

REPRESENTATIVE DEMOCRACY AS SOCIAL CHOICE *

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Abstract

Social Choice traditionally employs the preferences of voters or agents as primitives. However, in most situations of constitutional decision-making the beliefs of the members of the electorate determine their secondary preferences or choices. Key choices in US political history, such as the ratification of the Constitution in 1787 and the election of Lincoln in 1860, were conditioned by changing beliefs as regards the truth of propositions about the political universe. Preference-based models of election tend to conclude that candidates, or parties, converge to a vote-maximizing policy position at the “electoral center”. Empirical work suggests that such a conclusion is invalid. This chapter argues, on the contrary, that parties or candidates adopt positions that optimize, in a Nash equilibrium sense, with respect to both their beliefs over electoral response, and their beliefs over appropriate policy choices. The analysis indicates that political choices will be different depending on whether plurality (“first past the post”) or proportionality is used as the method of electoral representation.

Keywords

belief and preference aggregation, constitutional quandary, plurality and proportional electoral systems, political equilibrium

JEL classification: C71, C78, D72

1. Introduction: Constitutional choices

To inquire into the best form of government in the abstract . . . is not a chimerical, but a highly practical employment of scientific intellect. *J.S. Mill (1861)*

The deepest issue of social choice concerns the nature of the constitutional “bargain” made within a society. The arguments of Hobbes suggest the necessity of a Constitution, the Leviathan, to mitigate the consequences of anarchy or “Warre”, within which “the life of man [is] solitary poor, nasty, brutish and short”. [Hobbes (1651) Ch. 18, p. 234]. Hobbes may well have constructed his argument in response to the chaos of the civil war in Britain. But the people of Britain rejected the strong form of Leviathan in the person of Oliver Cromwell, Lord Protector. On Cromwell’s death, Charles Stuart was welcomed back as monarch. Even then the nature of the contract between the people and the monarch was deeply contested. In his *Two Treatises of Government* arguing against Filmer’s *Patriarcha* (1652), John Locke developed further the notion of the contract. “And thus every Man, by consenting with others to make one Body Politick, under one Government, puts himself under an Obligation to every one of that Society, to submit to the determination of the majority . . .” [Locke (1690) pp. 197–198].

The institutional innovations put in place in Britain after the “Glorious Revolution” and the assumption of the throne by William and Mary in 1688 led to the creation of a strong fiscal state that better equipped Britain to compete with France during the entire period until 1815 [Brewer (1988), North and Weingast (1989)]. France, in contrast, was unable to complete similar fiscal innovations. The distinguished French theorist and financier, Turgot, attempted to redesign the French fiscal apparatus prior to the American War of Revolution. However, aid provided by Louis XVI of France, in alliance with his minister of state, Vergennes, to the American colonists was enough to bankrupt France and force the calling of the Estates-General in 1789 [Norberg (1994)]. In the charged milieu thus created, the great social mathematician, Condorcet, hoped to create a constitutional monarchy, based on a Declaration of the Rights of Man. The faction supporting such a constitution, the Girondins, were expelled in the Jacobin Terror of 1794. As Schama (1989, p. 856) notes: “The great exponent of a state in which science and virtue would be mutually reinforcing, the Marquis de Condorcet, died in abject defeat”.

It is well known that Condorcet in his *Essai* of 1785 [McLean and Hewitt (1994)] laid the foundations of social choice theory. His famous “Jury Theorem”¹ [Rae (1969), Schofield (1972)] has recently excited considerable interest [Ladha (1992, 1993, 1995), Ladha and Miller (1996), Austen-Smith and Banks (1996), Feddersen and

¹ The theorem asserts that, in a situation of risky social choice between two alternatives, if the average probability, p , that a juror chooses the true option exceeds one half, then the jury, using majority rule, will choose the true option with probability greater than p . Moreover, this latter probability will approach unity as the size of the jury increases.

Pesendorfer (1998), Duggan and Martinelli (1999)]. Even better known is the notion of a Condorcetian “voting cycle”. This work by Condorcet was brought to the attention of political scientists and economists by Duncan Black (1958): see also the Introduction in Black’s *Collected Works* in 1998. The most famous result in this tradition of formal social choice theory is, of course, due to Kenneth Arrow (1951).

The two distinct ideas of *belief aggregation* associated with the Jury Theorem, and of *preference aggregation*, as utilized in social choice theory, seem to generate entirely different interpretations of the functioning of a democracy.

I have suggested elsewhere [Schofield (2002a)] that Condorcet’s result on the Jury Theorem influenced Madison’s arguments in his essay on “The Vices of the Political System of the United States” of 1786, and in his *Federalist X* of 1787 [Madison (1999, pp. 69–80 and pp. 160–167)]. Madison argued in these two essays that well-intentioned representatives could act as a “jury” in determining superior choices for the republic. [See Beer (1993) for discussion]. To better understand the process of the ratification of the American Constitution in 1787–88, it is also important to make sense of the decision by the colonial representatives in the Continental Congress to declare independence from Britain in 1776.

My view of these two crucial decisions in 1776 and 1787 was that they initially involved deep constitutional quandaries. In the early 1770s it was unclear to the colonists whether the British did indeed threaten tyranny. For their part the British cabinet was concerned with the effects on the Indian population of settler intrusion into the Ohio Valley. After the seven-years war of 1756–1763, a rebellion by a coalition of Indian tribes under Pontiac took two years to settle. The British tried first of all by an Act of Proclamation to close the Ohio Valley to settlement. When this proved ineffectual, the Quebec Act of 1774 transferred control of the whole trans-Appalachian region to Quebec. By this act, colonial expansion to the West was to be constrained. Although this provocation may have inflamed the colonial elite, the obvious military capability of Britain initially deterred a declaration of independence. However, the secret promise of aid to the colonists by Louis XVI changed the decision problem facing the Continental Congress from one of uncertainty to one of risk. While the members of the Congress may be regarded as risk-averse, in the dilemma presented between independence and acquiescence, they were willing to choose the risky option of war. Of course, not all of the colonial population agreed with this assessment. Nonetheless, this risky choice led to the successful outcome of independence. From the point of view of the American elite in 1786–87, the historical fact that the choice in 1776 proved to be “correct” would have given some empirical basis for believing in the ability of a representative body to choose wisely.

As Riker (1964) suggested many years ago, the weak form of confederation adopted by the American states after the Revolutionary War had proved to be ineffectual in dealing with the significant threat already posed by Spain in the Mississippi Valley in the early 1780s. Although a stronger federation was the obvious way to deal with this threat, such a constitutional transformation created an intellectual quandary.

A fundamental belief at this time was that democratic choice was beset by the problem of factional turbulence. Adam Smith (1776) had, for example, alluded to such a belief in the closing pages of *Wealth of Nations*.

Madison's argument in *Federalist X* dealt directly with this quandary. While recognizing the possibility of such turbulence, Madison argued that representatives of a heterogeneous republic could indeed make choices for the public good. If we interpret Madison's argument in terms of the Jury Theorem, then it only seems to be valid, however, if the choices to be made are "binary" (left–right or yes–no, for example).

