Leaders, Voters and Activists in the Elections in Great Britain 2005 and 2010*

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March 1, 2011

Abstract

Discussion of the relationship between parties and the electorate is often based on the notion of partisan constituencies, that parties adopt policy positions that correspond to the average position of the party supporters. In contrast, the Downsian "spatial model" assumes that parties are purely opportunistic and maneuver to gain as many votes as possible. A third, more empirical model, based on the early work of Stokes assumes that voter choice is based on the evaluation of each of the party leader's competence or ability to deliver policy success. Such an evaluation can be provided by overall assessment in terms of the leaders' character traits.

This paper attempts to relate these three classes of models by examining the elections in Great Britain in 2005 and 2010. Using the British Election Study, we construct spatial models of these elections in Great Britain as well as in the three regions of England, Scotland and Wales. The models incorporate the electoral perceptions of character traits. We compare the equilibrium vote maximizing positions with the *partisan positions*, estimated by taking the mean of each of the parties voters' preferred positions. We define an equilibrium to be a *stable attractor* if the vote share at the equilibrium exceeds the share at the partisan position by a significant proportion (determined by the implicit error of the stochastic model). We infer that none of the equilibria are stable attractors, and suggest that the partisan positions are also preferred by the party activists, the key supporters of each party.

*This paper is based on work supported by NSF grant 0715929 and a Weidenbaum Center grant. Earlier versions were first completed while Gallego was a visitor at the Center and later when Schofield was the Glenn Campbell and Rita Ricardo-Campbell National Fellow at the Hoover Institution, Stanford, 2010. This version is prepared for presentation at the Priorat Workshop in Theoretical Political Science, June 16-18, 2011.

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1 Introduction

In recent years there has been much discussion, using both the theoretical and empirical tools, about the fundamental electoral incentives of political leaders in democratic societies. One model that is employed has been based on *partisan constituencies*. The idea here is that party leaders can fairly easily obtain information about the policy positions of their supporters, and each can respond by advocating policies that are close to the mean of the preferences of their respective supporters. Such a feature would satisfy the ideological congruence between citizens and policy makers (Huber and Powell, 1994; Ezrow, 2010). The term "responsible parties" (Adams, 2001) has been used to characterize the divergent policy choices that are likely in such a system of political competition. It has also been shown by Bernhardt *et al.* (2009), in a variant of such a model, that the choice between different policy options, induced by responsible parties, can, under some circumstances, enhance electoral welfare.

On the other hand, the standard Downsian (1957) model of political competition is that of "opportunistic," office seeking parties. Each voter is assumed to choose the party whose policy position is closest while parties are assumed to maneuver so as to gain as many votes as possible. Within the context of the spatial model, there has been controversy over whether rational parties will converge to an electoral center, or whether elections will be fundamentally chaotic, as suggested by Riker (1980).

However, as Stokes (1963, 1992) emphasized many years ago, the non-policy judgments, or *valences*, of party leaders by the electorate are just as important as electoral policy preferences. As Sanders et al. (2011) comment, valence theory is based on the assumption that "voters maximize their utilities by choosing the party that is best able to deliver policy success." The authors go on to note that an overall assessment of a party leader by a voter "provides a simple affective heuristic for arriving at an evaluation of that leader's party."

There is now a substantial literature that shows that valence models contribute significantly to an understanding of voter choice. Earlier work based on electoral perceptions of leaders' *character traits* has formed the basis for recent extensive analysis of British, Canadian and US electoral response by Clarke, Kornberg and Scotto (2009) and Clarke, Sanders, Stewart and Whiteley (2005, 2009). For Britain, they argue that electoral responses

> were a reflection largely of [the] changing perceptions of the decision-making competence of the main political parties and their leaders. At any point in time, [the] preferences were strongly influenced by their perceptions of the capacity of the rival parties–the putative alternative governments of the day–to solve the major policy problems facing the country.

These works have shown that valence, as measured by the perceptions of the character traits of the candidates, or of party leaders, is a key element of election. We shall call a stochastic spatial model incorporating these empirical estimates of character traits a *traits model.* Clarke, Kornberg and Scotto (2009: 159) also compared a "Downsian" or pure spatial stochastic model of the 2000 and 2004 US presidential elections with a traits model of the same elections and found that "the two models have approximately equal explanatory power."

While the traits model has the virtue of statistical significance, and can be used to estimate the changing electoral perceptions in the lead-up to an election, it gives only one half of the relationship between voters and parties. The trait characteristics are presumed to be based, to some extent at least, on integrating the quality of policy decisions in the past, or by estimating the likelihood of good decisions in the future (Penn, 2009). Suppose these estimates are independent of current declared policies. If the spatial element is statistically relevant, then, as in the Downsian model, the party could make a policy move so as to increase its vote, perhaps by attracting voters who do not have a strong opinion about the quality of the party leaders.

It is possible, of course, that party leaders have strong policy preferences, as proposed in various models,[1] or are bound by constituency congruence. However, for an electoral system based on plurality rule, or first past the post, it is difficult to rationalize the refusal to seek out votes wherever possible.

One way to determine the importance of the vote motive is to construct a formal or empirical model that combines elements of valence and policy distance. A number of such formal models have been offered in recent years.[2] More recent work has developed mixed logit statistical models that incorporate some measure of valence. Such models can be used examine whether, in equilibrium, parties will converge to the electoral center, or diverge in some fashion.[3]

In this paper, we examine the robustness of the partisan constituency model in the last two general elections in Great Britain. We use the British Election Studies (BES) to construct, for each election, a two-dimensional policy space, X. Using vote information, we can infer a preferred policy point, x_i, in X, for each voter. Consistent with the partisan constituency model, we assume each party, j, is located at the mean position, z_j^*, of its voters. Knowing vote intentions we constructed various spatial logit models of the elections, based on these data. A key parameter of each model is the *spatial coefficient*, β, which is a measure of the importance of policy distance.[4] Simulation of the models then allows us to relate any vector of party positions, $\mathbf{z}$, to a vector of vote share functions $\mathbf{V}(\mathbf{z})=(V_1(\mathbf{z}),...V_p(\mathbf{z}))$, predicted by the particular model with p parties. Each party, j, is then characterized by a measure of *exogenous valence*, λ_j, namely the intercept term, associated with party j in the model. This model can be considered to be Downsian, since it is based on a pure spatial model.

In addition to exogenous valence, the we also incorporated *sociodemographic valences.* Whereas exogenous valence measures a common bias across all voters, sociodemographic valences, based on gender, domicile, income *etc.*, allow these

[1] Wittman (1977), Calvert (1985), Duggan and Fey (2005), and Duggan (2006)

[2] Ansolabehere and Snyder (2000), Groseclose (2001), Aragones Palfrey (2002),

[3] Adams and Merrill (1999, 2001, 2002), Adams et al. (2005)

[4] In the empirical models that we consider, the spatial coefficients were statistically significant at the 0.001 level.

perceptions of the party leaders to vary across relevant sociodemographic categories. We then extended the model by using the survey evaluations of party leaders to construct a trait index based on factor analysis of the responses. The trait index involves both the attractiveness (or valence) of the party leaders, but also the competence, trustworthyness and responsiveness of party leaders.

We repeated the analysis using the electoral perceptions of the traits of the party leaders, in all three regions of Great Britain. As in previous work, we found that the models involving traits are superior to the pure spatial models. However, as Tables 11 and 27 show, for both 2005 and 2010, the joint spatial models with traits give superior log-likelihoods than the pure traits models. We can therefore use variants of the spatial models to estimate whether parties can gain votes by moving from the partisan constituency positions.

To do this we use the notion of a "local Nash equilibrium" (LNE) under vote maximization A LNE is simply a vector of party positions with the property that no party may make a small unilateral move and yet increase utility (or vote share). We use $\mathbf{z}^{el}$ to denote the LNE. However, this definition of LNE does not require that the equilibrium is a *stable attractor*, in the sense that parties will shift their positions towards the equilibrium. We argue that an equilibrium in a spatial model has to present some obvious advantage to at least some of the party leaders. Assuming that parties are initially located at the *partisan constituency* vector, $\mathbf{z}^*$, say, then we can use the stochastic model to compare the estimate of the vote shares, $\mathbf{V}(\mathbf{z}^*)$, with the vote shares, $\mathbf{V}(\mathbf{z}^{el})$, at the LNE, $\mathbf{z}^{el}$. Because the structure of the formal model, it is convenient to use the criterion that the lowest valence leader, say $j = 1$, will be advantaged. However, the stochastic model involves statistical risk, and to deal with risk we assume the low valence parties are somewhat risk averse. We compute the lower expected vote share (at the 95% level) for a low valence party 1, say and let $V_1(\mathbf{z}^{el})$ denote this share. We define the *vote margin* for a low valence party to be $\delta = V_1(\mathbf{z}^{el}) - V_1(\mathbf{z}^*)$ and say that $\mathbf{z}^{el}$ is a *stable attractor* if $\delta > 0$.

We first examine the pure spatial model and determine whether the equilibrium, $\mathbf{z}_0$ is one where each party is located at the *electoral mean*, z_0, as suggested by the mean voter theorem.[5]

For each pure spatial model, we compute a coefficient, called the *convergence coefficient*, denoted c, that depends on various parameters, namely the exogenous valences $\{\lambda_j\}$, the electoral variance and the spatial coefficient. Previous work (Schofield, 2007) has shown that if $c < 1$, then the LNE is one where all parties adopt the same position at z_0.[6] On the other hand, if $c \geq w$,where w is the dimension of the space, then the LNE is one where all parties diverge away from the electoral mean. For 2005, we found for Britain that $c = 0.84$, while in 2010 we found $c = 0.98$. In both cases therfore the electoral mean is predicted to be a local equilibrium. We repeated the analysis for the regions, England, Scotland and Wales and found very similar results. To locate the LNE for the regional models with and without sociodemographic valence, we deployed a sim-

[5] Enelow and Hinich (1984a b)

[6] The definitions and all technical details and computations are in the Appendix.

ulation program. Not all equilibria were determined to be at the joint electoral mean, but the simulations did find them to be located very close to this mean. In all cases however, the (95%) lower predicted vote share of the low valence parties were lower than the sample vote share. We inferred that any centripetal tendency towards the electoral mean would be quite weak. By our definition, none of these equilibria were stable attractors.

We repeated the equilibrium exercise and constructed stochastic traits models of these elections. In 2005 we found the equilibrium positions and vote shares of the traits models to be only slight perturbations of the pure spatial models. In 2010, however, we found that the trait indices for Gordon Brown, the Labor leader, were much lower than the other two party leaders, Clegg and Cameron. Obviously enough, this was the reason why Labor did so badly in the election. Our equilibrium analysis suggests that the equilibrium vote maximizing position for Brown under the traits model was very close to his estimated position. However, we estimate that the Labor party vote share would be *lower* at this equilibrium than at the party constituency positions, $\mathbf{z}^*$. By our definition, such an equilibrium is not even an attractor.

While much recent research has modeled the trait characteristics of political leaders in a number of countries, we believe this is the first examination of the optimal response of leaders to these electoral perceptions. Since these perceptions are distributed in the electorate, a rational leader should adjust policies to take advantage of this distribution, if possible.

However, electoral success also depends on the resources made available by party activists. The preferred positions of activists can be assumed to influence the location of the parties. In our analysis we use various methods to estimate the positions of activists, and find these positions to be very similar to those of party partisans. This provides additional support for inferring that parties adopt positions at, or very close to, the partisan constituency positions. While equilibrium analysis have suggested that parties will tend to the electoral mean, we contend that these models do not provide an accurate picture of party positioning.

2 Models of British Elections

Earlier work by Schofield (2005) suggest that when Tony Blair took over from John Smith as leader of the Labor[7] Party, then the exogenous valence, $\lambda_{lab,}$, of the party increased up to the 1997 election. Conversely, the exogenous valence, λ_{con}, for the Conservatives, under John Major, declined.[8] Major resigned after the 1997 election, and William Hague became leader of the Conservatives.

The Appendix defines the notion of a gradient electoral pull for each party. For example, for the Labor Party the weighted electoral mean for the party is

[7] We shall use the American spelling of labor throughout this paper.

[8] For discussion of the nature of party competition in Britain from 1992 on see Clarke, Stewart and Whiteley (1997, 1998).

some position z^{el}_{lab}, determined by the valences $\lambda_{con}, \lambda_{lab}$, λ_{lib} of the the three party leaders. Then the *electoral pull* at any position, z_{lab}, is a gradient vector

$$\frac{d\mathcal{E}^*_{lab}}{dz_{lab}}(z^{el}_{lab}) = \left[z^{el}_{lab} - z_{lab}\right].$$

pointing from z_{lab} towards z^{el}_{lab}.

The Activist Balance Theorem in the Appendix asserts that for equilibrium to hold at a position z^{μ}_{lab} it is necessary to balance the electoral pull by a counter activist pull, $\frac{1}{2\beta}\frac{d\mu_{lab}}{dz_{lab}}$, where $\mu_{lab}(z^{\mu}_{lab})$ is the activist support for Labor. Thus the balance equation is

$$\left[z^{el}_{lab} - z^{\mu}_{lab}\right] + \frac{1}{2\beta}\left[\frac{d\mu_{lab}}{dz_{lab}}\right] = 0$$

Figure 1 gives an illustration based on an a stochastic model for Britain for recent elections. There are two dimensions, with the left-right axis determined by economic variables, such as taxes and the free market. The second, north-south axis is associated with attitudes to the European Union, immigration etc.

The Labor Party, under Blair, benefits from resources from two potential activist groups, with preferred policy positions at L and E. The contract curve, or activist catenary, connects the preferred positions of an activist group (L) on the economic left and an activist group (E), supporting membership of the European Union.

The optimal Labor position will be determined by a version of the above balance equation

$$\left[z^{el}_{lab} - z^{\mu}_{lab}\right] + \frac{1}{2\beta}\left[\frac{d\mu_{lab,L}}{dz_{lab}} + \frac{d\mu_{lab,E}}{dz_{lab}}\right] = 0$$

which equates the "electoral pull" against the two "activist pulls," generated by the two different activist functions, $\mu_{lab,L}$ and $\mu_{lab,E}$. In the same way, if there are two activist groups for the Conservatives, generated by functions $\mu_{con,C}$ for pro-British activists and $\mu_{con,B}$ for economic conservatives, then we obtain a balance equation:

$$\left[z^{el}_{con} - z^{\mu}_{con}\right] + \frac{1}{2\beta}\left[\frac{d\mu_{con,C}}{dz_{con}} + \frac{d\mu_{con,B}}{dz_{con}}\right] = 0.$$

Figure 1 presents a conservative party leader position in the upper right quadrant of the space, where the conservative electoral pull and activist pulls are equal and opposite. To apply this model, we need to estimate the weighted electoral means for all three major parties, z^{el}_{lab}, z^{el}_{con}, z^{el}_{lib}. We shall examine the elections of 2005 and 2010, to estimate the valences of party leaders, and thus the optimal party positions. This will allow us to estimate the influence of activists on the parties.

[Insert figure 1 here]

3 The Election in the United Kingdom in 2005

In the June 2001 election in the United Kingdom, the Labor Party, under Tony Blair repeated its election victories of 1997 by taking 413 (out of 646) seats against the Conservative Party, led by William Hague, and the Liberal Democrats, led by Charles Kennedy. Hague resigned after the election, and Iain Duncan Smith became leader of the Opposition. In need of more popular leadership, Michael Howard became leader of the Conservative Party in November, 2003. In the election of May, 2005, Blair again repeated his success by leading the party to victory with 356 seats. It was generally assumed that the Labor Party lost 57 seats, while the Conservatives gained 32, because of the British involvement in Iraq. Howard stepped down as opposition leader in December 2005 and David Cameron became leader of the Conservative Party. Tables 1, 2 and 3 give the election results for Great Britain as a whole, as well as separate Tables for England, Scotland, Wales and Northern Ireland in 2005. Figure 2 shows those constituencies won by the various parties.

[Insert Tables 1,2 and 3, and Figure 2 here]

We use the results from the British Election Study (BES) to construct a pure spatial model of the election. We argue that this model suggests that Labor won the election because of the significant valence difference between Blair and Howard. We also ran separate models for England, Scotland and Wales, incorporating the regional parties, the SNP in Scotland and Plaid Cymru in Wales. These models show that the valence differences between Blair and Howard were particularly pronounced in Scotland and Wales.

Tables 4 and 5 give the questionnaire from the BNES and the factor analysis. The first and second factors explain about 43% of the variance of the 13 question responses. As can be seen in Table 5, the first factor is strongly associated with the issue of "EU membership", "Immigrants", "Asylum seekers" and "Terrorism". We call this the *nationalism* dimension. We have oriented this axis so that a high value means stronger nationalism. The second dimension is *economic*. The items of "tax/spend", "free market", "international monetary transfer", "international companies" and "worry about job loss overseas" have strong influence in this dimension. In the economic dimension, a higher value indicates a pro-market oriented attitude. We used the economy as the x-axis and nationalism as the y-axis.

We also considered other questions measuring social values such as voters views on minority, gender role, censorship, environment, death penalty, but the loadings were less than 0.20 at most in the first two factors. The analysis is based on responses to these 13 questions, with 1564 respondents from England, Scotland and Wales. On the 0-10 scale, those who reported relatively stronger voting intention (> 7) for a party were taken as the party's voter. Respondents who said they "volunteered to get involved in politics" were coded as activists. The factor loading then allow us to estimate voters' positions along the two dimensions, thus giving a set $\{x_i\}$ of voter positions, which we regard as the voters ideal points.

The positions of Labor, the Conservative Party and the Liberal Democrats on the two dimensions were estimated using the result of the factor analysis and the respondents' voting intentions. The position of party j is denoted z_j and is estimated by taking the mean value, z_j, of those who voted for party j. For each party we used only those respondents who gave the high score for their intention to vote for the party. We call z_j the *partisan constituency* position of the party. Figure 3 presents the smoothed electoral distribution and the estimated partisan constituency positions of the parties.[9] The electoral distribution of activists, and the activist means by party are given in Figure 4.[10] while Figure 5 gives the electoral distribution and party positions in the three regions of England, Scotland and Wales.[11]

[Insert Tables 4 and 5, and Figures 3, 4 and 5 here].

We then constructed the pure spatial model using these data. Table 6 (model 1) gives the pure spatial model for just the three largest parties, in Great Britain, with a single $\beta-$coefficient.[12] We also estimated the spatial model with separate $\beta-$coefficients, $(\beta_{Econ}, \beta_{Nat}) = (0.39, 0.13)$, as in Table 6a. We did this because the electoral variances were very different on the two dimensions, namely $(1.646, 3.961)$, respectively. However, the results using these two $\beta-$coefficients were almost identical to the single $\beta-$coefficient result.

