative behavior takes as given the position and strengths of the parties. Because a centrally located party may dominate coalitional bargaining, and because such a party should be able to garner a large share of the vote, there would appear to be a strong *centripetal* tendency in all electoral systems.
- (v) However, estimates of party positions suggest that parties adopt quite heterogeneous positions (see Benoit and Laver 2006). This suggests that there is a countervailing or *centrifugal* force that affects all parties.
- (vi) While core parties can be observed in a number of polities with electoral systems based on proportional rule, the dominance of such central parties can be destroyed, particularly if there is a tendency to political fragmentation and social conflict.
- (vii) It is very unlikely that the heterogeneous positions of the parties can be accounted for in terms of a stochastic model of elections based on *intrinsic* valence alone. Empirical work on Italy, Netherlands Britain and the United States can be used to substantiate this inference (Giannetti and Sened 2004; Schofield 2005; Schofield and Sened 2006; Schofield, Claasen, Ozdemir and Zakharov 2009).



- (viii) This suggests that party location can be better modeled as a balancing act between the *centripetal electoral* pull and the *activist centrifugal pull*.
- (ix) Under *proportional electoral methods*, there need be no strong tendency forcing activist groups to coalesce, in order to concentrate their influence. If activist groups respond to this impulse, then activist fragmentation will result in party fragmentation. As the figures in Section 3 of this paper illustrate, parties tend to be scattered throughout the policy space. Activist groups, linked to small parties, may aspire to political office. This is indicated by the observation that the bargaining domain in the legislature (the heart) often includes small parties.
- (x) In some countries (such as Italy), a centrist core party can dominate the political landscape. To maintain dominance, such a party requires a high valence leader who can also maintain a flow of resources from a centrist activist group. By definition however, an activist group will tend to be located at a policy extreme. Thus a core party may need the support of an activist group that is not concerned about policy per se, but about monetary rewards. Thus there may be a link between core dominance and corruption. A development of this notion could give the underlying reason for the collapse of core dominance in Italy.
- (xi) Under *plurality rule*, small parties face the possibility of extinction. Unlike the situation in a polity based on proportional rule, an activist group linked to a small party in a plurality polity has little expectation of influencing government policy. Thus activist groups face increasing returns to size. The activist model of elections presented in Section 4 suggests that when there are two dimensions of policy, then there will tend to be at most four principal activist groups. The nature of the electoral contest generally forces these four principal activist groups to coalesce into at most two, as in the United States and the United Kingdom.
- (xii) In the United States, plurality rule induces the two party system, through this effect on activist groups. Although the two party configuration may be in equilibrium at any time, the tension within the activist coalitions induces a slow rotation, and thus political realignment. Presidential candidates must balance the centripetal electoral effect against the centrifugal valence effect. It is plausible that, in general, the relative electoral effect is stronger under plurality than under proportional rule. Of course, this depends on the intensity of the policy conflict between activist groups.
- (xiii) The well known relationship between proportional representation and a degree of political fragmentation (as measured by effective number) may be accounted for indirectly as a consequence of the logic forced on activist groups rather than parties themselves.

**Acknowledgment** This paper is based on work supported by National Science Foundation grant 0715929. The first version was presented at the Workshop on Simulation and Electoral Systems, University of Western Piedmont, Alessandria, Italy, June 2007.

A revision was completed while Schofield was the Leitner visiting professor at Yale University. This revision was completed while Schofield was the Glenn Campbell and Rita Ricardo-Campbell National Fellow at the Hoover Institution, Stanford in 2009.

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