Although it was not made explicit by Madison, an institutional apparatus that creates "binary" political competition may be a necessary condition for the validity of the argument. It has indeed been noted by many scholars [Schattschneider (1942), Riker (1953)] that politics in the USA nearly always collapses to two-party competition. As Riker observed, however, plurality rule² may be necessary for two-party concentration, but does not appear to be sufficient (consider the Liberal Democratic Party in present-day Britain). Indeed, probably the most crucial presidential election in US history took place in 1860, but involved four candidates. Lincoln (the Republican candidate) competed against Douglas (the "northern" Democrat), Bell (the "Whig"), and Breckinridge (the "southern" Democrat). Although Lincoln gained approximately 60% of the northern vote, he took only 40% of the total popular vote. Nonetheless, because of the plurality nature of the electoral college, Lincoln gained 180 electoral-college votes out of 303. The lead-up to this election also involved the deep quandary over the resolution of the compromise over slavery. In a sense this quandary was transformed into a dilemma for the northern electorate, as a result of the Dred Scott Decision by the Supreme Court in 1857. I have argued [Schofield (2002b)] that this decision was interpreted by Lincoln as a threat by the southern states to extend slavery to the North. Acquiescing to this potential threat could result in a great expected cost to northern voters. On the other hand, choosing Lincoln increased the probability of war, also inducing a terrible expected cost.

In a sense the institutional feature of plurality rule forced a choice, one way or another. As a thought experiment, imagine that the election of 1860 took place under proportional representation³ of some kind. Then it is plausible that the result would have been a "coalition government" of northern and southern Democrats, and Whigs. Under a "parliamentary government" of this kind, it is unlikely that war would have occurred. Instead, some kind of compromise would have been made, and the issue of slavery resolved in a different fashion.

This thought experiment is offered as a way of emphasizing different outcomes from different electoral laws. Arguments over the superiority of one form of representation

² Plurality rule means simply that the party or candidate with more votes than any other wins the political prize. Presidential elections in the USA are based on plurality rule in the electoral college.

³ Proportional representation (PR) means that representation of parties or candidates is approximately proportional to vote shares.

over another date back, certainly, to the nineteenth century. John Stuart Mill (1861) for example, suggested that a “proportional” electoral rule led to fairer outcomes, because all interests would be represented. In contrast Lawrence Lowell (1896) argued that the legislature must contain “two parties only, in order that the parliamentary form of government should permanently produce good results”. The arguments have persisted to the present time. Riker (1953), for example, put forward the case that the use of proportional representation tends to lead to increasing political fragmentation. In the Weimar Republic of Germany, the number of significant parties in the Reichstag increased from seven in 1919 to sixteen in 1930 [Mellen (1943)]. It is not implausible that this paved the way for Hitler. Karl Popper (1945) also wrote of the dangers of proportional representation, and much later [Popper (1988)] agreed with Duverger (1984) that plurality rule had the advantage of offering a clear choice to the electorate. [See Schofield (2001) for a review of Riker’s work].

One can, of course, contrast plurality and proportional electoral rules by a “macro-political” examination of any different effects that can be observed. A second, possibly more illuminating, procedure is to construct theoretical models designed to distinguish between different electoral rules. Unfortunately, most formal social choice models focus on two-party competition. The results in this research program are mixed. The so-called “chaos theorems” in spatial voting theory suggest that anything can happen. [See Riker (1980) for such an argument]. On the other hand a large body of literature on the spatial model of elections [Calvert (1985), Enelow and Hinich (1984), for example] suggests that candidates, under two-party competition, converge to an electoral center. The same result obtains moreover, under proportional representation, when candidates attempt to maximize “expected vote share” [Lin et al. (1999)]. Electoral models in this Downsian tradition [Downs (1957)] cannot therefore distinguish between the consequences of plurality and proportional rule. However, it is not at all obvious that candidates, in the “multi-party” situation characterized by proportional representation, will necessarily adopt policies simply to maximize votes. It seems much more likely that candidates come to the political arena with well-specified beliefs that they wish to implement.

This brings us to a more general theoretical point that is worth emphasizing. Almost all models of elections assume that voters are characterized simply by their political preferences, and that candidates desire to “win” (by maximizing votes perhaps). However, it is worth distinguishing between the fundamental preferences of a voter, and “secondary” preferences of the voter over acts or choices.

The connecting relationship between the two is created by the set of beliefs that the voter, or agent, has over the state of the world. These beliefs are subjective probability estimates concerning the truth values of various propositions [Savage (1954)]. Most social choice theory focuses on situations where the preferences are fixed. Clearly, however, new information about the state of the world will change beliefs, and thus secondary preferences. If Bayes’ Law is utilized, as the procedure by which beliefs are transformed, then one might expect beliefs and secondary preferences to change continuously, even slowly. The instances, mentioned above, of the transformation of

constitutional quandaries into choices, suggest, on the contrary, that beliefs can change dramatically under certain circumstances.

Implicit in the concluding section of Keynes' *General Theory of Employment* [Keynes (1936)] is the assertion that fundamental constitutional decisions must often be made in an uncertain environment. As Keynes observed:

By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence. *Keynes (1937)*

[See Schofield (1999a) for an elaboration on the relevance of uncertainty in Keynes' thought].

Uncertainty is more than just some variance in subjective probability estimates. In situations that I have described as quandaries, the costs of incorrect choices are high, while individuals will be unwilling to derive secondary preferences from their uncertain estimates of probabilities. How, then, can social choice be made? One feature that appears to characterise the constitutional choices described above is that a compelling argument was presented which had the effect of transforming uncertainty into risk. For example, in the Continental Congress in 1776, uncertain concerns over the intentions of the British and French were changed into coherent beliefs. Although the expected costs of acquiescence or rebellion were both high, the decision became one of risk, not uncertainty. It is reasonable to identify the consequent social decision as a *dilemma*. Although not all members of the Congress would necessarily have precisely the same probability estimates, it is clear that a majority chose rebellion. This outcome, based on secondary preferences, can be interpreted as a *core* (or unbeaten outcome under the institutional rule adopted). By analogy, the set of beliefs that underlay this decision can be described as a core belief.

In the same way, the set of beliefs over foreign threat, and over the possible resolution of the dangers of factionalism (held by the members of the Constitutional Convention in Philadelphia in 1787) led to a core belief in the wisdom of ratification.

Finally, the election of 1860 can be seen as the culmination of a process leading from electoral uncertainty, to a more precise appreciation of the dilemma (involving anticipated costs and probabilities) to a final choice. It is important to note that the set of beliefs, held in the northern electorate, was transformed, by the institutional mechanism of the electoral college, into a core belief that sustained the choice of Lincoln. [See Schofield (2002b) for further discussion].

As I have suggested above, when beliefs of the voters are the most significant primitives to be studied, then decision-making may be described in terms of a *jury*. In contrast, when the beliefs of the agents are held fixed, and preferences of the agents are taken as the primitives, then the object of study will be termed *committee decision-making*. In the body of the chapter that follows I shall review various models of elections that are derived from the theory of committee decision-making. Section 2 surveys the traditional view of an election as a committee choice. Typically,

this literature assumes that there are only two parties, or candidates, who attempt to adopt positions in order to win. When there are more than two parties, then coalition formation is necessary for the creation of government. Bargaining between parties (after the election) can also be interpreted in terms of committee decision-making [in Chapter 8 of this Volume, Peleg (2002) presents a somewhat different perspective on this process].