[Insert Tables 6 and 6a here]

We used the Lib Dem party as the baseline for the MNL pure spatial models, for Great Britain, with 1149 respondents (i.e., those who voted for the Labor, Conservative and Liberal Democratic parties in Great Britain). Below, we give the regional models for England, Scotland and Wales based on a sample size of 1564 (consisting of those respondents who voted for either the three major parties or the two regional parties, the Scottish National Party and Plaid Cymru in Wales). Because the exogenous valences and traits estimates were very different across regions, it was important to determine the regional LNE.

The sample proportions for Labor, Conservatives and Lib Dems in Great Britain were 41.5%, 34.0%, and 24.5%, respectively. These were similar to the actual vote shares (in Great Britain, excluding minor parties and Northern Ireland) of 39.4%, 36.0%, 24.6%, respectively.[13]

The estimated partisan constituency positions of the three major parties, and two regional parties in Great Britain, in 2005, were found to be:

$$\mathbf{z}^* = \begin{bmatrix} 2005 & Lab & LibDem & Cons & SNP & PC \\ Econ & -0.393 & -0.192 & 0.522 & -0.12 & -0.31 \\ Nat & -0.470 & -0.949 & 0.907 & -0.11 & 0.04 \end{bmatrix}.$$

[9] We use LAB (Labor), CON (Conservatives), LIB (Liberal Democrats) for the positions in Figure 3.

[10] The survey provided information on activist vote intentions.

[11] We use SNP (Scottish National Party) and PC (Plaid Cymru) for the estimated positions of these regional parties in Scotland and Wales.

[12] Table 6 includes other models with traits etc. that we define below.

[13] We can call these vote shares a three-way split of vote shares, since they give the shares just between these three parties.

as in Figure 5.[14] The activist means, for the three major parties, are

$$\mathbf{z}^{act} = \begin{bmatrix} 2005 & Lab & LibDem & Cons \\ Econ & -0.40 & -0.22 & 1.00 \\ Nat & -1.61 & -1.51 & 0.86 \end{bmatrix}.$$

This suggests that Conservative Party activists, on average, are much more right wing on economic issues (the $x - axis$) than Conservative voters, while activists for the Labor Party and Liberal Democratic Party tend to be less nationalistic (much more supportive of the EU). The voter mean in Britain was exactly at the origin $(0.0, 0.0)$

3.1 Pure Spatial Models for Great Britain for 2005

The computation section 8 in the Appendix gives the details of the pure spatial model in Great Britain with the three major parties. Note that, from Table 6, the $\beta-$coefficient is highly significant. The electoral mean for Great Britain is $(0, 0)$. The valence estimates from Table 6 are

$$(\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}, \beta) = (0.52, 0.27, 0, 0.15).$$

We computed the central estimate of the convergence coefficient, $c(\boldsymbol{\lambda},\beta)$ to be 0.84, implying that the joint mean $\mathbf{z}_0 = (0, 0)$, is an LNE. The predicted vote shares at $\mathbf{z}_0$ were computed to be

$$\boldsymbol{\rho}_s^{GB} = (\rho_{Lab}, \rho_{Con}, \rho_{Lib}) = (0.42, 0.33, 0.25).$$

Table 7 gives the upper and lower 95% values on these estimates as well as the upper 95% estimate on $c(\boldsymbol{\lambda},\beta)$.[15]

[Insert Table 7 here.]

These estimates compare with the three way split of sample shares:

$$(s_{Lab}, s_{Con}, s_{Lib}) = (0.415, 0.34, 0.245).$$

and the actual three-way split of vote shares among these parties:

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}) = (0.394, 0.36, 0.246).$$

Comparison of the estimated equilibrium vote shares under the pure spatial model and the sample shares at the estimated partisan constituency positions suggests that the Liberal Democrat Party would obtain a vote share, taken just with the three parties of 0.25 at the pure spatial equilibrium in comparison to three way sample vote share of 0.245. However because of the stochastic aspect of the vote model, the 95% bounds on ρ_{Lib} are $[0.22, 0.28]$. The party could expect

[14] These estimates are very similar to those obtained by Schofield (2005) for 1997, taking the means of MPs' responses to the BES for that year.

[15] The upper 95% bound on c was determined to be 0.97.

at least 22% of the vote, were the parties to move to such an equilibrium. The vote margin is

$$\delta = V_1(\mathbf{z}^{el}) - V_1(\mathbf{z}^{*}) = \rho_{Lib}^{lower} - s_{Lib} = 0.22 - 0.245 = -0.25$$

By our definition, the equilibrium for the pure spatial model is not a *stable attractor*. To move to such an equilibrium position at the electoral origin, the Liberal Party would have to change its support for the European Union. We now consider the regional models to determine if the local equilibria in these regions are stable attractors

3.1.1 Pure Spatial Models for England Scotland and Wales in 2005

Table 8 gives the results of the pure spatial model for the regions of England, Scotland and Wales, while Figure 5, above, gave the electoral distributions and party positions in the regions. The sample sizes in the three regions were 942, 362 and 260 respectively.

[Insert Table 8 here]

The computation section of the Appendix shows that the regional convergence coefficients can be estimated to be

$$(c^{eng}, c^{sct}, c^{wales}) = (0.75, 0.97, 0.80)$$

Table 9 shows the various lower and upper 95% bounds on the predicted vote shares and convergence coefficients. The calculations in the Appendix shows that, with 95% probability, all the Hessians of the low valence parties have negative eigenvalues in the three regional models. We infer that convergence to the joint mean, even in these regions, is an equilibrium prediction for all parties.[16]

[Insert Table 9 here]

All $\beta-$coefficients are significant at the 99% level, and these coefficients as well as the convergence coefficients take very similar values to those obtained for the analysis for Great Britain.

We can compare these estimated vote shares at the joint mean in these regional elections with the actual vote shares calculated by the three or four-way split between the parties in each region.

Vote shares at the regional joint means estimated by simulation of the pure spatial models are:

England	(Lab, Con, Lib)	$=(0.376, 0.360, 0.264)$
Scotland	(Lab, Con, Lib, SNP)	$=(0.403, 0.212, 0.202, 0.184)$
Wales	(Lab, Con, Lib, PC)	$=(0.416, 0.248, 0.221, 0.115)$

[16] In the estimations, the valences of the SNP in Scotland and Plaid Cymru in Wales are not significantly different from zero. Assuming valences of zero would give *higher* estimates of ρ_{snp}^{sct} and ρ_{con}^{wales}, and therefore *lower* values of c^{sct} and c^{wales}. Using the 95% bounds to construct appropriate bounds on our estimates of the convergence coefficients provides a more robust confirmation of this conclusion.

The three or four way sample vote shares are:

England	(Lab, Con, Lib)	$=(0.364, 0.384, 0.251)$
Scotland	(Lab, Con, Lib, SNP)	$=(0.406, 0.202, 0.207, 0.185)$
Wales	(Lab, Con, Lib, PC)	$=(0.412, 0.262, 0.208, 0.119)$

Actual three or four way vote shares are:

England	(Lab, Con, Lib)	$=(0.38, 0.38, 0.24)$
Scotland	(Lab, Con, Lib, SNP)	$=(0.41, 0.165, 0.24, 0.185)$
Wales	(Lab, Con, Lib, PC)	$=(0.448, 0.226, 0.194, 0.132)$

The valence of the Conservative Party is obviously much lower in Scotland ($\lambda_{Con}^{sct} = 0.046$) and in Wales ($\lambda_{con}^{wales} = 0.11$), than in England ($\lambda_{con}^{eng} = 0.31$), so we obtain $\rho_{con}^{eng} = 0.36$, $\rho_{con}^{sct} = 0.212$, $\rho_{con}^{wales} = 0.248$ for the estimated vote shares for this party if the parties were at the regional equilibria.[17] When the parties are at their partisan constituency positions, then the four-way actual vote shares of the Conservative Party are lower in Scotland (16.5%) and Wales (22.6%). We argue that the vote shares in Scotland and Wales are based on the position of this party, located far from the center in contrast to the Liberal Democrats, Scottish Nationals and Plaid Cymru. We have inferred above that the Conservative Party obtains almost its votes in England (about 8.1 million out of a total of 8.8 million), and essentially does not compete in Scotland or Wales.

The valence of the Labor Party is $\lambda_{Lab}^{sct} = 0.69$ in Scotland and $\lambda_{Lab}^{wales} = 0.63$ in Wales, which are both much higher than in England ($\lambda_{lab}^{eng} = 0.35$), giving $\rho_{lab}^{sct} = 0.40$, and $\rho_{lab}^{wales} = 0.42$, in contrast to $\rho_{lab}^{eng} = 0.376$. These estimations are close to the four-way sample vote shares for Labor in these regions (0.406 and 0.412 in Scotland and Wales, respectively). The 80 seats Labor wins in Scotland and Wales, as well as its relatively high valence in these regions gives it an electoral advantage in general.

Notice that the vote share of the SNP of 1.65% in Britain is due entirely to its four-way sample vote share in Scotland of 0.185 while $\rho_{snp}^{sct} = 0.184$. Similarly, the vote share of 0.65% for PC (Plaid Cymru) in Britain is due to a four-way sample vote share of 0.119 in Wales, while $\rho_{pc}^{wales} = 0.114$. We estimate that the two regional parties also gained because of their centrist positions in comparison to the more extreme position of the Conservative Party. Note that these two low valence parties have no incentive to move to the estimated regional equilibria obtained from the pure spatial model.

In Scotland and Wales the four-way vote shares for the Liberal Democrats are 0.27 and 0.208 respectively while $\rho_{lib}^{sct} = 0.202$ and $\rho_{lib}^{wales} = 0.226$. We infer that the difference between the actual vote shares and the estimated vote shares for this party at the joint mean is due to the location of the Liberal Democrats

[17] The 95% lower bound on ρ_{con}^{sct} is 0.16 and the lower bound on ρ_{con}^{wales} is 0.18. See Table 9i.

at a quite pro-Europe position on the Nationalism axis. In Scotland, the more extreme position of the Conservative Party puts it in fourth place. Notice that the seat share of the Liberal Democrats is much higher in Scotland than in England. We infer that this is because the relative valence of the Conservative Party is much lower in Scotland and this decreases the relative electoral disadvantage of the Liberal Democrats. In Wales, the Liberal Democrats lose votes because of the slightly more centrist positions of Plaid Cymru, and even Labor.

It is obvious from these estimates that Labor depends to a considerable degree on electoral support in Scotland and Wales, with its main competitors in Scotland being the Lib Dems followed by the SNP. In Wales, Labor has only weak competition from the Conservatives and Lib Dems. The electoral distributions suggest that voters in Scotland are slightly more supportive of Europe, and voters in Wales are slightly less supportive of Europe, than in Great Britain as a whole. Devolution and the increased valence for the SNP in Scotland and for the Lib Dems throughout Britain will affect the electoral chances for Labor.

In the 2010 election that we next examine Labor was able to retain its 41 seats in Scotland, but lost 3 of its seats in Wales.

For 2005, we see that in England, the lower 95% predicted vote share of the Lib Dems at the LNE was 0.23, which is below its sample vote share (0.25) at the vector of partisan constituency positions. By our definition, the electoral mean in the three way election in England is not a *stable attractor*. In Scotland, the lower 95% vote share of the SNP at the LNE is $\rho_{snp}^{sct} = 0.140$.which is less than the sample share of 0.19, while in Wales, the lower vote share of Plaid Cymru is $\rho_{pc}^{sct} = 0.08$. which is less than the sample share of 0.12.

None of the regional equilibria for the pure spatial models are a *stable attractors*.

[Insert Tables 9]

We now examine the effects of traits.

3.2 The Spatial Model with Traits for 2005

To extend the model, we used survey questions on the party leader traits (in Table 4c) to construct a trait index by factor analysis, as in Table 10. Notice that Q.1 in Table 4c refers to voters feelings about the party leaders. The response to this question gives an indication of the valence or attractiveness of the leader. However, Q. 2 and Q. 4 deal with competence and trust, while Q.3 asks whether the leader is responsive to voter concerns.

Table 6, above, compares the various models: pure spatial, pure traits, and the spatial model with traits, with and without sociodemographic variables.

Notice that by Table 6, the $\beta-$ coefficient is still highly significant in the spatial models just with traits and with both traits and sociodemographics, while the valence term for Blair is not statistically significant in the traits models, with or without sociodemographics.

We redid the traits analysis for the three regions, obtaining measures for the trait indices for the SNP leader, Salmond in Scotland and Llwyd in Wales (see

Tables 11 to 13) and analyzed all trait, spatial and sociodemographic models by region (see Tables 14 to 17). The loglikelihoods are obviously superior for the general models with traits and sociodemographics. For example, for England, the Log Likelihood (LL) of the pure spatial model is -945, while the traits model has a far superior value of -463. Adding the spatial model to traits we find the LL becomes -460, while adding the sociodemographics we obtain a LL of -440. In the regional traits models, the constant (or valence) term for Blair tends to be significant with just traits, but not when the sociodemographic terms are included. The AIC (Akaike information criterion) measures also drop significantly, as new variables are added.

[Insert Tables 11 to 17 here]

A later Table 25 , in the section on comparison of models, gives the differences in loglikelihoods between the models for the spatial, traits and joint models for Great Britain. This table suggests that the spatial and traits models do complement one another.

Since the spatial component is significant for these models, we can estimate local Nash equilibria under vote maximizing behavior by the parties for the various spatial traits models.

Using simulation on the spatial traits model we can compute the equilibrium to be

$$\mathbf{z}_{st}^{el} = \begin{bmatrix} 2005 & Lab & LibDem & Con \\ Econ & -0.05 & -0.01 & 0.17 \\ Nat & -0.31 & -0.25 & 0.19 \end{bmatrix}$$

with predicted vote share of $(\rho_{Lab}, \rho_{Con}, \rho_{Lib})_{st} = (0.41, 0.34, 0.25)$.

Similarly, the local equilibrium for the spatial traits model including sociodemographics is

$$\mathbf{z}_{sts}^{el} = \begin{bmatrix} 2005 & Lab & LibDem & Con \\ Econ & -0.07 & -0.04 & 0.16 \\ Nat & -0.31 & -0.20 & 0.14 \end{bmatrix}$$

with an identical expected vote share of

$$(\rho_{Lab}, \rho_{Con}, \rho_{Lib})_{sts} = (0.41, 0.34, 0.25).$$

We see the vote share of the LibDems at the estimated spatial trait equilibrium is almost identical to the sample vote share at the estimated positions at the partisan constituency positions. Since we have defined a stable attractor to be an equilibrium where the expected equilibrium gain in the vote share of low valence party is at least 10%. By this criterion, these joint traits equilibria are not stable attractors.

The estimations for the 2005 suggest that the equilibria of the traits models are almost identical to the pure spatial models. Although the traits models are statistically superior to the pure spatial models, the equilibria are little changed.

3.3 The election of May 2010.

Gordon Brown became leader of the Labor Party and Prime Minister on 27 June 2007 after the resignation of Tony Blair, while Nicholas William Peter

Clegg became leader of the Liberal Democrats on 18 December 2007. Brown's popularity fell dramatically as a result of various scandals involving the Labor Party and the deep economic and financial crisis.[18] The outcome of the May 6, 2010 election was a hung Parliament with no majority party. Gordon Brown formally resigned as Prime Minister on May 11 and David Cameron formed the next government, in alliance with the Liberal Democrats, with Clegg as deputy Prime Minister. (See Tables 18, 19 and 20 for the election results.)

[Insert Tables 18, 19 and 20 here]

The Cabinet comprised 18 Conservatives and 5 Liberal Democrats, roughly proportional to the two parties' strengths. Fairly obviously, the electorate lost any faith it may have had in Gordon Brown because of the dire consequences of the recession. On September 25, 2010, Edward Miliband was elected leader of the Labor Party.

After the election it became obvious that the United Kingdom faced a deficit (the public sector borrowing requirement) of £140 billion (about $240 billion, or 11% of GDP) and a total debt of £820 (about 56% of GDP).[19] The Queen's speech, at the opening of the new Parliament on May 25, laid out the policy plans for the new government, including a reduction of the budget deficit of $220 billion.

The new coalition government of Conservative and Liberal Democrat had to deal with the budget deficit, and by September had begun to propose various cuts in government spending, of about £83 billion (or $130 billion), including possibly a 20% cut in defense. The government announced in October that it intended to cut the child welfare benefit (about $32/week for the first child) for any family making over $70,000/annum. These cuts were followed by a proposed substantial reduction in the support for British universities, increase in student fees, and the elimination of hundreds of thousands of public sector jobs. In a vote on 9th December 2010, the coalition government voted 323 in favour and 302 against, with a majority of only 21, to raise university fees to a maximum of £9,000 per annum from £3000.[20] There were quite violent student protests on November 12 and later on December 9 in London.

Figure 6 gives an indication of the estimated positions of the three major parties in this election. The policy distance between the Conservative and Liberal Democrat positions suggest that there may well be conflicts in the coalition over government policies. Although Clegg and Cameron had come to an agreement about the budget cuts prior to the coalition agreement, many members of the Liberal Democrats were deeply opposed to such cuts.

[Insert Figure 7 here]

[18] The UK public net debt had increased from about 53% of GDP to about 68% in three years. This however was comparable to the US, at about 60%, and much lower than Germany, about 77% and Greece, 108%. The unemployment rate reached 8%, somewhat lower than that of the U.S. where it was 9.9%.

[19] The North Sea oil windfall of about $400 billion was simply dissipated. It is not unlikely that Scotland will in the future try to capture for itself some of the 30 billion barrels of oil estimated to still lie under the North Sea.

[20] Obviously, many Liberal Democrats voted against the government.

4 Modelling the election of 2010

We proceeded in the same way as for 2005 to construct a factor space based on the 2010 BES. Tables 21 gives the survey questions and Table 22 gives the Factor Analysis.

[Insert Tables 21 and 22 here]

To construct the factor space, we used the eight survey items specified in Table 22.[21] The sample (n=6409) included respondents who participated both in pre- and post-election surveys, voted for Lab, Con, Lib, SNP or PC, and were without missing data points in the variables regarding vote choice, issue dimensions, traits and sociodemographic. The sample contained 5466, 636 and 307 respondents from England, Scotland and Wales, respectively. The sample party votes shares were

$$(Lab, Con, Lib, SNP, PC) = (0.281, 0.400, 0.289, 0.025, 0.005).$$

As in 2005, one dimension is *Nationalism* and the second one is *Economy.* A larger value in the *Nationalism* dimension is strongly associated with disapproval to Britain's EU membership and disagreement with Britain's further cooperation with EU. On the *Economic* dimension, those who prefer tax-cut, disagree to increasing the tax-free allowance to £10,000, to a "mansion" property tax, to limiting pension tax relief, and to ecotax have higher values. In sum, a larger value on the first dimension indicates stronger nationalism and on the second dimension pro-market attitudes. As in the previous analysis we represent the economic dimension as the x-axis and the nationalism dimension as the y-axis.

Using the factor scores, we estimated the party positions. Each party position is estimated as the mean of the voters who intended to vote for the party before the election using the "Vote Intention item in pre-election surveys". The party positions were estimated to be

$$\mathbf{z}^* = \begin{bmatrix} 2010 & Lab & LibDem & Cons & SNP & PC \\ Econ & -0.21 & -0.35 & 0.39 & -0.32 & -0.07 \\ Nat & -0.44 & -0.43 & 0.45 & -0.21 & 0.23 \end{bmatrix}.$$

The mean of the voter positions was $(0.010, 0.003)$ in 2010.

To determine activists, we used the survey question '*on a scale of 0 to 10, how much attention do you generally pay to politics?*' Those who answered 6 or more to this question were regarded as activists. (n=746). Since the pre and post election surveys gave vote intentions and actual vote with regard to the mail parties, we could allocate activists to these three parties.The mean value of all respondents was (Economy, Nationalism)=$(0.010, 0.003)$ and the activist mean was $(-0.048, -0.277)$. Figure 6 shows the electoral distribution in Great Britain, while figure 7 shows activist distributions by party on the two

[21] We also included several other policy related items such as War-in-Afghan and Reducing crime vs. the rights of suspects. However, the contribution of these items was very low in either of the dimensions.

dimensions. Using the pre-election and post-election surveys, we can compare voters and activists as follows:

$$\text{voters by actual vote} \quad : \quad \begin{bmatrix} & Lab & LibDem & Con \\ Econ & -0.19 & -0.34 & 0.40 \\ Nat & -0.33 & -0.35 & 0.49 \end{bmatrix}$$

$$\text{activists by actual vote} \quad : \quad \begin{bmatrix} & Lab & LibDem & Con \\ Econ & -0.18 & -0.42 & 0.42 \\ Nat & -0.63 & -0.58 & 0.40 \end{bmatrix}$$

$$\text{activists by vote intention} \quad : \quad \begin{bmatrix} & Lab & LibDem & Con \\ Econ & -0.19 & -0.45 & 0.38 \\ Nat & -0.67 & -0.76 & 0.45 \end{bmatrix}$$

[Insert Figure 7 here]

We also used the trait perceptions of the party leaders, given in Table 21 to perform a factor analysis. Table 23 reports the factor loadings for the three major party leaders, while Table 24 reports the results for the logit models-pure spatial, pure traits, spatial with traits and joint (spatial, traits and sociodemographics). Table 24 again makes clear that the traits model is far superior to the pure spatial model. However, the difference in loglikelihoods between the spatial model with traits and the pure traits model is a significant +123. Adding sociodemographics gives a significant +37. The AIC (Akaike information criterion) measures also drop significantly, as new variables are added. (Tables 25a,b give the comparisons of log likelihoods of the various models for Great Britain for both elections.)

[Insert Tables 23, 24, and 25 here]

Comparing Table 6 (model 1), the pure spatial model for 2005 with Table 27(model 1), the pure spatial model for 2010, we see immediately that Brown has low exogenous valence ($\lambda_{Lab}^{2010} = -0.04$) relative to Clegg in 2010, and this value was much lower than Blair's exogenous valence ($\lambda_{Lab}^{2005} = +0.52$) relative to Kennedy in 2005. Even when trait perceptions are included, the *significantly positive*[22] valence estimates for Blair are higher for the two nested models with traits, and spatial with traits, than the *significantly negative valence* estimates for Brown in the same models.[23]

We now turn to an analysis of the pure spatial models for Great Britain and the regions.

4.1 Equilibria in the pure spatial models for Great Britain and the regions in 2010

The electoral mean for the election was $(0.010, 0.003)$ while the activist mean was $(-0.048, -0.277)$. The Appendix shows that the three party valences for

[22] Table 6 (models 2 and 3) show that ($\lambda_{Lab}^{2005} = +0.19; +0.18$) are both statistically significant at the 10% level.

[23] Table 24 (models 2 and 3) show that ($\lambda_{Lab}^{2010} = -0.96; -0.98$) are both statistically significant at the 99% level. The lower 95% bounds on Blair's valence are higher than the upper 95% bounds on Brown's valence. These comparisons are not strictly valid, but they are indicative.

the pure spatial model in Great Britain are

$$(\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}) = (-0.04, 0.17, 0)$$

Using Labor as the low valence party we compute the central estimate of the convergence coefficient to be 0.98. Since this is less than 1, then from the Valence Theorem, the sufficient condition for convergence is satisfied. Simulation of the model confirmed this inference.[24]

Comparing the vote shares we find the predicted equilibrium shares to be:

$$(\rho_{Lab}, \rho_{Con}, \rho_{Lib}) = (0.305, 0.376, 0.319).$$

The three-way sample vote shares were

$$(s_{Lab}, s_{Con}, s_{Lib}) = (0.29, 0.41, 0.30).$$

The three-way split of actual vote shares among these parties in Great Britain was

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}) = (0.30, 0.40, 0.30).$$

Note that the lower 95% estimate $\rho_{Lib}^{lower} = 0.29 = s_{Lib} = 0.29$. By our definition this LNE is not a stable attractor.

4.1.1 Regional Models for 2010

We also ran the pure spatial model for the three regions, England, Scotland and Wales, with sample sizes for the regional models of 5465, 636, and 307 respectively. Table 26 gives the results of the pure spatial model for the regions of England, Scotland and Wales. Figures 9 and 10 show the voter and activist distributions in the three regions along with the party positions, while Figure 11 gives the activist means by party. We partition the activists by party as before.. The numbers of activists are 718, 87 and 43 in England, Scotland, and Wales, respectively.

[Insert figures 9, 10 and 11 here]

See Table 27 for the various lower and upper bounds on predicted vote shares and sample shares.

[Insert Table 27 here]

For England we have an electoral mean of $z_0^{eng} = (0.022, 0.029)$. Using

$$(\lambda_{lab}, \lambda_{con}, \lambda_{lib}) = (-0.123, 0.209, 0),$$

we computed

$$(\rho_{Lab}, \rho_{Con}, \rho_{Lib})^{eng} = (0.284, 0.395, 0.321)$$

and determined c^{eng} to be 1.085 and verified that z_0^{eng} was a pure spatial equilibrium.[25]

[24] Table 27, below, gives the estimated lower and upper 95% bounds on the coefficients as well showing that the upper 95% bound on c for Great Britain was 1.10. We show in the Appendix that the Hessian still had negative eigenvalues.

[25] We also estimated the upper 95% bound on c to be $c^{eng} = 1.22$, and estimated the Hessian still to have negative eigenvalues.

For Scotland, $c^{sct} = 1.51$, and the electoral mean $z_0^{sct} = (-0.148, -0.206)$ was a LNE.[26]

In Wales the origin $z_0^{wales} = (-0.080, 0.089)$ was not an LNE, but a saddlepoint.[27] The LNE was simulated to be:

$$\mathbf{z}_s^{wales} = \begin{bmatrix} 2010 & Lab & LibDem & Cons & PC \\ Econ & -0.140 & -0.206 & 0.010 & 0.218 \\ Nat & -0.202 & -0.457 & 0.209 & 0.696 \end{bmatrix}.$$

with vote share

$$(\rho_{Lab}^{wales}, \rho_{Con}^{wales}, \rho_{Lib}^{wales}, \rho_{pc}^{wales})^* = (0.353, 0.252, 0.270, 0.126).$$

Using these results, we obtained the following predictions of vote shares at the regional LNE to be:

England	(Lab, Con, Lib)	$=(0.284, 0.395, 0.321)$
Scotland	(Lab, Con, Lib, SNP)	$=(0.360, 0.158, 0.233, 0.249)$
Wales	(Lab, Con, Lib, PC)	$=(0.353, 0.252, 0.270, 0.126)$

The sample vote shares in each region were:

England	(Lab, Con, Lib)	$=(0.268, 0.434, 0.298)$
Scotland	(Lab, Con, Lib, SNP)	$=(0.362, 0.162, 0.230, 0.247)$
Wales	(Lab, Con, Lib, PC)	$=(0.349, 0.293, 0.251, 0.107)$

As these figures and those in the Appendix, the equilibria in England and Scotland are not stable attractors. The divergent LNE in Wales could be stable, but only the PC has an incentive to move to this LNE. A move to this LNE would entail the PC moving to much more extreme positions on both axes.

Comparing the valences of Blair and Brown across the regions we see again that Brown's exogenous valences are significantly lower than Blair's valences. For Blair we have ($\lambda_{Lab}^{eng} = 0.35$, $\lambda_{Lab}^{sct} = 0.69$, $\lambda_{Lab}^{wales} = 0.63$) and for Brown, ($\lambda_{Lab}^{eng} = -0.12$, $\lambda_{Lab}^{sct} = 0.44$, $\lambda_{Lab}^{wales} = 0.33$).[28]

4.2 Traits Models for 2010

Table 28 gives the regional traits models. Comparing the valences of Blair in 2005 and of Brown in 2010 across the regions we see again that Brown's exogenous valences are significantly lower than Blair's. For the spatial traits models for Blair in 2005 we have ($\lambda_{Lab}^{eng} = -0.23$, $\lambda_{Lab}^{sct} = 0.89$, $\lambda_{Lab}^{wales} = 0.97$) and for Brown, in 2010, ($\lambda_{Lab}^{eng} = -1.02$, $\lambda_{Lab}^{sct} = -0.96$, $\lambda_{Lab}^{wales} = -0.37$).[29]

[26] We also estimated the upper 95% bound on c to be $c^{sct} = 1.98$, and estimated the Hessian still to have negative eigenvalues.

[27] Since the joint origin is a saddlepoint, the low valence party, the PC, has an incentive to move away from the origin, along the eigenvector with positive eigenvalue.

[28] These valences are all significant, and different, at the 10% level.

[29] As we mentioned before, these comparisons are not strictly valid. Nonetheless they are indicative.

The $\beta-$coefficients in the spatial traits models in Table 24 are significant at the 99% level for Great Britain, so we can examine the spatial traits model.

[Insert Table 28 here]

Using the results of the various models we determined the equilibria to be:

$$\mathbf{z}_{st}^{el} = \begin{bmatrix} 2010 & Lab & LibDem & Con \\ Econ & -0.21 & -0.13 & 0.04 \\ Nat & -0.34 & -0.13 & 0.16 \end{bmatrix}$$

for the spatial traits model with vote share

$$(\rho_{Lab}, \rho_{Con}, \rho_{Lib})_{st} = (0.29, 0.41, 0.30).$$

and

$$\mathbf{z}_{sts}^{el} = \begin{bmatrix} 2010 & Lab & LibDem & Con \\ Econ & -0.21 & -0.11 & 0.05 \\ Nat & -0.34 & -0.14 & 0.15 \end{bmatrix}$$

for the spatial traits model with sociodemographics, with vote share

$$(\rho_{Lab}, \rho_{Con}, \rho_{Lib})_{sts} = (0.30, 0.42, 0.29).$$

These compare with the estimated positions

$$\mathbf{z}^{*} = \begin{bmatrix} 2010 & Lab & LibDem & Con \\ Econ & -0.19 & -0.34 & 0.40 \\ Nat & -0.33 & -0.35 & 0.49 \end{bmatrix},$$

with the three-way sample vote shares of

$$(s_{Lab}, s_{Con}, s_{Lib}) = (0.33, 0.41, 0.26).$$

and with three-way actual vote shares

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}) = (0.30, 0.40, 0.30).$$

Again these LNE were not stable attractors.

Because the various estimates of Brown's traits scores are much lower than the other two leaders, we find that the Labor party equilibrium position under the spatial traits models is fairly close to its estimated partisan constituency position. However, the estimated Labor three way vote share at the LNE is lower than at the partisan position. Again these various LNE were not stable attractors.

5 Conclusion

Comparing 2005 and 2010, it is clear that Labor lost the 2010 election because of Brown's low exogenous valence, as measured in all nested models, including the regional models. In particular, the drop in Brown's popularity, as measured by the valence estimates of spatial traits model, meant that in England in 2010, the Conservatives took 43% of the vote to Labor's 30.6%. In 2005, these two parties each took about 35%. The Lib Dems increased their vote share in England from 22.8% in 2005 to 26.4%, in 2010, because both Brown and Cameron had exogenous valences lower than the Lib Dem leader, Clegg.[30]

The local vote maximizing equilibrium at the joint origin is confirmed for the pure spatial models for the election in Great Britain, in 2005, considering just the three major parties, as well as in the regions when small regional parties are included. We obtain the similar results for 2010, except possibly for Plaid Cymru in Wales. For both the 2005 and 2010 elections, the models involving traits are statistically superior to the pure spatial models. However, combining the spatial model with traits gives a model that has superior log-likelihoods. In 2010, the large traits differences between Brown and the other two leaders gives a divergent equilibrium, with Brown's equilibrium position relatively close to his estimated position. However, at this equilibrium, we estimate that the Labor party would lose vote share.

We have based this analysis on the supposition that parties are located at the partisan constituency positions. On this assumption we have shown that the spatial component adds statistical significance. We have performed the thought experiment to locate local equilibria to these various model. To deal with the stochastic uncertainty of the spatial model, we have argued that an equilibrium needs to be a stable attractor for a low valence party, so that such a party will be willing to shift position to the equilibrium. Our estimations suggest that only the non-centrist pure spatial local equilibrium in Wales in 2010 could be a stable attractor, and this would require the Plaid Cymru to move to much more extreme positions on both axes.

We suggest instead that the party partisan positions provide suitable estimates of the various positions of the parties. Moreover, because activists for the parties tend to have somewhat more extreme positions than the party voters, activists will exert themselves to maintain each party at a position that is more congenial to the activists.

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[30] In Blair's new book (Blair, 2010) he writes of Gordon Brown, "Political Calculation, yes. Political feelings, no. Analytical intelligence, absolutely. Emotional intelligence, zero. The British electorate appear to have had the same feelings.

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7 Appendix: The Formal Stochastic Model

The electoral model presented here is an extension of the multiparty stochastic model of McKelvey and Patty (2006), modified by inducing asymmetries in terms of valence. The justification for developing the model in this way is the empirical evidence that valence is a natural way to model the judgements made by voters of party leaders or candidates. There are a number of possible choices for the appropriate model for multiparty competition. The simplest one, which is used here, is that the utility function for the candidate of party j is proportional to the anticipated vote share, V_j, of the party in the election. [31]

With this assumption, we can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE). Because the vote share functions are differentiable, we use calculus techniques to obtain conditions for positions to be locally optimal. Thus we examine what we call *local pure strategy Nash equilibria* (LNE). From the definitions of these equilibria it follows that a PNE must be a LNE, but not conversely.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of policy choices, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that candidates cannot predict vote response precisely, but that they can estimate the effect of policy proposals on the expected vote share. Implicitly we assume that parties rationally anticipate the electoral outcome of any policy decision they make. Parties use focus groups in addition to regularly polling voters in order to determine the electoral response to local policy changes in the period leading up to the election. In the model with valence, the stochastic element is associated with the weight given by each voter, i, to the average perceived quality or valence of the candidate.

Definition 1.***The Stochastic Vote Model*** $\mathbb{M}(\boldsymbol{\lambda},\boldsymbol{\theta},\boldsymbol{\mu},\beta)$ ***with Activist Valence***.

The data of the spatial model is a distribution, $\{x_i \in X\}_{i\in N}$, of voter ideal points for the members of the electorate, N, of size n. We assume that X is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Without loss of generality, we adopt coordinate axes so that $\frac{1}{n}\Sigma x_i = 0$. By assumption $0 \in X$, and this point is termed the *electoral mean.* Each of the parties in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare prior to the specific election to be modeled. Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of candidate policy positions.

We define a stochastic electoral model, which utilizes sociodemographic variables and voter perceptions of character traits. For this model we assume that voter i utility is given by the expression

[31] For refining the model, and for empirical analysis, we can adapt the model so that parties choose positions to maximize their seat shares, relative to a given constituency structure. We adopt this simplifying assumption in order to present the essential structure of the formal model.

$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$ where

$$u_{ij}(x_i, z_j) = \lambda_j + (\theta_j \cdot \eta_i) + (\alpha_j \cdot \tau_i) - \beta\|x_i - z_j\|^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j. \quad (1)$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The constant term, λ_j, is the exogenous *valence* of party j, and the exogenous valence vector $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ is assumed to satisfy $\lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1$. In empirical multinomial logit models, the valence vector, $\boldsymbol{\lambda}$, is estimated by the intercept terms for each party. The points $\{x_i : i \epsilon N\}$ are the preferred policies of the voters and $\mathbf{z} = \{z_j : j \epsilon P\}$ are the positions of the parties. The term $\|x_i - z_j\|$ is simply the Euclidean distance between x_i and z_j. The error vector $\boldsymbol{\epsilon} = (\epsilon_1,...,\epsilon_j,..,\epsilon_p)$ is distributed by the type I extreme value distribution, as assumed in empirical MNL estimation (Train, 2003), and defined below. The variance of ϵ_j is fixed at $\frac{\pi^2}{6}$, so that by definition β has dimension $\frac{1}{L^2}$, where L is whatever unit of measurement is used in X.

Sociodemographic aspects of voting are modeled by $\boldsymbol{\theta}$, a set of k -vectors $\{\theta_j : j \in P\}$ representing the effect of the k different sociodemographic parameters (class, domicile, education, income, religious orientation, etc.) on voting for party j while η_i is a k-vector denoting the i^{th} individual's relevant "sociodemographic" characteristics. The compositions $\{(\theta_j \cdot \eta_i)\}$ are scalar products, called the *sociodemographic valences* for j.

The terms $\{(\alpha_j \cdot \tau_i)\}$ are scalars giving voter i's perceptions and beliefs about the character *traits* of party j. We could in principle incorporate individual beliefs about the ability of the parties to influence the state of the economy, etc. We let $\boldsymbol{\alpha} = (\alpha_p, ...\alpha_1)$. A *trait score* can be obtained by factor analysis from a set of survey questions asking respondents about the traits of the party leader, such as 'competent', 'responsive', 'trustworthy', etc. The perception of traits could be augmented with voter perception of the state of the economy, etc. in order to examine how anticipated changes in the economy affect each party's electoral support.