However, the choices made by political parties over leaders and policy positions are made prior to elections. Section 3 presents a formal perspective on the existence of equilibria in such a “political game”. Such equilibria depend on the beliefs held by the party principals over the nature of the electoral response to party declarations.

One way to model an election is to describe each voter by a stochastic choice function, which (given party positions) assigns a probability vector to the voter. The probability that voter i chooses party j can be interpreted as an estimate of the belief by i that party j is the superior choice (given the recent history of party actions). Econometric analyses can then be used to estimate the relationship between the vector of party positions and the expected vote shares of the various parties. A number of examples are offered which strongly indicate that parties do not adopt policy positions simply to maximize expected vote. Indeed, the results of Section 4 suggest that the elite members of parties choose their political leaders subject to complex beliefs over electoral response *and* preferable policy choices. In multiparty systems it is clear that the logic of such choices can lead to high political fragmentation.

The theoretical and empirical analyses presented in these three sections are not conclusive but they do suggest that plurality and proportional electoral systems generate quite different political choices. Because of the need to bargain over coalition government, proportional electoral systems may support political choices that are, in a sense, “risk-averse”. The concluding section of this chapter suggests that plurality electoral methods may endanger risky choices by some of the parties. It is possible that these “endogeneous” risk postures in different political systems can be connected to the quite different present-day economic characteristics of the United States and Britain on the one hand, and the multiparty polities of Continental Europe, on the other.

2. Voters as a “committee”

It has been traditional in the literature on the spatial voting model to suppose that the electorate comprises a committee. That is to say, each voter, i , has a “bliss” point x_i in a “policy” space, Z , say. It is usual to assume that Z is a compact, convex subset of Euclidean space \mathfrak{R}^w . The “utility”, u_i , for voter i from some policy point $y \in Z$ is typically assumed to be Euclidean, of the form $u_i = -\|x_i - y\|^2$, where $\| \cdot \|$ is the norm on \mathfrak{R}^w . Almost all the early work on this spatial voting model studied the induced strict majoritarian preference Q_f on Z . Let $f = \{x_i: i \in N\}$ denote the distribution of voter ideal points for the society N of size $n = |N|$. Let $Q_i: Z \rightarrow Z$ denote voter i 's

strict preference correspondence, given by $z \in Q_i(y)$ iff $\|z - x_i\| < \|y - x_i\|$. Q_f is given by $z \in Q_f(y)$ iff $z \in Q_i(y)$ for all i belonging to some majority coalition A of size $|A| > \frac{1}{2}n + 1$. The social equilibrium, or majority rule *voting core*, is

$$E(Q_f) = \{\bar{x}: Q_f(\bar{x}) = \emptyset\}. \quad (2.1)$$

Results by Plott (1967), McKelvey and Schofield (1986), Banks (1995) and Saari (1997) can be interpreted to imply that $E(Q_f)$ is empty for almost all $f \in \mathfrak{R}^w$, as long as $w \geq 3$. Moreover, the majoritarian preference Q_f will generally be *chaotic*. For almost any pair of points $y_0, y_r \in W$ there exists a finite voting trajectory y_1, \dots, y_{r-1} with $y_t \in Q_f(y_{t-1})$ for $t = 1, \dots, r$ [McKelvey (1976, 1979), Schofield (1977, 1978, 1985), Austen-Smith and Banks (1998, 1999)].

In a very influential book, William Riker (1982) discussed the relevance of these social choice results for democratic theory. Riker contrasted the populist (or Rousseauian) view of democracy, whereby liberty is obtained “by embodying the will of the people in the action of officials . . .” [Riker (1982) p. 11], with the liberal (or Madisonian) view that a sufficient condition for democracy is that “the government . . . is administered by people holding their offices during pleasure, for a limited period, as during good behavior”. [*Federalist XXXIX*, in Bailyn (1993)]. In Riker’s opinion, the chaos results from social choice theory made the populist view untenable. However, the liberal view was still compatible with these results. Nonetheless it did appear possible that even liberal democracy would be beset by instability or factional turbulence, of the kind discussed in the previous section in relation to Madison’s fears of the possible outcome of Union. It is worth mentioning that throughout the 1970s many scholars had also drawn pessimistic conclusions about the long-term survival of democracy in the face of the economic disorder induced by monetary instability and the oil crises of 1974 and 1979. Indeed, these fears can be described as a “democratic quandary”. I shall discuss this quandary in Section 5 below.

A number of scholars adopted a “neo-institutional” approach arguing that political institutions generated rules of the political game and that these rules would lead to equilibrium behavior by politicians [see for example Shepsle (1979), Shepsle and Weingast (1981) and Fiorina and Shepsle (1982)]. Riker’s response was that in any institutional setting, the “losers” would have an incentive to change the rules, in the hope of gaining power [Riker (1980)]. Overall, he argued that “The most important conclusion of [this] line of reasoning is that, in the long run, nearly anything can happen in politics” [Riker (1980)]. In his later work on the Ratification of the Constitution and the lead-up to the Civil War, Riker, in a sense, set out to explore this hypothesis [Riker (1984, 1986, 1996)].

However, arguments about the stability of democracy based on the chaos theorem have to make more precise the nature of the calculus of politicians themselves. The simplest assumption of a two-party (or two-candidate) model of elections is that each of the two candidates (j, k , say) simply adopts positions $z_j, z_k \in Z$ in an effort to defeat

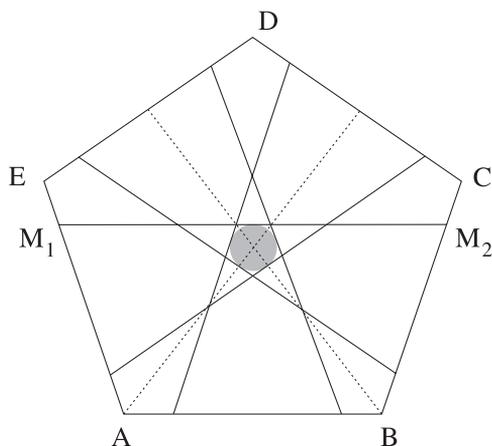


Fig. 1. Median Lines and the yolk under majority rule with a uniform voter distribution on the pentagon. (Figure drawn by Tsvetan Tsvetkov.)

the other. For example we could assume that the utility U_j of candidate j at the policy profile z_j, z_k was given by

$$U_j(z_j, z_k) = \begin{cases} 1 & \text{if } z_j \in Q_f(z_k), \\ -1 & \text{if } z_k \in Q_f(z_j), \\ 0 & \text{otherwise.} \end{cases} \quad (2.2)$$

A pure-strategy Nash (1950, 1951) equilibrium (PSNE) in this two-agent game is a pair (z_j^*, z_k^*) such that $U_j(z_j, z_k^*) > U_j(z_j^*, z_k^*)$ for no $z_j \in Z$ and $U_k(z_j^*, z_k) > U_k(z_j^*, z_k^*)$ for no $z_k \in Z$.