The partial models include a pure spatial model, $\mathbb{M}(\boldsymbol{\lambda}, \beta)$, a pure sociodemographic model, $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta},)$, a pure trait model, $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\alpha}, \beta)$, and *joint* models, with or without traits, $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \beta)$ and $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \beta)$.

In all models, voter behavior is modeled by a probability vector. The probability that a voter i chooses party j at the vector $\mathbf{z}$ is

$$\begin{aligned} \rho_{ij}(\mathbf{z}) &= \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j]. && (2) \\ &= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j]. && (3) \end{aligned}$$

Here Pr stands for the probability operator generated by the distribution assumption on ϵ. The *expected vote share* of party j is

$$V_j(\mathbf{z}) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(\mathbf{z}). \quad (4)$$

This definition assumes that each voter has equal weight, $\frac{1}{n}$. The following analysis can be carried out when voters have different weight. The differentiable function $V : X^p \to \mathbb{R}^p$ is called the *party profile function*.

Definition 2: ***The Type I Extreme Value Distribution,*** Ψ.

(i) The cumulative distribution, Ψ, has the closed form

$$\Psi(h) = \exp\left[-\exp\left[-h\right]\right],$$

with probability density function

$$\psi(h) = \exp[-h]\exp\left[-\exp\left[-h\right]\right]$$

and variance $\frac{1}{6}\pi^2$ (Train, 2003).

(ii) For each voter i, and party j, the probability that a voter i chooses party j at the vector $\mathbf{z}$ is

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{p} \exp u^*_{ik}(x_i, z_k)}. \tag{5}$$

In this stochastic electoral model it is assumed that each party j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, .., z_{j-1}, z_{j+1}, .., z_p)$.

Definition 3. ***Equilibrium Concepts.***

(i) A strategy vector $\mathbf{z}^*$=$(z_1^*, ..., z_{j-1}^*, z_j^*, z_{j+1}^*, ..., z_p^*)$ is a *local Nash equilibrium* (LNE) iff, for each party j, there exists a neighborhood X_j of z_j^* in X such that

$$V_j(z_1^*, ..., z_{j-1}^*, z_j^*, z_{j+1}^*, ..., z_p^*) \geq V_j(z_1^*, ..., z_j, ..., z_p^*) \text{ for all } z_j \in X_j.$$

(ii) A strategy vector $\mathbf{z}^*$=$(z_1^*, ..., z_{j-1}^*, z_j^*, z_{j+1}^*, ..., z_p^*)$ is a *pure strategy Nash equilibrium* (PNE) iff X_j can be replaced by X in (i).

We can also define *strict local Nash equilibria* (SLNE) and *strict Nash equilibria* (SPNE) by requiring strict inequalities in the definitions.

From the definitions, it follows that if $\mathbf{z}^*$ is a PNE it must be an LNE.

Notice that in this model, each party is uncertain about the precise electoral outcome, because of the stochastic component of voter choice.

In real life, parties use focus groups and opinion poll data to estimate the effect of their policy decisions on their vote shares at the time of election. The model essentially assumes that parties utilize such information by searching for a "local equilibrium" policy position in order to gain as many votes as possible.

It follows for the model $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\mu}, \beta)$, that for voter i, with ideal point, x_i, the probability, $\rho_{ij}(\mathbf{z})$, that i picks j at $\mathbf{z}$ is given by

$$\rho_{ij}(\mathbf{z}) = [[1 + \Sigma_{k \neq j}[\exp(f_{kj})]]^{-1} \tag{6}$$

where

$$f_{kj} = u^*_{ik}(x_i, z_k) - u^*_{ij}(x_i, z_j). \tag{7}$$

$$\text{Thus } \frac{d\rho_{ij}}{dz_j} = \{2\beta(x_i - z_j) + \frac{d\mu_1}{dz_1}(z_j)\}[\rho_{ij} - \rho_{ij}^2]. \tag{8}$$

We use this gradient equation to show that the first order condition for $\mathbf{z}^*$ to be a LNE is given by

$$\frac{dV_j(\mathbf{z})}{dz_j} = \frac{1}{n}\sum_{i\in N}\frac{d\rho_{ij}}{dz_j} = 0. \tag{9}$$

7.1 The Model without traits.

We first consider the model $\mathbb{M}(\boldsymbol{\lambda},\beta)$,by setting $\boldsymbol{\mu} = \boldsymbol{\theta} = \mathbf{0}$.

In this case, the first order condition is

$$\begin{aligned}\frac{dV_j(\mathbf{z})}{dz_j} &= \frac{1}{n}\sum_{i\in N}\frac{d\rho_{ij}}{dz_j} &(10)\\ &= \frac{1}{n}\sum_{i\in N}\{2\beta(x_i - z_j)\}[\rho_{ij} - \rho_{ij}^2] &(11)\end{aligned}$$

Suppose that all z_j are identical. Then all ρ_{ij} are independent of $\{x_i\}$ and thus of i, and ρ_{ij} may be written as ρ_j. Then for each fixed j, the first order condition is

$$\frac{dV_j(\mathbf{z})}{dz_j} = 2\beta[\rho_j - \rho_j^2]\sum_{i\in N}[(x_i - z_j)] = 0 \tag{12}$$

Thus, when there is only exogenous valence, then for all j, balance solution satisfies $z_j = \frac{1}{n}\sum_{i\in N} x_i$, the *electoral mean.* We denote by $\mathbf{z}_0$ the vector where each z_j is given by the electoral mean, and call this vector the *joint electoral mean.*[32]

Definition 4: ***The Convergence Coefficient of the Model*** $\mathbb{M}(\boldsymbol{\lambda},\beta)$ when the space X has dimension w.

(i) Define

$$\rho_1 = \left[1 + \sum_{k=2}^{p}\exp\left[\lambda_k - \lambda_1\right]\right]^{-1}. \tag{13}$$

(ii) Let X be endowed with an orthogonal system of coordinate axes $(1,\ldots,s,\ldots,t,\ldots,w)$. For each coordinate axis let $\xi_t = (x_{1t}, x_{2t},\ldots,x_{nt}) \in \mathbb{R}^n$ be the vector of the t^{th} coordinates of the set of n voter ideal points. Let $(\xi_s,\xi_t) \in \mathbb{R}$ denote scalar product. The covariance between the s^{th} and t^{th} axes is denoted $(\sigma_s,\sigma_t) = \frac{1}{n}(\xi_s,\xi_t)$ and $\sigma_s^2 = \frac{1}{n}(\xi_s,\xi_s)$ is the electoral variance on the s^{th} axis. Note that these variances and covariances are taken about the electoral means on each axis.

(iii) The symmetric $w \times w$ *electoral covariance matrix* ∇_0 is defined to be $\frac{1}{n}\left[(\xi_s,\xi_t)\right]_{t=1,..w}^{s=1,..w}$.

[32] Since we can renormalize and set $\Sigma x_i = 0$,we can write $\mathbf{z}_0 = (0,...,0)$ and call this vector the joint origin.

(iv) The *electoral variance* is

$$\sigma^2 = \sum_{s=1}^{w} \sigma_s^2 = \frac{1}{n} \sum_{s=1}^{w} (\xi_s, \xi_s) = \text{trace}(\nabla_0).$$

(v) The $w \times w$ *characteristic matrix* of party 1 is given by

$$C_1 = 2\beta(1 - 2\rho_1)\nabla_0 - I. \tag{14}$$

(vi) The *convergence coefficient of the model* $\mathbb{M}(\boldsymbol{\lambda},\beta)$ is

$$c \equiv c(\boldsymbol{\lambda},\beta) = 2\beta[1 - 2\rho_1]\sigma^2. \tag{15}$$

Observe that the $\beta-$parameter has dimension $L^{-2} = \frac{1}{L^2}$, so that c is dimensionless. We can therefore use c to compare different models.

Note also that party 1 is by definition the party with the lowest valence, and ρ_1, as defined above, is the probability that a generic voter will choose this party when all parties are located at the mean. The estimate of the probability ρ_1 depends only on the comparison functions $\{f_{kj}\}$, and these can be estimated in terms of the valence differences.

The following result is proved in Schofield (2007).

Mean Voter Valence Theorem.

(i) The joint mean $\mathbf{z}_0$ satisfies the first order condition to be a LNE for the model $\mathbb{M}(\boldsymbol{\lambda},\beta)$.

(ii) The necessary and sufficient second order condition for SLNE at $\mathbf{z}_0$ is that C_1 has negative eigenvalues.[33]

(iii) A *necessary* condition for $\mathbf{z}_0$ to be a SLNE for the model $\mathbb{M}(\boldsymbol{\lambda},\beta)$ is that $c(\boldsymbol{\lambda},\beta) < w$.

(iv) A *sufficient* condition for convergence to $\mathbf{z}_0$ in the two dimensional case is that $c < 1$. $\square$

Notice that (iii) follows from (ii) since the condition of negative eigenvalues means that trace $(C_1) = 2\beta[1 - 2\rho_1]\sigma^2 - w < 0$. In the case $c(\boldsymbol{\lambda},\beta) = w$, then trace$(C_1) = 0$, which means either that all eigenvalues are zero, or some are positive. This degenerate situation requires examination of C_1. The additional condition $c < 1$ is sufficient to guarantee that $det(C_1) > 0$, which ensures that both eigenvalues are negative.

The expression for C_1 has a simple form because of the assumption of a single distance parameter β. It is possible to use a model with different coefficients $\boldsymbol{\beta} = \{\beta_1, \beta_2 ..., \beta_w\}$ on each dimension. That is we assume the spatial component of the utility of voter i has the form

$$-\sum_{k=1}^{w} \beta_k (x_{ik} - z_{jk})^2$$

[33] In the usual way, the condition for an LNE is that the eigenvalues are negative semi-definite.

In this case the characteristic matrix can readily be shown to be

$$\mathbf{C}_1 = 2(1-2\rho_1)\boldsymbol{\beta}\nabla_0\boldsymbol{\beta} - \boldsymbol{\beta}, \tag{16}$$

where $\boldsymbol{\beta}$ is the diagonal matrix of the the β coefficients, while $\boldsymbol{\beta}\nabla_0\boldsymbol{\beta}$ is the covariance matrix where each axis is weighted by the coefficients $\boldsymbol{\beta} =\beta_1, \beta_2, .., \beta_w$. The necessary condition in this case is that trace$(\mathbf{C}_1) < 0$, or

$$2(1-2\rho_1)trace(\boldsymbol{\beta}\nabla_0\boldsymbol{\beta}) <\beta_1 + \beta_2 ... + \beta_w.$$

and we can take

$$c(\boldsymbol{\lambda}, \boldsymbol{\beta}) = \frac{2(1-2\rho_1)trace(\boldsymbol{\beta}\nabla_0\boldsymbol{\beta})}{\frac{1}{w}(\beta_1 + \beta_2 ... + \beta_w)} \tag{17}$$

again giving the necessary condition of $c(\boldsymbol{\lambda}, \boldsymbol{\beta}) \leq w$, for a LNE.

Note that if C_1 has negative eigenvalues, then the Hessians of the vote shares for all parties are negative definite at the joint mean. When this is true, then the joint mean is a candidate for a PNE, and this property can be verified by simulation.

When the convergence condition $c(\boldsymbol{\lambda},\beta) < w$ is violated the joint mean cannot be a SPNE. In the degenerate case $c(\boldsymbol{\lambda}, \boldsymbol{\beta}) = w$ it is necessary to examine the characteristic matrix to determine whether the joint mean can be a PNE.

To determine whether an LNE is a stable attractor, it is important to estimate the 95% upper and lower bounds on ρ_j. If we let h be the 95% bound on λ_j then we we use Taylor's Theorem, which asserts that

$$\rho_j(\lambda_j \pm h) = \rho_j(\lambda_j) \pm h\frac{d\rho_j}{d\lambda_j} = \rho_j(\lambda_j)[1 \pm h(1-\rho_j(\lambda_j)]. \tag{18}$$

7.2 The Model with activist valence functions

We also consider an activist model $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \beta)$ given by the utility assumption,

$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$ where

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) + (\theta_j \centerdot \eta_i) + (\alpha_j \centerdot \tau_i) - \beta\|x_i - z_j\|^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j. \tag{19}$$

The terms $\{\mu_j : j\epsilon P\}$ are the endogenous *activist valence functions*.

Definition 5: The balance solution for the model $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\mu},\beta)$.

Let $[\rho_{ij}(\mathbf{z})] =[\rho_{ij}]$ be the n by p matrix of voter probabilities at the vector, which we denote by $\mathbf{z}^{\boldsymbol{\mu}}$, and let

$$[\varpi_{ij}] = \left[\frac{\rho_{ij} - \rho_{ij}^2}{\Sigma_{k=1}^n(\rho_{kj} - \rho_{kj}^2)} \right] \tag{20}$$

be the $n \times p$ matrix of weighting coefficients.

The *balance equation* for z_j^{μ} is given by expression

$$z_j^{\boldsymbol{\mu}} = \frac{1}{2\beta}\frac{d\mu_j}{dz_j} + \sum_{i=1}^{n} \varpi_{ij} x_i. \tag{21}$$

The vector $\sum_i \varpi_{ij} x_i$ is a convex combination of the set of voter ideal points. This vector is called the *weighted electoral mean* for j. Define

$$z_j^{el} = \sum_i \varpi_{ij} x_i. \tag{22}$$

The balance equations for $j = 1, ..., p$ can then be written as

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j^{\boldsymbol{\mu}}) + \frac{1}{2\beta}\frac{d\mu_j}{dz_j}(z_j^{\boldsymbol{\mu}}) = 0. \tag{23}$$

The first term in this equation is termed the *centripetal marginal electoral pull of* party j and is defined at z_j by

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j) = \left[z_j^{el} - z_j\right].$$

It is a gradient vector pointing from z_j towards the *weighted electoral mean*, z_j^{el}, of the party. This weighted electoral mean is that point where the electoral pull is zero. Notice that the each entry in the vector $\mathbf{z}^{el} = (z_1^{el}, z_2^{el}, ...z_p^{el})$ depends on all other entries. The vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for party* j. In vector notation we write:

$$\frac{d\mathbf{E}^*}{d\mathbf{z}}(\mathbf{z}^{\boldsymbol{\mu}}) = \left[\mathbf{z}^{el} - \mathbf{z}^{\boldsymbol{\mu}}\right].$$

$$\frac{d\mathbf{E}^*}{d\mathbf{z}}(\mathbf{z}^{\boldsymbol{\mu}}) + \frac{1}{2\beta}\frac{\mathbf{d}\boldsymbol{\mu}}{\mathbf{dz}}(\mathbf{z}^{\boldsymbol{\mu}}) = 0.$$

If $\mathbf{z}^{\boldsymbol{\mu}}$ satisfies the system of balance equations, for all j, then call $\mathbf{z}^{\boldsymbol{\mu}}$ a *balance solution*.

The following theorem is proved in Schofield (2006).

Activist Balance Theorem. Consider the electoral model $\mathbb{M}(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\mu}, \beta)$ based on the Type I extreme value distribution, and including both exogenous and activist valences.

(i) The first order condition for $\mathbf{z}^{\boldsymbol{\mu}}$ to be an LNE is that it is a balance solution.

(ii) If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then a balance solution will be a LNE. □

Notice that if X is open, then this first order condition at $\mathbf{z}^{\boldsymbol{\mu}}$ is necessary for $\mathbf{z}^{\boldsymbol{\mu}}$ to be a PNE. We implicitly assume that any relevant $\mathbf{z}^{\boldsymbol{\mu}}$ will lie in the interior of X.

In versions of these models we shall write $\mathbf{z}^{el}$ for the LNE. This may or may not be equal to $\mathbf{z}_0$.

8 Computations

8.1 Pure Spatial Models for Great Britain for 2005

The Appendix defines the electoral covariance matrix for any election. From the 2005 survey this was estimated to be

$$\nabla_0 = \begin{bmatrix} & x & y \\ x & 1.646 & 0.00 \\ y & 0.00 & 3.961 \end{bmatrix}.$$

with esd (electoral standard deviation) = $\sigma = 2.36$. The pure spatial model in Table 6 gives

$$(\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}, \beta) = (0.52, 0.27, 0, 0.15).$$

Thus the probability a generic voter picks the Liberal Democratic party, when all parties are at the mean, is:

$$\rho_{lib} = \frac{\exp(0)}{\exp(0.518) + \exp(0.272) + \exp(0)} = 0.25,$$

which is similar to the actual share of 24.6% and the sample share of 24.5% (with respect to the three major parties) in Great Britain. The probabilities that a generic voter picks the various parties when all three parties are located at the electoral mean is given by the vector:

$$\boldsymbol{\rho}_s^{GB} = (\rho_{Lab}, \rho_{Con}, \rho_{Lib}) = (0.42, 0.33, 0.25).$$

Table 7 compares these estimates with the three way party shares, and also gives the lower and upper 95% estimates on ρ.

[Insert Table 7 here]

These values give the best estimate, on the basis of the pure spatial model, of the three-way vote shares when all parties are at the mean. The actual three-way split of vote shares among these parties was

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}) = (0.394, 0.36, 0.246)$$

and the split of sample shares was

$$(s_{Lab}, s_{Con}, s_{Lib}) = (0.415, 0.34, 0.245).$$

The estimated vote shares at the joint mean, and the actual and sample three party vote shares are quite close. We now show formally that the joint mean is an LNE. Regarding the Lib Dems as the low valence party, then from the

Appendix we obtain:

$$\begin{aligned} C_{Lib} &= 2\beta(1-2\rho_{lib})\nabla_0 - I \\ &= 2(0.150)(0.5)\begin{bmatrix} 1.646 & 0 \\ 0 & 3.961 \end{bmatrix} - I = \begin{bmatrix} 0.246 & 0 \\ 0 & 0.593 \end{bmatrix} - I \\ &= \begin{bmatrix} -0.754 & 0 \\ 0 & -0.407 \end{bmatrix}, \\ \text{with } c^{GB} &= 2\beta(1-2\rho_{lib})trace(\nabla_0) \\ &= 2(0.15)(0.5)(5.6) = 0.84. \end{aligned}$$

The trace is negative and the determinant is positive. From the Valence Theorem, the sufficient condition for convergence is satisfied, and we estimate that the joint mean is an LNE for the pure spatial model with three parties.