Obviously enough if $z \in E(Q_f)$ then a possible PSNE is the pair $(z, z) \in Z \times Z$. On the other hand, the chaos theorem implies that PSNE in this game generally do not exist. However, there may be situations in which mixed-strategy Nash equilibria (MSNE) exist [Kramer (1978)]. In the 1980s interest focused on characterizing MSNE in such spatial election models.

To illustrate these results consider Figure 1. Assume that the distribution, f , of voter ideal points is given by the uniform distribution on the pentagon, Z in the figure. A median line (such as M_1M_2 in Figure 1) exactly divides the distribution in two. If the infinite family of median lines were to intersect, then $E(Q_f)$ would be non-empty. [For example if one had, instead of the pentagon, the disc with the uniform distribution, then $E(Q_f)$ would be non-empty. See Arrow (1969), for an illustration.] In Figure 1 the medians do not intersect. But they do generate a disc, called the *yolk*: the smallest disc that touches all medians. The positioning of the yolk in Figure 1 suggests that its location and size provide a measure of the spherical symmetry of the distribution, f . Various alternative "attractors" of the voting process have been considered. One of these is the uncovered set of Q_f [Miller (1980)]. The general idea is to modify Q_f so that it exhibits no cycles. Thus, the covering relation, \hat{Q}_f induced by Q_f is defined by $z \in \hat{Q}_f(y)$ iff $z \in Q_f(y)$ and $Q_f(z) \subseteq Q_f(y)$. The uncovered set, $\tilde{E}(Q_f)$ of Q_f , is given

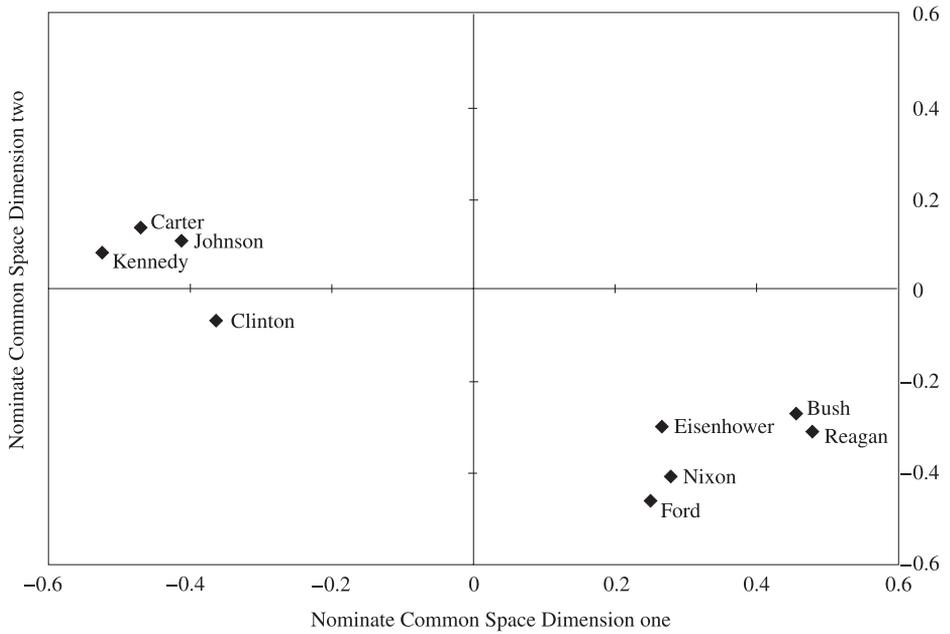


Fig. 2. “Positions” of US presidents from Eisenhower to Clinton in the two-dimensional policy space of Poole and Rosenthal (1997).

by $\tilde{E}(Q_f) = E(\tilde{Q}_f) = \{z \in Z: \tilde{Q}_f(z) = \phi\}$. It is known that the support of MSNE for the two-candidate game just described, under the electoral distribution f , is contained within the uncovered set of Q_f [Banks, Duggan and Le Breton (1998, 2002)]. Earlier studies of the uncovered set [McKelvey (1986), Cox (1987)] show that the uncovered set is “centrally located”, and converges to the social equilibrium $E(Q_f)$, when this exists. For example in Figure 1 the uncovered set is bounded by a disc with radius approximately twice the yolk radius.

An alternative notion, the “heart” [Schofield (1999b–d)], is a local version of the uncovered set. In Figure 1 the heart, defined by Q_f , is the set bounded by the family of median lines. The heart contains both the yolk and the uncovered set, and can usually be easily calculated. More importantly, the heart is “lower hemi-continuous” in all parameters. I shall use the idea of the heart in the next section.

The conclusion of the general Downsian model [Downs (1957)] of two-party competition is that parties, or candidates, “converge” to a domain, centrally located with respect to the electoral center [Calvert (1985)]. However, there is very little evidence that two-party competition induces convergence of this kind.

For example, Figure 2 presents estimates of US presidential policy positions from Eisenhower to Clinton. The space, Z , is obtained from estimates of Poole and Rosenthal (1991, 1997) and from National Election Survey Data. The left–right dimension is a

usual economic dimension, while the north–south dimension concerns social policy/civil rights. Clearly there is little indication that Democrat and Republican candidates converge to the electoral origin.

This convergence phenomenon is a theoretical property of a committee of voters, choosing “deterministically” in the manner described. An alternative voter model is “probabilistic” or stochastic. Here the “utility” for voter i from party j at position z_j takes the form $u_i(z_j) = -\|z_j - x_i\|^2 + \epsilon_j$, where ϵ_j is a normal random variable, with mean 0 and variance σ^2 . Voter i chooses party j , at z_j , over party k at z_k iff $u_i(z_j) > u_i(z_k)$. Thus voter choice is described by a probability pair (ψ_{ij}, ψ_{ik}) , where ψ_{ij} , the probability that i chooses j , is given by $\text{Prob}(-\|z_j - x_i\|^2 + \epsilon_j > -\|z_k - x_i\|^2 + \epsilon_k)$. It is usual to assume that (ϵ_j, ϵ_k) are both normal and independent, and that party j , say, adopts a position to maximize the expectation $\frac{1}{n} \sum_{i \in N} \psi_{ij}$.

As Enelow and Hinich (1984) and Coughlin (1992) have shown, for such a model there will typically exist a PSNE (z^*, z^*) where each party adopts an identical position, z^* , at the mean of the voter distribution. Figure 2 suggests that both the deterministic and the probabilistic voter model are inappropriate as theoretical devices to understand elections under plurality, or majority rule. We shall return to this point in Section 4 below, where I shall argue that modeling voter choice as a committee (based on *preferences*) is an invalid approach.

3. A “committee” of politicians

The deterministic voting model described in the previous section assumes that elections are riskless: both parties know the relationship between the pair of party declarations (z_j, z_k) and the electoral outcome. Even the probabilistic model assumes that each party attempts to maximize the *expectation* of the vote share. This is tantamount to ignoring the variance in the “stochastic” vote share variables.