We can use the 95% confidence interval of $\beta \in [0.15 \pm 1.96 \times 0.01] = [0.13, 0.17]$ and $\rho_{lib} \in [0.22, 0.28]$, the lower and upper bounds of c and C are given by,

$$c^{GB} = [2(0.13)(1-2\times0.28)(5.61), 2(0.17)(1-2\times0.22)(5.61)] = [0.62, 1.08]$$

$$C_{lib} = 2(0.13)(1-2\times0.28)\begin{bmatrix} 1.65 & 0.00 \\ 0.00 & 3.96 \\ & \end{bmatrix} - I, \quad 2(0.17)(1-2\times0.22)\begin{bmatrix} 1.65 & 0.00 \\ 0.00 & 3.96 \\ & \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.82 & 0.00 \\ 0.00 & -0.56 \\ & \end{bmatrix}, \quad \begin{bmatrix} -0.68 & 0.00 \\ 0.00 & -0.24 \\ & \end{bmatrix}$$

Again,the eigenvalues are negative and we can confirm that $\mathbf{z}_0$ is the LNE with probability over 95%.

Using Taylors Theorem gives the 95% bounds on ρ_{lib} to be approximately ±0.03 so the lower 95% bound on $\rho_{lib} = 0.22$ while $s_{Lib} = 0.245$, giving a negative vote margin, so the LNE is not a stable attractor.

Comparison of the model with separate $\beta-$coefficients, $(\beta_{Econ}, \beta_{Nat})$ gave a difference in loglikelihoods is +28, suggesting that this model is superior to one with a single $\beta-$coefficient. We obtain:

$$c(\boldsymbol{\lambda}, \boldsymbol{\beta}) = \frac{2(1-2\rho_{lib})trace(\boldsymbol{\beta}\nabla_0\boldsymbol{\beta})}{\frac{1}{w}(\beta_1 + \beta_2 ... + \beta_w)}$$

with $\frac{1}{2}(\beta_{Econ} + \beta_{Nat}) = \frac{1}{2}(0.388 + 0.131) = 0.255$ and $\rho_{lib} = 0.25$, we find

$$\begin{aligned} c(\boldsymbol{\lambda}, \boldsymbol{\beta}) &= \frac{2(0.5)}{0.255} trace \begin{bmatrix} (0.388)^2 1.646 & 0 \\ 0 & (0.131)^2 3.961 \end{bmatrix} \\ &= (3.92) trace \begin{bmatrix} 0.24 & 0 \\ 0 & 0.08 \end{bmatrix} = 1.25, \end{aligned}$$

while

$$
\begin{aligned}
C_{lib} &= 2(1-2\rho_1)\boldsymbol{\beta}\nabla_0\boldsymbol{\beta} - \boldsymbol{\beta} \\
&= \begin{bmatrix} 0.24 & 0 \\ 0 & 0.08 \end{bmatrix} - \begin{bmatrix} 0.388 & 0 \\ 0 & 0.131 \end{bmatrix} \\
& \begin{bmatrix} -0.148 & 0 \\ 0 & -0.05 \end{bmatrix}
\end{aligned}
$$

Again we find an LNE at the joint mean $\mathbf{z}_0$, but we also find that the LNE for this model is not a stable attractor.

8.1.1 England in 2005

For England

$$
\begin{aligned}
(\lambda_{lab}, \lambda_{con}, \lambda_{lib}, \beta) &= (0.35, 0.31, 0, 0.14) \\
\boldsymbol{\rho}_s^{eng} &= (\rho_{lab}, \rho_{con}, \rho_{lib}) \\
&= (0.38, 0.36, 0.26)
\end{aligned}
$$

The joint mean is $z_0^{eng} = (0.95, 0.156)$. We obtain:

$$
\begin{aligned}
\nabla_0^{eng} &= \begin{bmatrix} & x & y \\ x & 1.663 & 0.033 \\ y & 0.033 & 4.009 \end{bmatrix}, \\
\sigma^{eng} &= 2.38 \\
(\lambda_{Lab}^{eng}, \lambda_{Con}^{eng}, \lambda_{Lib}^{eng}, \beta^{eng}) &= (0.35, 0.31, 0, 0.14). \\
\rho_{lab}^{eng} &= 0.376, \rho_{con}^{eng} = 0.360, \rho_{lib}^{eng} = 0.264, \\
c^{eng} &= 2\beta(1-2\rho_{lib}^{eng})trace(\nabla_0^{eng}) \\
&= 2(0.14)(0.48)(5.67) = 0.75,
\end{aligned}
$$

again using the Lib Dems as the low valence party.

Using the 95% confidence interval of $\beta \in [0.14 \pm 1.96 \times 0.01] = [0.12, 0.16]$ and $\rho_{lib} \in [0.23, 0.30]$ the lower and upper bounds of c and C are given by,

$$
c^{eng} = [2(0.12)(1-2\times 0.30)(5.67), 2(0.16)(1-2\times 0.23)(5.67)] = [0.53, 1.00]
$$

$$
C_{Lib} = 2(0.12)(1-2\times 0.30) \begin{bmatrix} 1.65 & 0.00 \\ 0.00 & 3.96 \\ & \end{bmatrix} - I, \quad 2(0.16)(1-2\times 0.23) \begin{bmatrix} 1.65 & 0.00 \\ 0.00 & 3.96 \\ & \end{bmatrix} - I
$$

$$
= \begin{bmatrix} -0.85 & 0.00 \\ 0.00 & -0.63 \\ & \end{bmatrix}, \quad \begin{bmatrix} -0.71 & 0.01 \\ 0.01 & -0.29 \\ & \end{bmatrix}
$$

implying that z_0^{eng} is the regional LNE with probability greater than 95%.

Table 8 shows that the three-way sample votes shares in England are

$$(s_{Lab}, s_{Con}, s_{Lib}) = (0.364, 0.384, 0.251).$$

Just as for the previous example, $\rho_{lib}^{lower} = 0.23 < s_{Lib} = 0.251$ so $(0.95, 0.156)$ is not a stable attractor.

8.1.2 Scotland in 2005

For Scotland the joint mean is $z_0^{sct} = (-0.99, -0.309)$. We obtain

$$\begin{aligned}
\nabla_0^{sct} &= \begin{bmatrix} & x & y \\ x & 1.597 & 0.097 \\ y & 0.097 & 3.88 \end{bmatrix}, \\
\sigma^{sct} &= 2.34 \\
(\lambda_{Lab}^{sct}, \lambda_{Con}^{sct}, \lambda_{Lib}^{sct}, \lambda_{snp}^{sct}, \beta^{sct}) &= (0.69, 0.05, 0, -0.10, 0.139). \\
\rho_{Lab}^{sct} &= 0.403, \rho_{Con}^{sct} = 0.212, \rho_{Lib}^{sct} = 0.202, \rho_{snp}^{sct} = 0.184 \\
c^{sct} &= 2\beta(1 - 2\rho_{snp}^{sct}) trace(\nabla_0^{sct}) \\
&= 2(0.14)(0.63)(5.47) = 0.97,
\end{aligned}$$

with the SNP as the low valence party.

Using the 95% confidence interval of $\beta \in [0.14 \pm 1.97 \times 0.02] = [0.09, 0.19]$ and $\rho_{snp} \in [0.14, 0.24]$, the lower and upper bounds of c^{sct} and C_{snp} are given by

$$c^{sct} = [2(0.09)(1 - 2 \times 0.14)(5.48), 2(0.19)(1 - 2 \times 0.24)(5.48)] = [0.53, 1.47]$$

$$C_{snp} = 2(0.09)(1 - 2 \times 0.14) \begin{bmatrix} 1.58 & 0.10 \\ 0.10 & 3.88 \\ & \end{bmatrix} - I, \quad 2(0.19)(1 - 2 \times 0.24) \begin{bmatrix} 1.58 & 0.10 \\ 0.10 & 3.88 \\ & \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.85 & 0.01 \\ 0.01 & -0.63 \\ & \end{bmatrix}, \quad \begin{bmatrix} -0.57 & 0.03 \\ 0.03 & 0.04 \\ & \end{bmatrix}$$

Thus z_0^{sct} is the regional LNE with probability greater than 95%. At the equilibrium $z_0^{sct} = (-0.99, -0.309)$ the four way vote shares are predicted to be :

$$\boldsymbol{\rho}_s^{sct} = (\rho_{Lab}, \rho_{Con}, \rho_{Lib}, \rho_{snp})^{sct} = (0.403, 0.212, 0.202, 0.184)$$

The sample shares are

$$(s_{Lab}, s_{Con}, s_{Lib}, s_{snp}) = (0.406, 0.202, 0.270, 0.185).$$

Since the lower 95% bound is $\rho_{snp}^{sct} = 0.14$ while $s_{snp} = 0.185$, the margin $\delta = -0.045$, so z_0^{sct} is not a stable attractor.

8.1.3 Wales in 2005

For Wales the electoral mean is $z_0^{wales} = (-0.126, 0.276)$. We obtain

$$\begin{aligned}
\nabla_0^{wales} &= \begin{bmatrix} & x & y \\ x & 1.41 & 0.16 \\ y & 0.16 & 3.50 \end{bmatrix}, \\
\sigma^{wales} &= 2.2 \\
\lambda_{Lab}^{wales}, \lambda_{Con}^{wales}, \lambda_{Lib}^{wales}, \lambda_{pc}^{wales}, \beta^{wales}) &= (0.63, 0.11, 0, -0.66, 0.106). \\
\rho_{lab}^{wales} &= 0.416, \rho_{con}^{wales} = 0.248, \rho_{lib}^{wales} = 0.222, \rho_{pc}^{wales} = 0.114 \\
c^{wales} &= 2\beta(1 - 2\rho_{pc}^{wales}) trace(\nabla_0^{wales}) \\
&= 2(0.106)(0.78)(4.91) = 0.80,
\end{aligned}$$

with Plaid Cymru as the low valence party. Using the 95% confidence interval of $\beta \in [0.11 \pm 1.97 \times 0.03] = [0.05, 0.16]$ and $\rho_{pc} \in [0.08, 0.17]$, the lower and upper bounds of c^{wqles} and C_{pc} are given by

$$c^{wales} = [2(0.05)(1 - 2 \times 0.17)(4.91), 2(0.16)(1 - 2 \times 0.08)(4.91)] = [0.35, 1.30]$$

$$\begin{aligned}
C_{pc} &= 2(0.05)(1 - 2 \times 0.17) \begin{bmatrix} 1.41 & 0.16 \\ 0.16 & 3.50 \\ & \end{bmatrix} - I, \quad 2(0.16)(1 - 2 \times 0.08) \begin{bmatrix} 1.41 & 0.16 \\ 0.16 & 3.50 \\ & \end{bmatrix} - I \\
&= \begin{bmatrix} -0.90 & 0.01 \\ 0.01 & -0.75 \\ & \end{bmatrix}, \quad \begin{bmatrix} -0.63 & 0.04 \\ 0.04 & -0.08 \\ & \end{bmatrix}
\end{aligned}$$

Again we compute C_{pc} to have negative eigenvalues, with probability in excess of 95%, and $z_0^{wales} = (-0.126, 0.276)$ is the regional LNE.

The sample shares are

$$(s_{Lab}, s_{Con}, s_{Lib}, s_{pc}) = (0.412, 0.262, 0.208, 0.119).$$

The lower 95% estimate of $\rho_{pc}^{wales} = 0.08$ and $s_{pc} = 0.12$,so the margin is $\delta = 0.08 - 0.12 = -0.04 < 0$.

These arguments indicate that, with 95% probability, all the Hessians of the low valence parties have negative eigenvalues. We infer that convergence to the joint mean, even in these regions, is an equilibrium prediction for all parties.[34] However, these regional means are not stable attractors.

[34] In the estimations, the valences of the SNP in Scotland and Plaid Cymru in Wales are not significantly different from zero. Assuming valences of zero would give *higher* estimates of ρ_{snp}^{sct} and ρ_{con}^{wales}, and therefore *lower* values of c^{sct} and c^{wales}. Using the 95% bounds to construct appropriate bounds on our estimates of the convergence provides a more robust confirmation of our conclusion.

8.2 Pure spatial models for Great Britain and the regions in 2010

The covariance matrix for Great Britain in 2010 is

$$\nabla_0 = \begin{bmatrix} & x & y \\ x & 0.60 & 0.07 \\ y & 0.07 & 0.86 \end{bmatrix}.$$

with electoral standard deviation $\sigma = 1.21$. The pure spatial model in Table 24 gives

$$\begin{aligned} (\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}, \beta)^{GB} &= (-0.04, 0.17, 0, 0.86) \\ \text{so } (\rho_{Lab}, \rho_{Con}, \rho_{Lib})^{GB} &= (0.305, 0.376, 0.319). \end{aligned}$$

Labor is the lowest valence party according to the pure spatial model . If all three parties were located at the mean, then the Labor party vote share would be 0.305.

We can compute the convergence coefficient as follows:

$$\begin{aligned} C_{Lab} &= 2\beta(1-2\rho_{Lab})\nabla_0 - I \\ &= 2(0.834)(0.388) \begin{bmatrix} & x & y \\ x & 0.60 & 0.07 \\ y & 0.07 & 0.86 \end{bmatrix} - I \\ &= \begin{bmatrix} -0.584 & 0.028 \\ 0.028 & -0.436 \end{bmatrix} \\ \text{with } c^{GB} &= 2\beta(1-2\rho_{Lab})trace(\nabla_0) \\ &= 2 \cdot (0.857)(1-2(0.305)) \cdot (1.462) = 0.976. \\ &= 2(0.834)(0.388)(1.51) = 0.98 \end{aligned}$$

The convergence coefficient is less than 1, and from the Valence Theorem, the sufficient condition for convergence is satisfied.

Using the 95% confidence interval of $\beta \in [0.86 \pm 1.96 \times 0.02] = [0.81, 0.90]$ and $\rho_{lab} \in [0.29, 0.32]$, the lower and upper bounds of c^{GB} and C_{Lab} are given by,

$$c^{GB} = [2(0.81)(1-2\times 0.32)(1.46), 2(0.90)(1-2\times 0.29)(1.46)] = [0.86, 1.10]$$

$$C_{Lab} = 2(0.81)(1-2\times 0.32) \begin{bmatrix} 0.60 & 0.07 \\ 0.07 & 0.86 \\ & \end{bmatrix} - I, \quad 2(0.17)(1-2\times 0.25) \begin{bmatrix} 0.60 & 0.07 \\ 0.07 & 0.86 \\ & \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.65 & 0.04 \\ 0.04 & -0.49 \\ & \end{bmatrix}, \quad \begin{bmatrix} -0.55 & 0.05 \\ 0.05 & -0.35 \\ & \end{bmatrix}$$

Both have negative eigenvalues.

The three-way split of actual vote shares among these parties in Great Britain was

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}) = (0.30, 0.40, 0.30).$$

and the three-way sample vote shares were

$$(s_{Lab}, s_{Con}, s_{Lib})^{GB} = (0.29, 0.412, 0.298).$$

The lower 95% estimate of $\rho_{Lab}^{GB} = 0.29 = s_{Lab}^{GB} = 0.29$. Thus $\mathbf{z}_0^{GB}$ is not a stable attractor.

To compare 2005 and 2010, note that in 2005,

$$\beta\nabla_0 = (0.15)\begin{bmatrix}1.646 & 0 \\ 0 & 3.961\end{bmatrix} = \begin{bmatrix}0.24 & 0 \\ 0 & 0.59\end{bmatrix}$$

whereas in 2010

$$\beta\nabla_0 = (0.857)\begin{bmatrix}0.60 & 0 \\ 0 & 0.86\end{bmatrix} = \begin{bmatrix}0.20 & 0 \\ 0 & 0.73\end{bmatrix}$$

We also ran the pure spatial model for the three regions, England, Scotland and Wales, with sample sizes for the regional models of 5465, 636, and 307 respectively. Table 30 gives the results of the pure spatial model for the regions of England, Scotland and Wales.

8.2.1 Model for England in 2010

For England the joint mean is $z_0^{eng} = (0.022, 0.029)$, with $n = 5465$.

$$\begin{aligned}
\nabla_0^{eng} &= \begin{bmatrix} & x & y \\ x & 0.61 & 0.06 \\ y & 0.06 & 0.85\end{bmatrix} \\
\sigma^{eng} &= 1.235 \\
(\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}, \beta^{eng}) &= (-0.123, 0.209, 0, 0.859). \\
\rho_{lab}^{eng} &= 0.28, \rho_{con}^{eng} = 0.40, \rho_{lib}^{eng} = 0.32 \\
c^{eng} &= 2\beta(1 - 2\rho_{lab}^{eng})trace(\nabla_0^{eng}) \\
&= 2 \cdot (0.859) \cdot (1 - 2(0.284)) \cdot (1.462) \\
&= 1.085
\end{aligned}$$

$$\begin{aligned}
C^{eng} &= 2\beta(1 - 2\rho_{lab})\nabla_0^{eng} - I \\
&= 2 \cdot (0.86) \cdot (0.44) \cdot \begin{bmatrix} & x & y \\ x & 0.61 & 0.06 \\ y & 0.06 & 0.85\end{bmatrix} - I \\
&= \begin{bmatrix} & x & y \\ x & -0.55 & 0.04 \\ y & 0.04 & -0.37\end{bmatrix}
\end{aligned}$$

This Hessian has negative eigenvalues and z_0^{eng} is an LNE. Using the 95% confidence interval of $\beta \in [0.86 \pm 1.96 \times 0.02] = [0.81, 0.91]$ and $\rho_{lab} \in [0.27, 0.30]$, the lower and upper bounds of c^{eng} are

$$[2(0.81)(1-2\times 0.30)(1.46), 2(0.91)(1-2\times 0.27)(1.46)] = [0.96, 1.22]$$

The upper estimate of C^{eng} still has negative eigenvalues, so $z_0^{eng} = (0.022, 0.029)$ is the regional LNE with probability greater than 95%.

The split of sample shares was

$$(s_{Lab}, s_{Con}, s_{Lib})^{eng} = (0.27, 0.43, 0.3).$$

and the lower 95% bound on $\rho_{lab} = 0.27 = s_{Lab}$ so $\mathbf{z}_0^{eng}$ is not a stable attractor.

8.2.2 Model for Scotland in 2010

The electoral mean in Scotland is $z_0^{sct} = (-0.148, -0.206)$, $n = 636$.

$$\begin{aligned}
\nabla_0^{sct} &= \begin{bmatrix} & x & y \\ x & 0.54 & 0.05 \\ y & 0.05 & 0.89 \end{bmatrix} \\
\sigma^{sct} &= 1.18 \\
(\lambda_{Lab}, \lambda_{Con}, \lambda_{Lib}, \lambda_{snp}, \beta) &= (0.44, -0.44, 0, 0.07, 0.78) \\
\rho_{Lab}^{sct} &= 0.36, \rho_{Con}^{sct} = 0.15, \rho_{Lib}^{sct} = 0.23, \rho_{snp}^{sct} = 0.25 \\
c^{sct} &= 2\beta(1-2\rho_{con}^{sct}) trace(\nabla_0^{sct}) \\
&= 2\cdot(0.78)\cdot(1-2(0.15))\cdot(1.43) \\
&= 1.51
\end{aligned}$$

The conservatives are the lowest valence party so we compute:

$$\begin{aligned}
C_{con}^{sct} &= 2\beta(1-2\rho_{con})\nabla_0^{sct} - I \\
&= \begin{bmatrix} & x & y \\ x & -0.45 & 0.05 \\ y & 0.05 & -0.04 \end{bmatrix}
\end{aligned}$$

This Hessian has negative eigenvalues. Using the 95% confidence interval of $\beta \in [0.78 \pm 1.96 \times 0.08] = [0.63, 0.93]$ and $\rho_{con} \in [0.12, 0.19]$, the lower and upper bounds of c^{sct} are given by

$$[2(0.63)(1-2\times 0.19)(1.39), 2(0.93)(1-2\times 0.12)(1.43)] = [1.07, 1.98]$$

The split of sample shares was

$$(s_{Lab}, s_{Con}, s_{Lib}, s_{snp}) = (0.362, 0.162, 0.230, 0.247).$$

and actual vote shares

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}, \nu_{snp}) = (0.430, 0.171, 0.194, 0.205).$$

Note that the lower 95% bound on $\rho_{con} = 0.12 < s_{con} = 0.162$, so this LNE failed to be a stable attractor.

8.2.3 Model for Wales in 2010

For Wales the electoral mean is $z_0^{wales} = (-0.080, 0.089)$, $n = 307$.

$$\begin{aligned}
\nabla_0^{wales} &= \begin{bmatrix} & x & y \\ x & 0.59 & 0.15 \\ y & 0.15 & 0.89 \end{bmatrix} \\
\sigma^{wales} &= 1.217 \\
(\lambda_{lab}, \lambda_{con}, \lambda_{lib}, \lambda_{pc}, \beta) &= (0.33, -0.02, 0, -0.85, 0.92). \\
\rho_{lab}^{wales} &= 0.37, \rho_{con}^{wales} = 0.26, \rho_{lib}^{wales} = 0.26, \rho_{pc}^{wales} = 0.11 \\
c^{wales} &= 2\beta(1 - 2\rho_{pc}^{wales}) trace(\nabla_0^{wales}) \\
&= 2 \cdot (0.92) \cdot (1 - 2(0.11)) \cdot (1.48) \\
&= 2.12
\end{aligned}$$

$$\begin{aligned}
C_{pc}^{wales} &= 2\beta(1 - 2\rho_{pc})\nabla_0^{wales} - I \\
&= 2 \cdot 0.92) \cdot (0.78) \cdot \begin{bmatrix} & x & y \\ x & 0.59 & 0.15 \\ y & 0.15 & 0.89 \end{bmatrix} - I \\
&= \begin{bmatrix} & x & y \\ x & -0.16 & 0.21 \\ y & 0.21 & +0.27 \end{bmatrix}.
\end{aligned}$$

This Hessian has a saddlepoint at the joint mean. Using the 95% confidence interval of $\beta \in [0.92 \pm 1.97 \times 0.11] = [0.71, 1.14]$ and $\rho_{pc} \in [0.09, 0.14]$,the 95% bounds on c^{wales} are

$$[2(1.14)(1 - 2 \times 0.14)(1.48), 2(0.71)(1 - 2 \times 0.09)(1.48)] = [1.53, 2.75]$$

giving either a maximum or a saddlepoint.

Using the upper estimates we find a pure spatial equilibrium by simulation:

$$\mathbf{z}_s^{wales} = \begin{bmatrix} Party & Lab & LibDem & Cons & PC \\ x & -0.140 & -0.206 & 0.010 & 0.218 \\ y & -0.202 & -0.457 & 0.209 & 0.696 \end{bmatrix}.$$

with vote shares

$$(\rho_{Lab}^{wales}, \rho_{Con}^{wales}, \rho_{Lib}^{wales}, \rho_{pc}^{wales})^* = (0.353, 0.252, 0.270, 0.126).$$

The split of sample shares was

$$(s_{Lab}, s_{Con}, s_{Lib}, s_{pc}) = (0.349, 0.293, 0.251, 0.107).$$

and actual vote shares

$$(\nu_{Lab}, \nu_{Con}, \nu_{Lib}, \nu_{pc}) = (0.39, 0.28, 0.21, 0.12).$$

The 95% lower vote share of the PC at the mean is $0.09 < s_{pc} = 0.107$. The lower vote share of the PC at $\mathbf{z}_s^{wales}$ is $0.126 > 0.107$ so this LNE could be a stable attractor. On the other hand, the lower estimate for $\rho_{Con}^{wales} = 0.19 < 0.293$, so only the PC has an incentive to move to this LNE.

9 Tables for 2005

Table 1. 2005 UK Election: Great Britain			
Party	Vote[1] %	Seats[1]	Seat %
Conservative Party:	32.3	198	30.7
Labor Party	35.3	356	55.1
Liberal Democrat Party	22.1	62	9.6
Scottish National Party	1.5	6	0.9
Plaid Cymru	0.6	3	0.45
Total	91.8[2]	625+3[2]	96.7

[1] Percentage of total UK vote, including approx. 670,000 votes (2.6%) in N.Ireland.

[2] 3 others: 1 Independent, 1 Respect,Health Concern, 1 Green with about 5.4% vote and 0.05% seats, plus18 seats (2.8%) in N.Ireland, as in Table 3.

Table 2. 2005 Great Britain Election by Region									
	England			Scotland			Wales		
Party[1]	Vote %	Seats	Seat %	Vote %	Seats	Seat %	Vote %	Seats	Seat %
Con	35.6	194	36.8	15.8	1	1.7	21.4	3	7.5
Lab	35.4	286	54.3	39.5	41	69.5	42.5	29	72.5
LibDem	22.8	47	8.9	22.6	11	18.6	18.5	4	10.0
SNP	-			17.7	6	10.2			
PC	-						12.6	3	7.5
Total	93.8	527	100	95.6	59	100	95.2	39	100

[1] Con: Conservative Party; Lab: Labor Party; LibDem: Liberal Democrat Party
SNP: Scottish National Party; PC: Plaid Cymru

Table 3 Election in Northern Ireland in 2005			
Party	Vote share[1] %	Seat	Seat share[2] %
Independent	-	-	-
Democratic Unionist	0.9	9	1.3
Ulster Unionists	0.1	1	0.15
Social Dem and Labor	0.5	3	0.46
Sinn Féin	0.6	5	0.77
Total	2.1	18	2.8

[1] Percentage of total UK vote

[2] Seat share as percentage of total UK seats

Table 4a. Survey Questions for Britain in 2005

1. Thinking of the Euro, which of the following statements on this card would come closest to your own view?

2. The first issue is Britain's membership in the European Union. You'll see on this show card that the end of the scale marked 0 means that Britain should definitely get out of the EU, and the end of the scale marked 10 means that Britain should definitely stay in the EU. Where would you place yourself on this scale?

3. Using the 0 to 10 scale on this card, where the end marked 0 means that government should cut taxes and spend much less on health and social services, and the end marked 10 means that government should raise taxes a lot and spend much more on health and social services, where would you place yourself on this scale?

Please tick one box on each line to show how much you agree or disagree with each of these statements:

4. Immigrants make Britain more open to new ideas and cultures.

5. Immigrants take jobs away from people who were born in Britain.

6. Private enterprise is the best way to solve Britain's economic problems.

7. The government has the right to put people suspected of terrorism in prison without trial.

8. Immigrants increase crime rates.

9. Immigrants generally are good for Britain's economy.

10. Most asylum seekers who come to Britain should be sent home immediately.

11. The ability of banks and companies to move money across borders seriously undermines the British government's ability to manage the economy.

12. Big international companies are a threat to democratic government in Britain.

13. I am very concerned about the loss of British jobs to countries overseas.

Voters and Activists

14. Voters. Using the scale of 0 to 10 where 0 means very unlikely and 10 means very likely, how likely it is that you would ever vote for the following parties?... Vote choice was given by a response >7 to this question.

15. Activists: Over the past few years, have you ever volunteered to get involved in politics or community affairs? Those who answered yes were coded as activists. Their response to Q.14 was used to allocate the activists to different parties.

Total sample size for regional models= 1564,
Total sample size for voters for major parties in Great Britain= 1149.
Sample size for activists=210.

Table 4b. Sociodemographic Survey items in 2005
1. **Age** What is your year of birth? We subtracted the year from 2010.
2. **Gender** What is your gender?(1) Male (2) Female
3. **Education** At what age did you finish full-time education? (1) 15 or younger - (5) 19 or older Those who are still at school or university are recoded as (5), since all the respondents are older than 19.
4. **Income** Which of the following represents the total income of your household from all sources before tax-including benefits, saving and so on? (1) Less than £5,000 - (16) More than £100,000

Table 4c. Survey items used for party leader traits in 2005
1. **Feeling.** Using a scale from 0 to 10, where 0 means strongly dislike and 10 means strongly like, how do you feel about .....?
2. **Competence.** Using a scale from 0 to 10, where 0 means a very incompetent leader and 10 means a very competent leader, what do you think of..?..
3. **Responsive**. Using a scale from 0 to 10 to indicate the extent to which the different leaders respond to voters' concerns, what do you think ..?
4. **Trustworthy.** Using a scale from 0 to 10 scale to indicate how much trust you have for each of the party leaders, where 0 means no trust and 10 means a great deal of trust, how much do you trust ....?

Table 5. 2005 Factor Loadings for British Election

$n = 1149$	Nationalism	Economy
1. Euro	0.30	-0.17
2. EU membership	-0.32	0.14
3. Tax/Spend	-0.10	0.39
4. Immigrant/ Culture	0.32	-0.03
5. Immigrant Jobs	-0.34	-0.00
6. Free Market	-0.07	0.40
7. Terrorism	-0.28	-0.04
8. Immigrant crime	-0.38	0.02
9. Immigrant economy	0.36	-0.03
10. Asylum seekers	-0.38	0.01
11. Int. economy	-0.17	-0.48
12. Int. companies	-0.04	-0.53
13. Job loss overseas	-0.24	-0.34
variance	0.31	0.12
Cumulative variance	0.31	0.43

Table 6. 2005 Model Comparisons for Great Britain (base LibDem)					
Models		Pure spatial (1)	Traits only (2)	Spatial+Traits (3)	Spatial+Traits +Socios (4)
Party	Variable	Est[1] (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)
	β	0.15***	-	0.06***	0.08***
		(12.56)		(3.71)	(4.73)
Lab	λ_{Lab}	0.52	0.19	0.18***	0.70
		(6.84)	(1.84)	(1.68)	(1.43)
	Blair trait		1.72***	1.72***	1.74***
			(12.83)	(12.87)	(12.86)
	Howard trait		-0.63***	-0.64***	-0.64***
			(5.25)	(5.34)	(5.30)
	Kennedy trait		-0.74***	–0.71***	-0.70***
			(6.78)	(6.42)	(6.21)
	Age				-0.01
					(1.66)
	Education				0.03***
					(0.39)
	Gender				-0.11
					(0.60)
	Income				0.0
					(0.04)
Con	λ_{Con}	0.27***	-0.28*	-0.26***	-2.63**
		(3.22)	(2.32)	(2.18)	(4.42)
	Blair trait		-0.83***	-0.72***	-0.66***
			(6.46)	(5.48)	(5.04)
	Howard trait		1.90***	1.79***	1.72***
			(12.25)	(11.29)	(10.67)
	Kennedy trait		-1.31***	-1.15***	-1.16***
			(10.26)	(8.56)	(8.35)
	Age				0.02**
					(2.91)
	Education				0.13
					(1.69)
	Gender				0.05
					(0.24)
	Income				0.14***
					(4.08)
n		1149	1149	1149	1149
Log Likelihood (LL)		-1136	-754	-748	-728
AIC		2279	1518	1505	1475
McFadden's R^2		0.08	0.39	0.40	0.41

[1] In this paper we use the standard convention: *:$prob<0.05$; **:$prob<0.01$; ***:$prob<0.001$.

Table 6a. 2005 Spatial Models for Great Britain (base LibDem)

		Spatial	Different β coefficients
Party	**Var**	**Est.** (|t-stat|)	**Est.** (|t-stat|)
	β	0.15 *** (12.56)	
	β_{Econ}		0.39 *** (10.73)
	β_{Nat}		0.13 *** (10.77)
Lab	λ_{Lab}	0.52 *** (6.84)	0.50 *** (6.62)
Con	λ_{Con}	0.27 ** (3.22)	0.25 ** (2.96)
n		1149	1149
Log Likelihood		-1136	-1108
AIC		2279	2224
McFadden's R^2		0.08	0.10

Table 7. 2005 Sample Vote Shares and ρ in Great Britain

Party	Sample Vote[1]	ρ	[**L**,**U**][2]
Labor Party	0.41	0.42	$[0.39, 0.46]$
Conservative Party	0.34	0.33	$[0.29, 0.36]$
Liberal Democrat Party[5]	0.25	0.25	$[0.22, 0.28]$
c^3	$[0.62, 0.84, 1.08]$		

[1] Ignoring all other parties, [2] Lower and upper 95% bound on ρ.

[3] Lower 95% bound, best estimate and Upper 95% bound on c.

Table 8. 2005.Pure Spatial Model for the Regions in Great Britain			
base LibDem	**England**	**Scotland**	**Wales**
Var	**Est** (|t-stat|)	**Est** (|t-stat|)	**Est** (|t-stat|)
β	0.14 *** (11.32)	0.14 *** (5.93)	0.11 *** (4.13)
λ_{Lab}	0.35 *** (4.17)	0.69 *** (4.82)	0.63 *** (3.75)
λ_{Con}	0.31 *** (3.42)	0.05 (0.27)	0.11 (0.55)
λ_{SNP}		-0.10 (0.56)	
λ_{PC}			-0.66 ** (2.92)
n	942	362	260
LL	−945	−460	−327
AIC	1896	928	663
McFadden's R^2	0.09	0.05	0.04

Table 9 2005 Sample Vote Shares and ρ by Region

	Great Britain			England		
Party	S.vote[1]	ρ	[**L,U**][2]	S.vote[1]	ρ	[**L,U**][2]
Lab	0.41	0.42	[0.39,0.46]	0.36	0.38	[0.34,0.42]
Con	0.34	0.33	[0.29,0.36]	0.38	0.36	[0.32,0.40]
LibDem	0.25	0.25	[0.22,0.28]	0.25	0.26	[0.23,0.30]
c^3	[0.62,0.84,1.08]			[0.53,0.75,1.00]		

	Scotland			Wales		
Party	S.vote[1]	ρ	[**L,U**][2]	S.vote[1]	ρ	[**L,U**][2]
Lab	0.41	0.40	[0.34,0.47]	0.41	0.42	[0.34,0.50]
Con	0.20	0.21	[0.16,0.27]	0.26	0.25	[0.20,0.32]
LibDem	0.21	0.20	[0.16,0.26]	0.21	0.22	[0.17,0.29]
SNP	0.19	0.18	[0.14,0.24]	-		
PC				0.12	0.12	[0.08,0.17]
c^3	[0.53,0.97,1.47]			[0.35,0.80,1.30]		

[1] sample vote shares among respective parties. [2] Lower and upper 95% bounds on ρ.

[3] Lower 95% bound, best estimate and upper 95% bound on c.

Table 10. 2005 Factor Loadings for Traits in Great Britain			
	Blair traits	**Howard traits**	**Kennedy traits**
Blair feeling	0.91	-0.12	
Blair competent	0.79		0.20
Blair responsive	0.86		0.13
Blair trustworthy	0.94		
Howard feeling	-0.18	0.82	
Howard competent		0.87	0.11
Howard responsive		0.78	0.17
Howard trustworthy		0.90	
Kennedy feeling			0.82
Kennedy competent	0.13		0.85
Kennedy responsive	0.14		0.83
Kennedy trustworthy	0.15	0.13	0.85
variance	0.26	0.24	0.24
Cumulative variance	0.26	0.51	0.75

Table 11. 2005 Factor Loadings for Traits in England			
Leader	**Blair**	**Howard**	**Kennedy**
Labor Party			
Blair feeling	0.90	-0.13	
Blair competent	0.77		0.19
Blair responsive	0.85		0.12
Blair trust	0.93	0.10	
Conservative Party			
Howard feeling	-0.15	0.83	
Howard competent		0.86	0.12
Howard responsive		0.77	0.18
Howard trust		0.92	
Liberal Democratic Party			
Kennedy feeling			0.80
Kennedy competent	0.12		0.86
Kennedy responsive	0.13		0.83
Kennedy trustworthy	0.14	0.14	0.85
% variance	0.26	0.24	0.24
Cumulative % variance	0.26	0.50	0.74

Table 12. 2005 Factor Loadings for Traits in Scotland				
Leader	**Blair**	**Salmond**	**Howard**	**Kennedy**
Labor Party				
Blair feeling	0.88			
Blair competent	0.74		-0.11	
Blair responsive	0.86			
Blair trustworthy	0.94			
Conservative Party				
Howard feeling	-0.21		0.82	
Howard competent		0.12	0.85	0.12
Howard responsive			0.69	0.19
Howard trustworthy			0.87	
Liberal Democratic Party				
Kennedy feeling		0.25		0.79
Kennedy competent		0.20	0.16	0.78
Kennedy responsive	0.23	0.18	0.11	0.71
Kennedy trustworthy	0.16	0.22	0.17	0.81
Scottish National Party (SNP)				
Salmond feeling	-0.14	0.85		0.19
Salmond competent	-0.12	0.86		0.28
Salmond responsive		0.83	0.11	0.16
Salmond trustworthy		0.87	0.17	0.24
% variance	0.195	0.195	0.171	0.170
Cumulative % variance	0.195	0.390	0.561	0.730