Cox (1984) proposed an alternative model that emphasized the stochastic nature of the electoral response. First note that there are three states of the world resulting from the election when just two parties or candidates compete. Let us use \mathcal{D}_0 to denote a draw (where both parties or candidates gain equal shares of votes, or seats), \mathcal{D}_j to denote the state in which party j wins and \mathcal{D}_k to denote the state where k wins. Each party, j say, is assumed to have a “Euclidean” utility from the policy z of the form $-\|z - y_j\|^2$, where y_j is party j ’s bliss point. In state \mathcal{D}_j party j also gains government perquisites, say $\alpha_j \delta$.

Prior to the election, j, k declare policy positions (z_j, z_k) . If state \mathcal{D}_j ensues, then j implements z_j , and takes the government perquisite $\alpha_j \delta$.

If state \mathcal{D}_0 occurs, then a lottery, say $\tilde{g}_0(z_j, z_k)$ results: that is, $\{j, k\}$ bargain together and implement a randomized policy $\frac{1}{2}(z_j + z_k)$ dividing the perquisite equally. Let $\pi_0, \pi_j, \pi_k: Z^2 \rightarrow [0, 1]$ denote the electoral probability functions, so, for example, the probability of a draw when (z_j, z_k) is declared is $\pi_0(z_j, z_k)$.

Table 1
The Election in Britain, May 1, 1997

Party	Seats	% Seats	% Vote
Labour	419	63.6	44.4
Conservatives	165	25.0	31.4
Liberal Democrats	46	7.0	17.2
Scottish National Party	6	0.9	2.0
Plaid Cymru	4	0.6	0.5
Indep./Others	19 ^a	2.9	4.5
<i>Northern Ireland (of which 18)</i>			
Ulster Union	10		
UK Union	1		
Democratic Union	2		
SDLP	3		
Sinn Fein	2		
Effective Number		2.12	3.12

^a Of these nineteen, 1 is an independent (*i.e.*, a constituency in Cheshire). The other 18 are in Northern Ireland, as indicated.

Then the utility to j , say, of the underlying game form \tilde{g} is:

$$\begin{aligned}
 U_j(\tilde{g}(z_j, z_k)) = & \pi_j(z_j, z_k) [\alpha_j \delta - \|z_j - y_j\|^2] \\
 & + \pi_k(z_j, z_k) [-\|z_k - y_j\|^2] \\
 & + \pi_0(z_j, z_k) [\frac{1}{2} \alpha_j \delta - \|\frac{1}{2}(z_j + z_k) - y_j\|^2].
 \end{aligned} \tag{3.1}$$

In this formulation, when party k wins, then party j receives no government perquisites. Cox (1984) suggested that, under fairly natural conditions, including continuity of the probability functions, there would exist a PSNE for this game form. Of course, most parliamentary elections involve more than two parties. Indeed, many years ago Duverger (1954) suggested that electoral systems based on proportional representation would result in numerous parties. Conversely, he argued that so called “first past the post”, or plurality elections, based on single member districts, would tend to result in two-party politics. Britain is the obvious example of a plurality electoral system. As Table 1 illustrates, in the election in May 1997 in Britain, about 44% of the popular vote for Labour was sufficient to give it over 60% of the seats. However, the vote for the small center Liberal Democrat Party (at 17%) was by no means negligible. The empirical question of determining “the probability of winning” in such an electoral system is extremely difficult. Using survey techniques it is of course possible to

estimate the popular vote shares, but these do not translate directly into seat shares. I return to this question in the next section.

By contrast, Table 2 presents the outcomes in the last four elections in Israel. The electoral system is extremely proportional, in that vote shares and seat shares are almost identical. (Small parties with less than 2% of the popular vote received no seats).

To understand political motivations in such an electoral system, it is necessary to note that no party may expect to win the election (if this is interpreted to mean “gain a majority of the seats”).

However, it is possible to model post-election bargaining between parties. To understand the nature of the policy space in Israel, surveys conducted by Arian and Shamir (1995, 1999) for the 1992 and 1996 elections were utilized. Factor analysis of responses by a sample of voters (of size approximately 1000) gave a two-dimensional space, Z . The left–right dimension correlated with attitudes to the PLO, and the north–south dimension described beliefs regarding the proper relationship between the state and the Jewish faith. The pre-election declarations (or manifestos) of the various parties were processed using the Arian–Shamir questionnaire. The “positions” or declarations of the various parties are marked in Figure 3 for 1992 and in Figure 4 for 1996. [See Schofield, Sened and Nixon (1998) and Schofield and Sened (2002) for further details].

To extend Cox’s model of electoral competition, let us regard the situation after the 1992 election as a committee comprising the ten parties with seat strengths as given in Table 2. Let us use \mathcal{D}_1 to denote the family of “winning” or majority coalitions after the election of 1992. Obviously Labor, together with the three small parties (on the left), with 61 seats, form a majority. Likud, and the religious parties, together with Tzomet, have only 59 seats, and so need another party, such as Meretz, to attain a majority.

Further, let us ignore government perquisites for the moment and assume that each party has Euclidean utility based on an ideal policy point given by its declaration (as illustrated in Figure 3). With these party preferences and voting weights, the “social preference” in the Knesset can be denoted $\sigma_{\mathcal{D}_1}$. It should be obvious that the “parliamentary” core, or voting equilibrium $E(\sigma_{\mathcal{D}_1})$ is non-empty, and located at the Labor party declaration. To see this, note that the “compromise set” for each coalition is simply the convex hull of the ideal points of the coalition members. Clearly all “compromise sets” for the various winning coalitions intersect at a single point, namely that of the Labor party. A model of political bargaining proposed by Banks and Duggan (2000) suggests that the only possible outcome of bargaining in the coalition situation \mathcal{D}_1 is precisely this equilibrium or core point. Moreover Laver and Schofield (1990) argue that Labor would form a minority government. In fact, Labor under the leadership of Rabin did form a minority government, with implicit support from Meretz. As a consequence, the peace accords with the PLO were initiated.

After the election of 1996 a very different coalition structure, \mathcal{D}_2 , was created. It should be clear from Table 2 that under \mathcal{D}_2 , the coalition excluding Labor and the

Table 2
Elections in Israel

Party	Knesset Seats			
	1988	1992	1996	1999
Left:				
Labor	39	44	34	28
Meretz	–	12	9	10
Shinu	2	–	–	6
Others	9	–	–	–
Dem Arab	1	2	4	5
Commun.	4	3	5	3
Balad	–	–	–	2
<i>Subtotal</i>	<i>55</i>	<i>61</i>	<i>52</i>	<i>54</i>
Center:				
Olim	–	–	7	6
Yisrael Beiteinu	–	–	–	4
Third Way	–	–	4	–
Center	–	–	–	6
Right:				
Likud	40	32	32	19
Tzomet	2	8	–	–
<i>Subtotal</i>	<i>42</i>	<i>40</i>	<i>43</i>	<i>35</i>
Religious:				
Shas	6	6	10	17
Aguda	5	4	–	–
Yahadut	2	–	4	5
NRP	5	6	9	5
Moledet	2	3	2	4
Techiya	3	–	–	–
<i>Subtotal</i>	<i>23</i>	<i>19</i>	<i>25</i>	<i>31</i>
Total	<i>120</i>	<i>120</i>	<i>120</i>	<i>120</i>

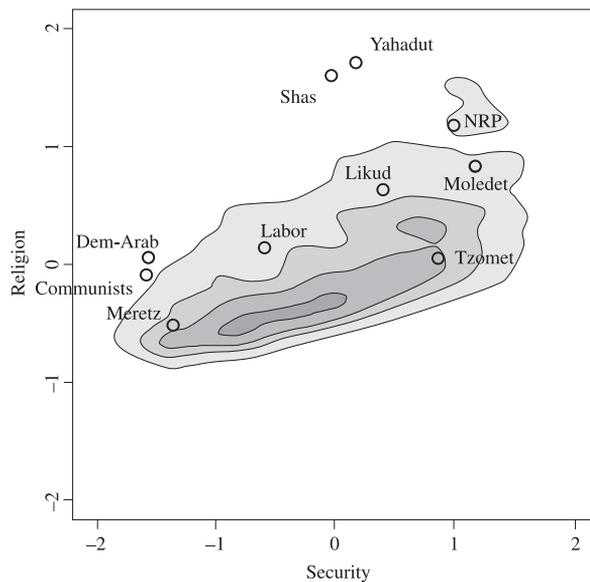


Fig. 3. Party Positions in the Israeli Knesset in 1992, also showing the 95%, 75%, 50% and 10% contour lines of the highest-density plot of the voter sample distribution [from Schofield and Sened (2002)].

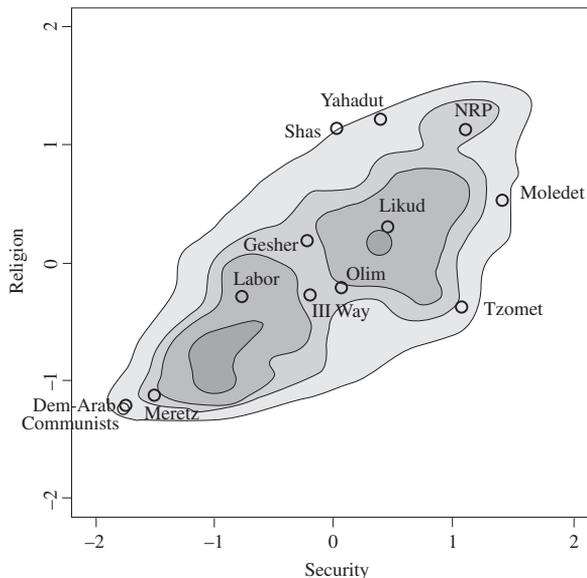


Fig. 4. Party Positions in the Israeli Knesset in 1996, also showing the 95%, 75%, 50% and 10% contour lines of the highest-density plot of the voter sample distribution [from Schofield and Sened (2002)].

“left” controlled a majority of 68 seats. In this case, the party preferences and voting weights give a different “social preference”, $\sigma_{\mathcal{D}_2}$ say. It is evident that the core $E(\sigma_{\mathcal{D}_2})$ is empty. Essentially there are, in principle, three different coalition governments possible after 1996: one based on Likud, Shas, and the religious parties; a national unity government of Likud and Labor; and a coalition of Labor, and the “left” together with Shas. Since the equilibrium, or core, $E(\sigma_{\mathcal{D}_2})$ is empty there is no possibility of a minority Likud government. In fact, Netanyahu (leader of Likud) won a separate prime-ministerial election against Peres (leader of Labor). Although Labor was the larger of the two parties, Netanyahu constructed a majority coalition that depended on Shas.

I suggest that bargaining between the parties is constrained to a set known as the “heart”⁴. Under Euclidean policy preferences, the “parliamentary” heart after 1996 is simply the convex hull of the declarations of Labor, Likud and Shas. Because Netanyahu won the separate election, the outcome after this election can be seen to be a point on the arc joining the Likud and Shas positions. However, because the Likud position was not an equilibrium, the Likud-led government was eventually defeated under a vote of no-confidence. (One aspect of this defeat was the defection of Gesher, a small centrist party, out of its alliance with Likud. Gesher is not listed in Table 2, since its seats counted as part of Likud.) After the 1999 election, a majority coalition led by Barak, of Labor, was constructed that collapsed two years later.

It is possible to construct a Nash equilibrium model that can be used to explain how parties choose declarations prior to an election. Firstly, suppose that $\{\mathcal{D}_1, \dots, \mathcal{D}_T\}$ denotes the collection of all possible coalition structures that can occur at the time of the election. Let P denote the set of parties, $\{1, \dots, p\}$, contesting the election. Suppose that the various parties have selected their leaders, and that the vector of leader positions is denoted $z = (z_1, \dots, z_j, \dots, z_p)$.

Assume that each leader j has a smooth, strictly convex preference correspondence $q_j(z_j): Z \times \Delta_p \rightarrow Z \times \Delta_p$.

This is derived from a utility function of the form $u_j(y, \delta) = -\|y - z_j\|^2 + \alpha_j \delta$, where δ represents government perquisites. The symbol Δ_p denotes the $(p - 1)$ -dimensional simplex. Let $q^\alpha(z)$ denote the profile of leader preferences, which we can regard as a profile on $W = Z \times \Delta_p$.

Once the election outcome (in terms of the coalition structure \mathcal{D}_i) is known, then this, together with the profile of leader preferences, generates a committee preference, which I shall now denote as $\sigma_{\mathcal{D}_i}^\alpha(z)$. It may be the case (as in the example of the Knesset in 1992) that the parliamentary core $E(\sigma_{\mathcal{D}_i}^\alpha(z))$ is non-empty. Even when the core is empty, however, it has been shown [Schofield (1999d)] that the more general solution, the “parliamentary heart”, is non-empty. Because the preferences of the committee are

⁴ A point x is in the “heart” defined by the “parliamentary” preference relation $\sigma_{\mathcal{D}}$ iff there is a neighborhood V of x such that $V \cap \tilde{\sigma}_{\mathcal{D}}(x) \equiv \phi$. Here $\tilde{\sigma}_{\mathcal{D}}$ is the covering relation induced from the preference $\sigma_{\mathcal{D}}$.

defined on the space $W = Z \times \Delta_p$, the heart $\mathcal{H}_t^\alpha(z)$, say, is a subset of W . For the fixed coalition structure, \mathcal{D}_t , the heart can be regarded as a correspondence $\mathcal{H}_t^\alpha: Z^p \rightarrow W$. Moreover, this correspondence admits a continuous selection [Michael (1956)]. I now assume that for each z , and \mathcal{D}_t , the outcome of coalition bargaining is a lottery $g_t^\alpha(z)$, whose support is contained within the heart. I further assume that for each fixed \mathcal{D}_t , this outcome function $g_t^\alpha: Z^p \rightarrow \tilde{W}$ is smooth. Here \tilde{W} denotes the space of lotteries on W , endowed with the weak topology [Parthasathy (1967)]. The function g_t^α is meant to represent the beliefs that the political elite have about the nature of the bargaining game, at the fixed coalition structure, \mathcal{D}_t . However, prior to the election the elite cannot be sure which coalition structure will occur. I assume, however, that at the vector, z , of party-leader positions, the political elite can estimate the probability $\pi_t(z)$ that \mathcal{D}_t occurs. I also assume that each electoral probability function $\pi_t: Z^p \rightarrow [0, 1]$ is smooth. With these smoothness assumptions on $g^\alpha = \{g_t^\alpha\}$, and on $\pi = \{\pi_t\}$, it is possible to show that party principals can rationally choose party positions.