Table 13. 2005 Factor Loadings for Traits in Wales				
Leader	Blair	Llwyd	Howard	Kennedy
Labor Party				
Blair feeling	0.94			
Blair competent	0.89		-0.11	0.19
Blair responsive	0.90		-0.15	
Blair trustworthy	0.93			
Conservative Party				
Howard feeling	-0.18	0.15	0.83	
Howard competent	-0.13	0.11	0.93	
Howard responsive		0.11	0.86	
Howard trustworthy			0.91	
Liberal Democratic Party				
Kennedy feeling		0.25		0.85
Kennedy competent		0.24		0.85
Kennedy responsive		0.16		0.90
Kennedy trustworthy	0.11	0.17		0.86
Plaid Cymru (PC)				
Llwyd feeling		0.85	0.12	0.17
Llwyd competent		0.90	0.10	0.19
Llwyd responsive		0.93		0.24
Llwyd trustworthy		0.88	0.15	0.23
variance	0.22	0.21	0.20	0.20
Cumulative variance	0.22	0.43	0.63	0.83

Table 14. 2005 MNL pure traits models by Region (base LibDem)							
		England $n = 717$		**Scotland** $n = 241$		**Wales** $n = 108$	
Party	**Variable**	**Est**	|t-stat|	**Est**	|t-stat|	**Est**	|t-stat|
Lab	λ_{Lab}	-0.21	1.46	0.88 ***	3.47	1.01 **	2.75
	Blair Traits	1.83 ***	10.07	1.55 ***	5.33	1.26 ***	3.40
	Howard Traits	-0.61 ***	3.96	-0.45	1.66	0.01	0.03
	Kennedy Traits	-0.63 ***	4.52	-0.88 **	3.25	-0.65	1.87
	Salmond Traits			-0.20	0.77		
	Llwyd Traits					-0.51	1.31
Con	λ_{Con}	-0.08	0.60	-0.59	1.63	-0.93	1.49
	Blair Traits	-0.99 ***	5.94	-0.43	1.38	-0.03	0.08
	Howard Traits	2.02 ***	10.35	1.89 ***	4.82	2.71 ***	3.83
	Kennedy Traits	-1.22 ***	7.99	-1.31 ***	3.84	-2.41 ***	3.63
	Salmond Traits			-0.76 *	2.50		
	Llwyd Traits					-0.78	1.28
SNP	λ_{SNP}			-0.12	0.40		
	Blair Traits			-0.44	1.50		
	Howard Traits			0.10	0.34		
	Kennedy Traits			-1.30 ***	4.22		
	Salmond Traits			0.95 **	3.10		
PC	λ_{PC}					-0.81	1.30
	Blair Traits					0.13	0.28
	Howard Traits					1.24 *	2.30
	Kennedy Traits					-1.85 **	2.78
	Llwyd Traits					2.37 **	3.20
LL		-463		-205		-84	
AIC		943		440		199	
McFadden's R^2		0.40		0.35		0.41	

Table 15. 2005 Spatial and Traits models by Region (base LibDem)							
		England		**Scotland**		**Wales**	
Party	**Variable**	**Est**	|t-stat|	**Est**	|t-stat|	**Est**	|t-stat|
	β	0.05 **	2.64	0.100 **	2.59	0.03	0.53
Lab	λ_{Lab}	-0.23	1.55	0.89 ***	3.50	0.97 **	2.60
	Blair Traits	1.84 ***	10.13	1.51 ***	5.20	1.24 ***	3.35
	Howard Traits	-0.63 ***	4.05	-0.46	1.69	0.02	0.04
	Kennedy Traits	-0.59 ***	4.25	-0.83 **	3.07	-0.62	1.77
	Salmond Traits			-0.18	0.71		
	Llwyd Traits					-0.50	1.28
Con	λ_{Con}	-0.07	0.48	-0.42	1.14	-0.97	1.54
	Blair Traits	-0.85 ***	4.98	-0.40	1.27	-0.00	0.00
	Howard Traits	1.89 ***	9.49	1.75 ***	4.35	2.66 ***	3.75
	Kennedy Traits	-1.06 ***	6.60	-1.09 **	3.06	-2.27 **	3.21
	Salmond Traits			-0.78 *	2.49		
	Llwyd Traits					-0.75	1.22
SNP	λ_{SNP}			-0.09	0.29		
	Blair Traits			-0.48	1.62		
	Howard Traits			0.05	0.16		
	Kennedy Traits			-1.20 ***	3.89		
	Salmond Traits			0.94 **	3.02		
PC	λ_{PC}					-0.88	1.38
	Blair Traits					0.13	0.28
	Howard Traits					1.21 *	2.26
	Kennedy Traits					-1.79 **	2.68
	Llwyd Traits					2.41 **	3.23
LL		-460		-201		-84	
AIC		938		435		201	
McFadden's R^2		0.41		0.36		0.41	

Tables 16. 2005 Pure Sociodemographic MNL model by Region (base LibDem)							
		England		**Scotland**		**Wales**	
Party	**Variable**	**Est**	|t-stat|	**Est**	|t-stat|	**Est**	|t-stat|
Lab	λ_{Lab}	1.14	1.87	2.37 *	2.00	1.06	0.64
	Gender (F)	-0.12	0.60	-0.28	0.72	0.30	0.52
	Age	-0.01	1.26	0.01	0.86	-0.00	0.18
	Educ	-0.08	1.15	-0.35 *	2.44	-0.15	0.71
	Income	0.01	0.35	-0.09	1.49	-0.02	0.19
Con	λ_{Con}	-1.64 **	2.59	-1.66	1.18	-3.96	1.77
	Gender (F)	0.03	0.16	0.04	0.08	1.21	1.77
	Age	0.03 ***	3.79	0.05 **	2.72	0.03	1.29
	Educ	-0.05	0.73	-0.19	1.13	-0.28	1.09
	Income	0.11 ***	3.38	-0.03	0.36	0.23	1.92
SNP	λ_{SNP}			2.29	1.69		
	Gender (F)			-1.05 *	2.28		
	Age			0.02	0.88		
	Educ			-0.23	1.40		
	Income			-0.10	1.40		
PC	λ_{PC}					-4.52 *	2.01
	Gender (F)					1.15	1.64
	Age					0.02	1.03
	Educ					0.24	0.10
	Income					0.16	1.42
LL		-756		-295		-132	
AIC		1531		620		294	
McFadden's R^2		0.03		0.06		0.08	

Table 17. 2005 Spatial, Traits and Sociodem. models by Region (base LibDem)							
		England		**Scotland**		**Wales**	
Party	**Variable**	**Est**	|t-stat|	**Est**	|t-stat|	**Est**	|t-stat|
	β	0.09 ***	4.11	0.08	1.72	0.07	1.11
Lab	λ_{Lab}	0.58	0.78	1.54	1.16	1.27	0.61
	Blair traits	1.84 ***	10.10	1.44 ***	4.64	1.36 ***	3.48
	Howard traits	-0.61 ***	3.90	-0.47	1.64	-0.01	0.02
	Kennedy traits	-0.60 ***	4.14	-0.76 **	2.66	-0.72	1.85
	Salmond traits			-0.19	0.74		
	Llwyd traits					-0.50	1.26
	Gender (F)	-0.21	0.90	-0.14	0.30	0.27	0.36
	Age	-0.01	1.54	0.01	0.41	-0.02	0.73
	Educ	0.06	0.75	-0.11	0.65	-0.11	0.45
	Income	0.00	0.01	-0.07	1.00	0.11	0.81
Con	λ_{Con}	-2.85 **	3.28	-2.12	1.22	-3.80	1.23
	Blair traits	-0.85 ***	4.78	-0.64	1.86	0.30	0.58
	Howard traits	1.90 ***	9.10	1.79 ***	4.25	2.45 ***	3.35
	Kennedy traits	-1.13 ***	6.62	-1.03 **	2.79	-2.37 **	3.15
	Salmond traits			-0.76 *	2.36		
	Llwyd traits					-0.65	0.95
	Gender (F)	-0.05	0.20	-0.07	0.12	0.56	0.58
	Age	0.02 *	2.07	0.04	1.86	0.01	0.29
	Educ	0.28 **	2.96	-0.12	0.48	-0.12	0.30
	Income	0.17 ***	3.83	0.00	0.02	0.34	1.92
SNP	λ_{SNP}			0.92	0.62		
	Blair traits			-0.66 **	2.06		
	Howard traits			0.12	0.35		
	Kennedy traits			-1.18 ***	3.56		
	Salmond traits			0.91 **	2.87		
	Gender (F)			-1.33 **	2.51		
	Age			0.03	1.31		
	Educ			-0.07	0.36		
	Income			-0.04	0.44		
PC	λ_{PC}					-4.92	1.65
	Blair traits					0.42	0.85
	Howard traits					1.20	1.91
	Kennedy traits					-2.10 **	2.68
	Llwyd traits					2.40 **	3.14
	Gender (F)					1.24	1.31
	Age					0.01	0.23
	Educ					0.04	0.13
	Income					0.30	1.95
LL		-440		-193		$-79.$	
AIC		914		441		214	
McFadden's R^2		0.43		0.39		0.45	

10 Tables for 2010

Table 18. 2010 UK Election: Great Britain			
Party	Vote[1] %	Seats[1]	Seat %
Conservative Party:	36.1	306	47.0
Labor Party	29.0	258	39.6
Liberal Democrat Party	23.0	57	8.8
Scottish National Party	1.7	6	0.9
Plaid Cymru	0.6	3	0.46
Total	90.4[3]	630[2]+1[3]	96.76

[1] Percentage of total UK vote, including approx. 675,000 votes (2.2%) in N.Ireland,

[2]One election to be contested later the seat at Thirsk and Malton, due to the death of the UKIP candidate.

[3]Greens have 1 seat. Together with other parties (British National Party, UK Independence Party, etc. but no seats) they have 7.4% vote in total,in Great Britain. In addition, 2.2% of the vote went to parties in N.Ireland with 18 seats (2.3%).See Table 22.

Table 19. 2010 Election in Great Britain by Region									
	England			Scotland			Wales		
Party[1]	Vote[2] %	Seats	Seat[3] %	Vote[4] %	Seats	Seat[3] %	Vote[4] %	Seats	Seat[3] %
Con	43.0	297	55.9	16.7	1	16.9	26.1	8	20.0
Lab	30.6	191	36.0	42.0	41	69.4	36.2	26	65.0
LibDem	26.4	43	8.1	18.9	11	18.6	20.1	3	7.5
SNP	-	-		19.9	6	10.1	-	-	
PC	-	-					11.3	3	7.5
Total	100	531		97.5	59	100	93.7	40	100

[1] Con: Conservative Party; Lab: Labor Party; LibDem: Liberal Democrat Party
SNP: Scottish National Party; PC: Plaid Cymru

[2]Percentage regional vote share across three parties

[3]Percentage regional seat share

[4]Percentage regional vote share across all parties

Table 20 Election in Northern Ireland in 2010			
Party	Vote share[1] %	Seat	Seat share[2] %
Alliance	0.1	1	0.15
Democratic Unionist	0.6	8	1.20
Independent (N.Down)	0.1	1	0.15
Social Dem and Labor	0.4	3	0.45
Sinn Féin	0.6	5	0.75
Total	1.8	18	2.8

[1]Percentage total UK vote

[2]Percentage total UK seat share

Table 21a. Survey Questions for Britain in 2010

For the May 2010 British election, we use the result of BES Campaign Internet Panel Survey (BES CIPS), which was released on May 31, 2010. Both pre- and post-election surveys are utilized. The questions used in this analysis are the following:

Issue dimensions from both pre- and post-election surveys
1. Overall, do you approve or disapprove of Britain's membership in the European Union? (1) Strongly approve - (5) Strongly Disapprove
Please indicate if you agree or disagree with the following policy proposals where (1) Strongly agree - (5) Strongly disagree
2. Have Britain co-operate more closely with the European Union.
3. Scrap Britain's Trident nuclear deterrent.
4. Using the 0 to 10 scale, where the end marked 0 means that government should cut taxes a lot and spend much less on health and social services, and the end marked 10 means the opposite where would you place yourself on this scale?
Please indicate if you agree or disagree with the following policy proposal where (1) Strongly agree - (5) Strongly disagree
5.Exempt the first £10,000 of earnings from income tax.
6.Charge a 'mansion' tax on properties worth over £2million.
7.Limit tax relief on pensions to the basic rate of tax.
8.Introduce new economic taxes including a fuel tax for airline flights.

Voting and Activists.
Vote choice from post-election surveys
Which party did you vote for in the General Election?
(1) Labour (2) Conservative (3) Liberal Democrat (4) Scottish National Party (5) Plaid Cymru
Vote intention from pre-election surveys
If 'yes' to the question "Have you decided which party you will vote for?", which party is that?
If 'no' to the question, which party do you think you are most likely to vote for?
(1) Labor (2) Conservative (3) Liberal Democrat (4) Scottish National Party (5) Plaid Cymru

Activists and Political Influence On a scale from 0 to 10, where 10 means a great deal of influence and 0 means no influence, how much influence do you have on politics and public affairs?
Those who responded > 5 were coded as activists.

21b. Sociodemographic Survey Items from pre-election surveys
1. Age What is your year of birth? We subtracted the year from 2010.
2. Gender What is your gender? (1) Male (2) Female
3. Education At what age did you finish full-time education? (1) 15 or younger - (5) 19 or older
Those who are still at school or university are recoded to (5), since all the respondents are older than 19.

4. Income Which of the following represents the total income of your household from all sources before tax-including benefits, saving and so on? (1) Less than £5,000 - (16) More than £100,000

21c. Traits from both pre- and post-election surveys
1. Feeling Using a scale that runs from 0 to 10, where 0 means strongly dislike and 10 means strongly like, how do you feel about [ ]?
2. Competence Using a scale that runs from 0 to 10, where 0 means a very incompetent leader and 10 means a very competent leader, how would you describe [ ]?
3. Knowledge When you listen to what [ ] has to say, do you think that in general he knows what he is talking about, or that he doesn't know? Please use the following scale where 0 means that [..] really doesn't know what he is talking about and 10 means he does know very much what he is talking about.
4. Interests When you listen to what [ ] has to say, do you think he has your best interests in mind, or that he does not think about your best interests? Please use the following scale where 0 means that [ ] never has your best interests in mind, and 10 means that he always does.
5. Trustworthy When you listen to what [ ] has to say, do you think generally that he tells the truth, or that he does not tell the truth? Please use the following scale where 0 means that he never tells the truth and 10 means that he always tells the truth.

Table 22: 2010 Factor Analysis		
	Nationalism	Economy
1.EU membership	0.89	
2.EU cooperation	0.85	0.18
3.Nuclear plan	0.28	0.41
4.Tax-spend	-0.34	-0.37
5.Tax exemption		0.39
6.Mansion tax	0.13	0.64
7.Tax relief		0.30
8.Ecotax	0.28	0.39
n	6409	
variance	0.224	0.142
Cumulative Variance	0.224	0.366

Table 23. 2010 Factor Loadings for Traits in Great Britain				
		Brown Trait	Cameron Trait	Clegg Trait
Brown	Feeling	0.87	-0.35	
	Competence	0.88	-0.30	
	Knowledge	0.81	-0.22	0.14
	Interests	0.87	-0.26	
	Trustworthy	0.87	-0.24	0.10
Cameron	Feeling	-0.38	0.83	
	Competence	-0.27	0.82	0.11
	Knowledge	-0.23	0.83	0.11
	Interests	-0.27	0.85	
	Trustworthy	-0.20	0.84	
Clegg	Feeling			0.82
	Competence			0.84
	Knowledge			0.82
	Interests	0.16		0.76
	Trustworthy	0.13	0.16	0.71
n		6218		
variance		0.28	0.26	0.21
Cumulative Variance		0.28	0.54	0.75

Table 24. 2010 Models for Great Britain (base LibDem)					
Models		Pure spatial (1)	Traits only (2)	Spatial+Traits (3)	Spatial+Traits +Socios (4)
Party	Variable	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)
	β	0.86*** (38.45)		0.47*** (14.87)	0.47*** (14.71)
Lab	λ_{Lab}	-0.04 (1.31)	-0.96*** (15.20)	-0.98*** (15.59)	-0.78** (3.26)
	Brown trait		1.76*** (27.25)	1.77*** (27.32)	1.77*** (27.09)
	Cameron trait		-0.71*** (12.86)	-0.74*** (13.37)	-0.74*** (13.22)
	Clegg trait		-0.97*** (18.50)	-0.94*** (18.07)	-0.93*** (17.65)
	Age				0.01* (2.49)
	Education				-0.21*** (6.71)
	Gender				0.07 (0.85)
	Income				-0.01 (0.61)
Con	λ_{Con}	0.17*** (4.50)	-0.52*** (9.25)	-0.55*** (9.46)	-0.34** (2.85)
	Brown trait		-1.60*** (25.03)	-1.28*** (19.22)	-1.26*** (18.53)
	Cameron trait		2.75*** (32.40)	2.45*** (28.23)	2.42*** (27.71)
	Clegg trait		-1.41*** (21.86)	-1.15*** (17.24)	-1.16*** (17.23)
	Age				-0.01** (2.74)
	Education				-0.05 (1.29)
	Gender				0.17 (1.73)
	Income				0.05*** (3.32)
n		6218	6218	6218	6218
LL		-5490	-3421	-3298	-3261
AIC		10983	6850	6606	6540
McFadden's R^2		0.19	0.49	0.51	0.52

Table 25a. 2005 Comparison of Log Likelihood for Great Britain 2005		$\mathbb{M}_2$		
		Traits	Spatial + Traits.	Joint[1]
	Traits	na	-7	-26
$\mathbb{M}_1$	Spatial and Traits	7	na	-19
	Joint[1]	26	19	na

Table 25b. Comparison of Log Likelihood for Great Britain 2010		$\mathbb{M}_2$		
		Traits	Spatial +Traits.	Joint[1]
	Traits	na	-123	-160
$\mathbb{M}_1$	Spatial +Traits	123	na	-37
	Joint[1]	160	37	na

[1] Joint=spatial model with traits and sociodemographics

Table 26. 2010 Pure Spatial Model for the Regions			
base=LibDem	England	Scotland	Wales
Var	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)
β	0.86***	0.78***	0.92***
	(36.12)	(10.17)	(8.39)
λ_{Lab}	-0.12***	0.44***	0.33*
	(3.40)	(4.16)	(2.22)
λ_{Con}	0.21***	-0.44**	-0.02
	(5.36)	(2.90)	(0.10)
λ_{SNP}		0.07	
		(0.60)	
λ_{PC}			-0.85***
			(4.03)
n	5465	636	307
LL	-4769	-784	-341
AIC	9545	1575	690
McFadden's R^2	0.19	0.08	0.16

Table 27 2010 Sample Vote Shares and ρ by Region

	Great Britain			England		
Party	S.vote[1]	ρ	$[\mathbf{L},\mathbf{U}]^2$	S.vote[1]	ρ	$[\mathbf{L},\mathbf{U}]^2$
Lab	0.29	0.31	[0.29,0.32]	0.27	0.28	[0.27,0.30]
Con	0.41	0.38	[0.36,0.39]	0.43	0.40	[0.38,0.41]
LibDem	0.30	0.32	[0.30,0.33]	0.30	0.32	[0.31,0.34]
c^3	[0.86,0.98,1.10]			[0.96,1.09,1.22]		

	Scotland			Wales		
Party	S.vote[1]	ρ	$[\mathbf{L},\mathbf{U}]^2$	S.vote[1]	ρ	$[\mathbf{L},\mathbf{U}]^2$
Lab	0.36	0.36	[0.32,0.41]	0.35	0.37	[0.30,0.44]
Con	0.16	0.15	[0.12,0.19]	0.30	0.26	[0.19,0.34]
LibDem	0.23	0.23	[0.20,0.28]	0.25	0.26	[0.21,0.33]
SNP	0.25	0.25	[0.21,0.30]			
PC				0.11	0.11	[0.09,0.14]
c^3	[1.07,1.51,1.98]			[1.53,2.12,2.75]		

[1] sample vote shares among respective parties. [2] Lower and upper 95% bounds on ρ.
[3] Lower 95% bound, best estimate and upper 95% bound on c.