To be more precise, I assume that the leader of party j is chosen by a *principal* of the party (that is one of the elite party members). The principal cares about final government policy, and about government prerequisites. The game form g^α and the electoral probability, π , are assumed to be common knowledge. The preference of the principal of party j is described by a von Neumann–Morgenstern utility function U_j , which represents the preference held by the principal over both policy and government prerequisites. At a vector, z , of declarations, the induced utility, U_j^S , of principal j , is given by

$$U_j^S(z) = \sum_{t=1}^T \pi_t(z) U_j(g_t^\alpha(z)). \tag{3.2}$$

Equation (3.2) can be seen to be an extension of Cox’s Equation (3.1) to the more complex case of multiparty competition. A local Nash equilibrium (LNE) of the game $(\{U_j: j \in P\}, g^\alpha)$ is a vector $z^* \in Z^p$, such that for each j there is a neighborhood V_j of z_j^* , with

$$z_j^* = \operatorname{argmax}\{U_j^S(z_1^*, \dots, z_j, \dots, z_p^*): z_j \in V_j\}. \tag{3.3}$$

Schofield and Sened (2002) have shown that a LNE “generically” exists, under the assumptions that have been made on g^α and π .

The model that has just been proposed attempts to incorporate a number of features that seem important for political decision-making. First of all, any policy declaration made by a party must be the result of compromise of some sort among the heterogeneous party elite. I assume above that this compromise party preference can be identified with that of a party principal. This principal then chooses a leader to present to the electorate. The leader’s policy preferences will be credible to that electorate. In choosing the leader, the principal must balance the effect the leader will have on both the electoral response, and on post-election coalition bargaining. Obviously enough, if

government perquisites are highly valued, then the principal may focus on choosing a leader who is centrally located, or electorally popular. If policy is more highly valued, then the principal may choose a “radical” leader, capable of vigorous bargaining with other parties over government policy.

To implement such a model requires both an estimation of the electoral risk functions $\{\pi_t\}$, as well as a determination of party-leader positions, and of principals’ preferences (or at least the policy preferences within each party).

The next section reports on recent work that has been undertaken on estimating π in different countries over a number of elections. The results suggest that elections more closely resemble jury choice, in that the choices of voters are determined by changing beliefs, rather than fixed preferences. Moreover, the choices of party positions (or leaders) seem not to be determined by attempts to maximize expected vote, but rather to influence government policy.

4. Elections as methods of belief aggregation

The existence of a “political equilibrium” proposed in the previous section depends on the smoothness of the electoral probability functions $\{\pi_t\}$. Since the model is one of political choice prior to an election, these probability functions should be regarded as beliefs of the political elite, concerning the nature of electoral response. To model these beliefs, we can use survey data to estimate an electoral model in the following fashion. Given a vector $z = (z_1, \dots, z_p)$ of party declarations, we can assume that the utility u_i of voter i is represented by a $(p \times 1)$ vector

$$u_i(z) = V_i + X\beta_i + S\gamma_i + \epsilon. \quad (4.1)$$

In a general multinomial probit (MNP) model, the $(p \times 1)$ vector ϵ consists of a disturbance term, which is multivariate normal, with mean 0, and a variance/covariance matrix Σ . In the simpler multinomial logit model (MNL) the disturbances are iid (independent and identically distributed). The $(p \times m)$ matrix X can incorporate socio-structural/individual features (such as class, education). The $(p \times 1)$ vector V_i describes policy aspects. In the pure spatial model V_i is simply the vector $(-d_{ij}^2)$, where $d_{ij} = \|z_j - x_i\|$ is the policy distance between party j and voter i ’s ideal point. The matrix S_i includes choice features such as the “likability” of party leaders, past economic performance of parties, *etc.* Voter i chooses party j with probability $\Psi_{ij}(z) = \text{Prob}[u_{ij}(z) > u_{ik}(z) \text{ for all } k \neq j]$. Thus the model is stochastic and voter i ’s behavior is described by a probability vector $\Psi_i(z) = (\Psi_{i1}(z), \dots, \Psi_{ip}(z))$.

If this empirical model is based on a sample, N , of size n , then the estimated vote share of party j at Z is obviously $\frac{1}{n} \sum_{i \in N} \Psi_{ij}(z)$. However, because $\Psi = \{\Psi_i\}_N$ are random variables, the vote shares will have significant variance. Intrinsic to this model is the existence of electoral risk. By modeling the electoral response, Ψ , it is possible to construct estimates of the probability functions $\{\pi_t\}$ used in the previous section. Such a model generates smooth functions $\{\pi_t\}$.

Because the Arian–Shamir survey data for 1992 and 1996 on Israel included voter intentions, it was possible to construct a pure spatial model of the 1992 election. In order to do so, it was necessary to incorporate constant terms (in the matrix S) for each of the parties. As indicated above, these terms described non-policy aspects of the parties (such as popularity of the party leaders). In 1992 the constant was largest for Labor, smaller for Likud, and much smaller for Meretz and the other parties [Schofield, Sened and Nixon (1998)]. We can infer from this that voters understood that Labor and Likud were the principal protagonists in the 1992 election. Voters whose preferred policy positions were closer to one of the religious parties apparently voted for Likud [see Cox (1997), for an analysis of strategic voting]. An examination of the expected vote shares for Likud and Labor suggested that these two parties were close to positions that maximized expected vote shares. However, the smaller religious parties, according to our estimation, could have dramatically increased their expected vote share by moving to the electoral center. The model proposed in the previous section suggests a reason why parties may adopt local Nash equilibrium positions that do not maximize expected vote share.

Consider for example the religious party Shas. As I indicated previously, under the coalition structure \mathcal{D}_2 , Shas could expect to belong to government. The bargaining model proposed by Banks and Duggan (2000) suggests that Shas, by adopting a policy position far from the center, could bargain more effectively with Likud or Labor. Thus, if the subjective probability π_2 held by Shas increases, the model suggests that Shas will adopt a more radical position. [A more detailed formal analysis of this observation is offered in Schofield and Parks (2000).]

Figures 3 and 4 do indeed suggest that the parties adjust their policy positions from one election to another. Note, however, that these adjustments are small, which is in keeping with the emphasis on *local* Nash equilibria made in the previous section.

It has been argued that, under vote maximization, parties will converge to an electoral center [Lin, Enelow and Dorussen (1999)]. There is no evidence from the Israeli examples presented here that convergence in the Downsian sense occurs. In fact, the density plot of the sample voter distribution in Figure 4 suggests that this distribution is strongly “bipolar”. The “center” of the electoral distribution is located between two peaks of the distribution, and only small parties (such as Olim or Gesher) adopted positions near this center.

Indeed, Israel appears to exemplify a hypothesis offered by Duverger. As he wrote “the centre does not exist in politics: there may well be a Centre party, but there is no centre tendency, no centre doctrine . . . Every Centre is divided against itself . . .” [Duverger (1954) p. 215].