Table 28. 2010 Major Party Traits models for the regions (base=LibDem)

		Pure trait			Spatial+trait		
		England	Scotland	Wales	England	Scotland	Wales
Party	**Var**	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)	Est (|t-stat|)
	β				0.48 ***	0.30 *	0.57 ***
					(14.30)	(2.37)	(3.72)
Lab	λ_{lab}	-0.99 ***	-0.95 ***	-0.33	-1.02 ***	-0.96 ***	-0.37
		(14.71)	(4.16)	(1.16)	(15.08)	(4.21)	(1.29)
	Brown	1.75 ***	1.87 ***	1.72 ***	1.76 ***	1.86 ***	1.73 ***
		(25.02)	(8.44)	(6.06)	(25.10)	(8.45)	(5.97)
	Cameron	-0.73 ***	-0.68 ***	-0.37	-0.76 ***	-0.70 ***	-0.39
		(12.26)	(3.71)	(1.48)	(12.74)	(3.81)	(1.56)
	Clegg	-0.97 ***	-0.85 ***	-1.27 ***	-0.94 ***	-0.83 ***	-1.19 ***
		(17.14)	(4.90)	(4.69)	(16.77)	(4.81)	(4.43)
Con	λ_{con}	-0.51 ***	-1.00 ***	-0.24	-0.53 ***	-1.07 ***	-0.40
		(8.55)	(4.38)	(0.87)	(8.63)	(4.47)	(1.34)
	Brown	-1.64 ***	-1.24 ***	-1.23 ***	-1.31 ***	-1.00 ***	-0.77 *
		(23.89)	(5.02)	(4.14)	(18.41)	(3.78)	(2.44)
	Cameron	2.78 ***	2.78 ***	2.22 ***	2.47 ***	2.56 ***	1.96 ***
		(30.61)	(7.95)	(6.20)	(26.64)	(7.18)	(5.21)
	Clegg	-1.42 ***	-1.63 ***	-1.28 ***	-1.16 ***	-1.44 ***	-0.84 *
		(20.65)	(5.97)	(4.27)	(16.30)	(5.14)	(2.50)
n		5465	479	274	5465	479	274
LL		-2983	-269	-157	-2869	-266	-149
AIC		5884	545	323	5746	539	307
McFadden's R^2		0.49	0.46	0.47	0.51	0.47	0.50

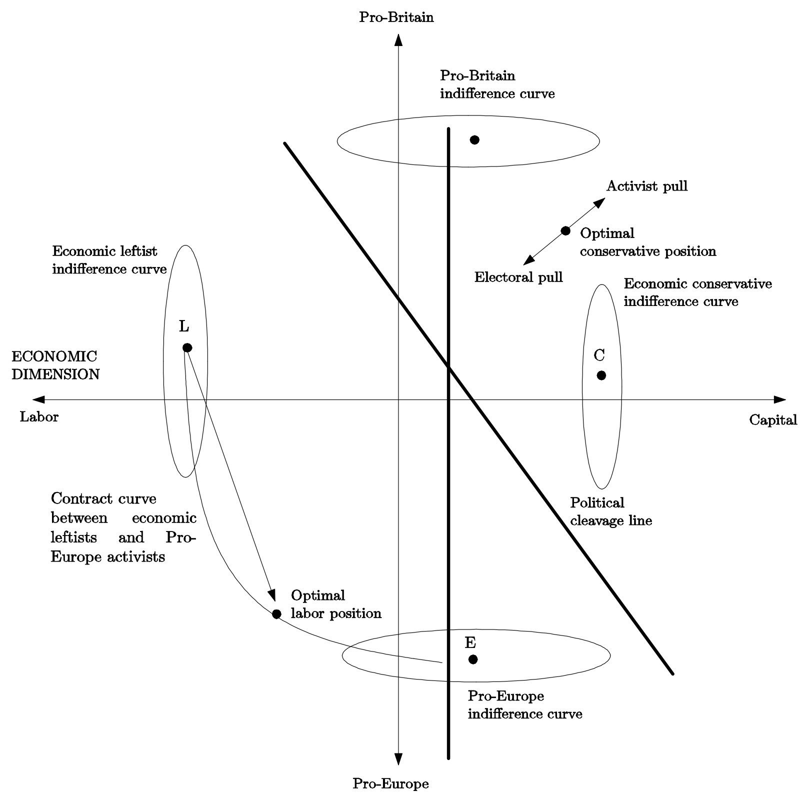

Figure 1: Influence of activists on Labor and Conservative Parties

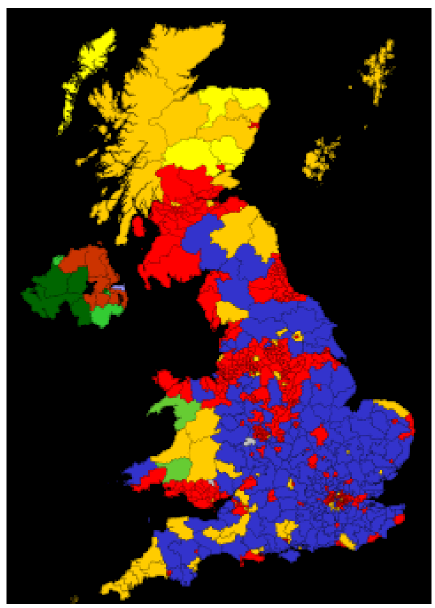

Figure 2: The electoral map in the UK 2005, with conservative constituencies in blue, Labor in red, Lib Dems in amber, SNP in yellow, and PC in green.

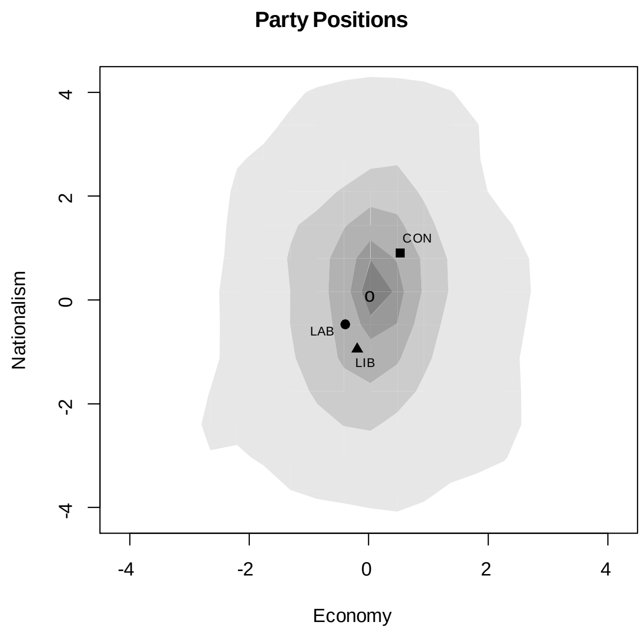

Figure 3: Electoral distribution and estimated party positions in Britain in 2005

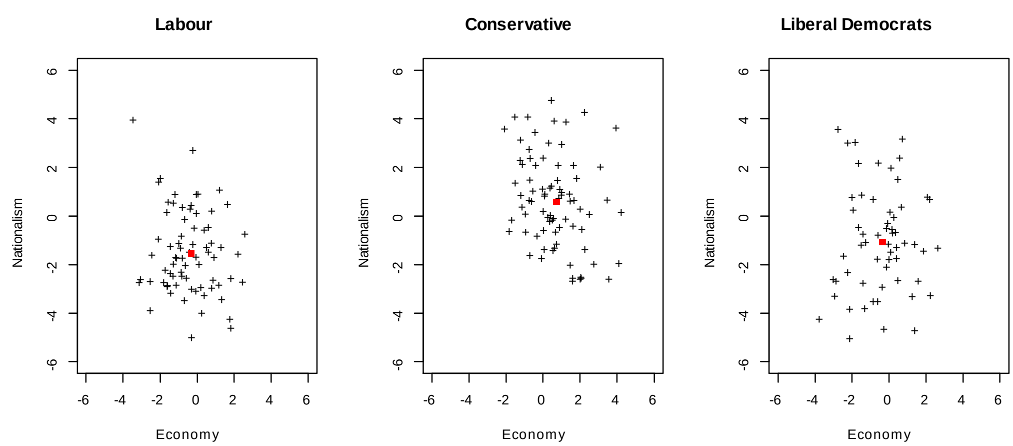

Figure 4: Activists and activist means by party in 2005

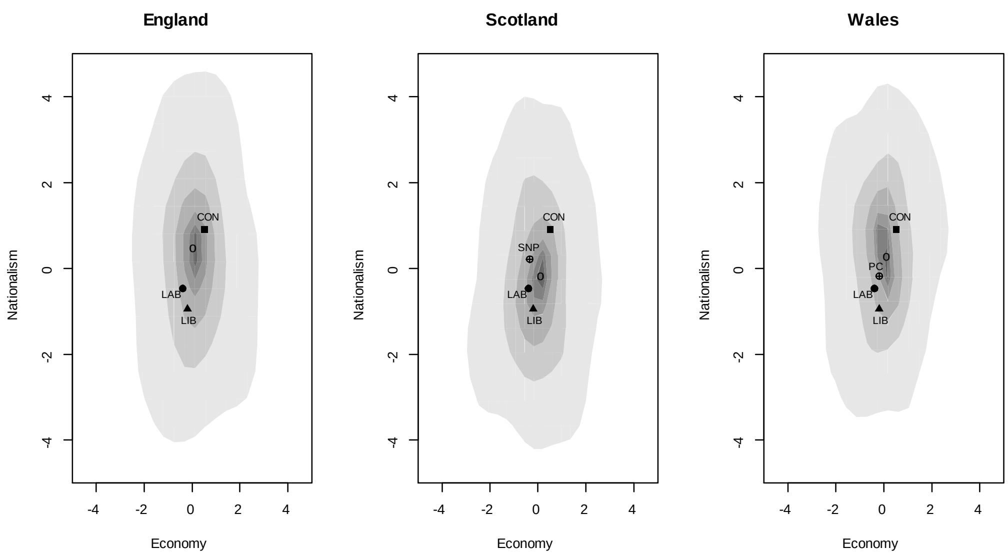

Figure 5: Party locations in the regions in 2005

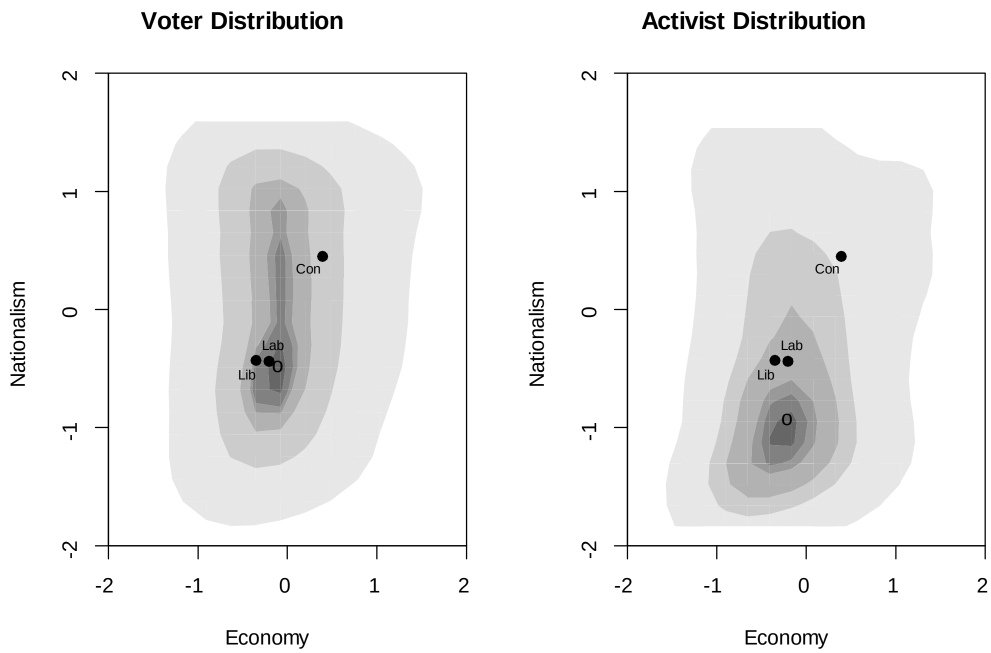

Figure 6: Actual respondent and activist positions in Britain in 2010

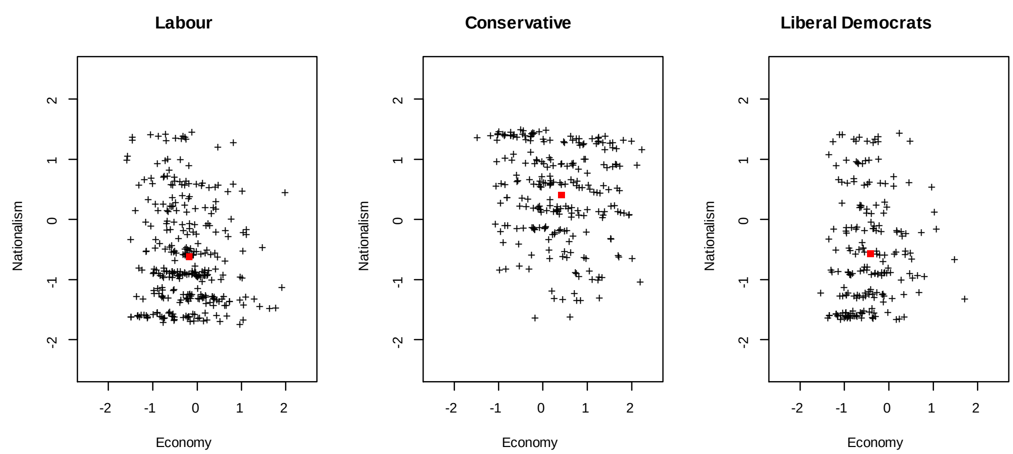

Figure 7: Activists and activist means by party (shown by red dot) in 2010

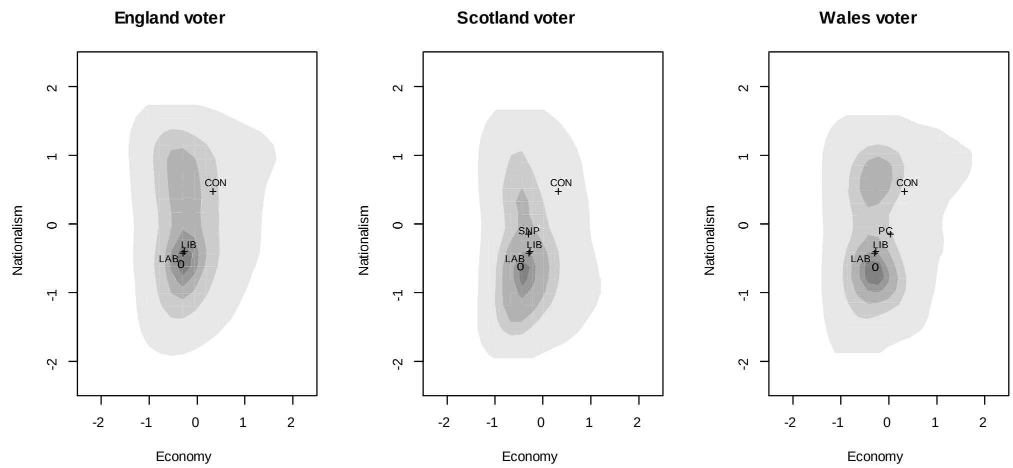

Figure 8: Party locations and vote distributions in the regions in 2010

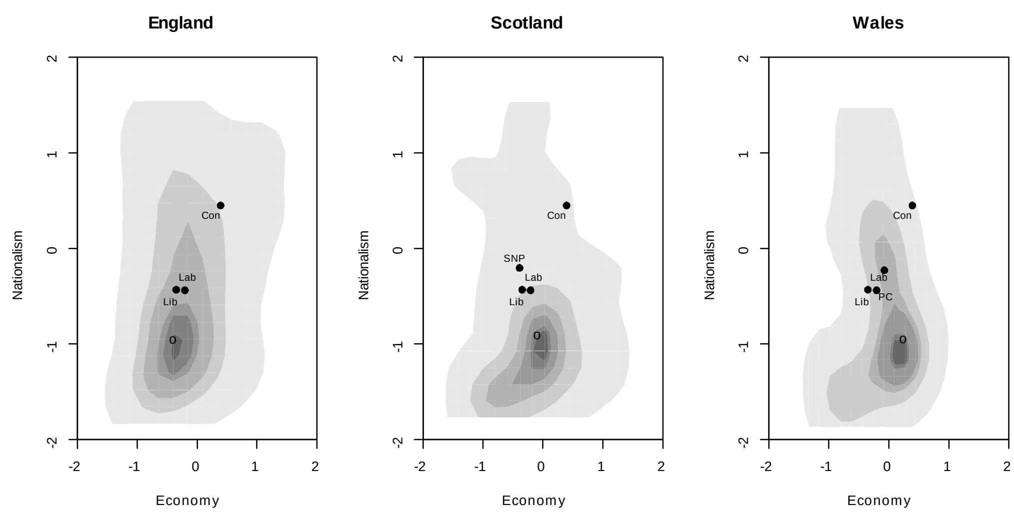

Figure 9: Activist distributions in the regions in 2010