A further inference can be drawn from Figures 3 and 4. It appears that the sample distribution of voter ideal points changes over time. In a sense the religious dimension has become more important, as the empirical correlation between attitudes on security and religion has declined. We can infer that voter choices are derived not so much from preferences, which are relatively stable over time, but from beliefs that change in accordance with a changing understanding of the nature of the political world. It

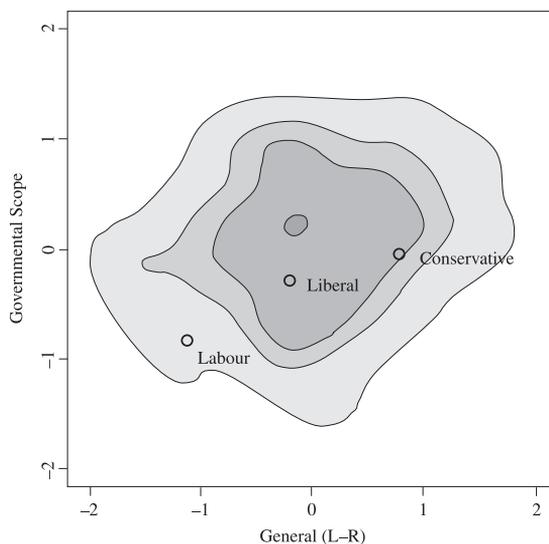


Fig. 5. Party Positions in the British Parliament in 1979, also showing a highest-density plot of the voter sample distribution [from unpublished research by Quinn, Martin and Whitford (1999)].

should also be observed that recent elections in Israel illustrate Riker's (1953) point that proportional representation may increase political fragmentation. While compromise is necessary in such a PR system, increasing fragmentation may make such agreement very difficult to attain.

Other empirical work on voting in proportional electoral systems suggests that voter choice is best understood as a method of belief aggregation rather than preference aggregation. Larger parties in countries such as the Netherlands and Germany tend not to converge to the electoral center. Voter choice is partially determined by policy considerations, but also by non-policy considerations such as leader dependability, *etc.* [Schofield, Martin, Quinn and Whitford (1998)].

Modeling voter choice in a plurality electoral system such as Britain is made difficult because the electoral model gives vote shares, rather than seat shares. However, a recent MNP analysis [Quinn, Martin and Whitford (1999)] is suggestive.

Figure 5 presents a picture of British politics in 1979. The background represents the distribution of voter ideal points (the contour lines give the 95%, 75%, 50% and 10% density regions). The first left-right dimension describes a general economic factor, and the second delineates preferences over the "scope of government". This distribution is estimated from a factor analysis of a voter sample undertaken by Rabier and Inglehart (1981). To obtain party positions a survey of political elites [ISEIUM (1983)] was utilized. This latter sample gave party-specific distributions of political elite responses. The two-dimensional median within each party was utilized to estimate each party position. Note that this estimated position can be interpreted as the position of the party "principal" (as utilized in the previous section), not the party "leader".

A classical socio-structural model of voting utilizes individual voter properties to

Table 3
Elections and Estimations for Britain, 1979

Party	Election vote (%) ^a	Election seats (%) ^b	Sample %	Estimation	95% Con. Int. ^c
Labour	36.9 (39.0)	43.45	35.41	35.24	(30.2, 40.5)
Conservatives	43.9 (46.4)	54.75	53.58	53.55	(48.0, 59.0)
Liberals	13.8 (14.6)	1.75	11.01	11.21	(7.7, 15.0)
Scot Nationals	1.6				
Welsh National	0.4				
N. Ireland, <i>etc.</i>	3.9				
<i>Effective Number</i>	2.80	2.15			

^a Numbers in parentheses are vote shares ignoring small parties and independents.

^b For simplicity, these are recalibrated as percentage of seats of the total number held by the three parties. Other, regional, parties only controlled 2.5% of parliamentary seats.

^c Abbreviation: Con. Int., Confidence Interval.

estimate voter choice. In Equation (4.1), this is equivalent to assuming the vector $V_i \equiv 0$. Utilizing variables such as religion, income, education, it is possible to estimate how the sample (of size 426) voted. Actual voter choice was known, and the estimation correctly predicted 49% of the votes. A pure spatial model assumes $X \equiv 0$, and estimates V_i making use of voter and party positions. In the estimate for 1979 a small positive constant term for the Conservatives, and a negative term for the Liberals, were incorporated. Table 3 gives the actual election votes share together with the estimated vote response.

Because the model is fitted to the sample response, the estimated expected vote shares are very close to the sample vote shares. What is perhaps more relevant is that approximately 49% of the sample choices were correctly predicted. In a joint model, utilizing both spatial and socio-structural variables, the correct prediction increased to 51%. Analysis of the Bayes factors [Kass and Raftery (1995)] suggested that the pure spatial model was the best statistical estimator of voter response in Britain.

This analysis raises a number of interesting questions about voter choice under a plurality electoral system such as Britain's.

Firstly, constant terms [as designated by S in Equation (4.1)] were utilized. However, estimated confidence intervals on these constants gave no indication that they were statistically different from zero.

Secondly, the spatial model did correctly predict about 50% of the voter choices, but was poor at correctly assessing the choice of voters near the center of the electoral distribution. Such voters would be expected to exhibit a high probability of voting for the Liberals. It is probable, of course, that "centrist" voters strategically chose one of the two large parties. However, the wide confidence intervals on the party-

specific constant terms suggest that modeling the strategic component of voter choice is extremely difficult.

Thirdly, the modeling exercise did not utilize party declarations or leader positions, but rather estimates of the average position of the elite in each party. It was obvious, however, that the two large parties could potentially have increased their “expected vote shares” by modifying their policy positions. Since there is no evidence that they did this, it seems implausible that the usual Downsian vote-maximization interpretation can be valid.

To further explore the nature of party choice in Britain, the British General Election Studies of the elections of 1992 and 1997 were utilized. The questions on the surveys covered topics such as taxes, nationalization, redistribution, the European Community, women’s rights and Scottish Nationalism (for Scottish voters). Moreover, the questionnaire asked voters to (subjectively) locate the three major parties (Labour, Conservative and Liberal Democrat). Factor analysis gave scores to each of the questions, so that a policy space could be constructed. Unlike the analysis for 1979, a single dimension was constructed. A key aspect of this policy dimension was the extent of redistributive policies.

Figure 6 presents an estimate of the distribution of the sample of voters (excluding those from Scotland) in this policy space, together with the perceived positions of the three main parties, in 1992 and 1997.

It is, of course, difficult to directly compare the figures for 1992 and 1997, since the composition of the policy space may have changed slightly. Nonetheless, the figures suggest that between 1992 and 1997 Labour was perceived to have moved nearer the center of the distribution, while the Conservative Party moved further away.

However, again there is no evidence whatsoever that the two main parties “converged” towards an electoral center. Clearly leadership choice in Britain involved an element of risk-taking.

The empirical analyses discussed in this section give some hints about the nature of political choice in electoral systems based on proportional and plurality systems. One point should perhaps be made on the differences between plurality electoral systems, such as Britain’s, and those based on proportional representation. The game described by Equation (3.2) in the previous section is intended to be relevant to both types of political systems. However, the nature of electoral risk is different under plurality and proportionality. Relatively small changes in electoral support under plurality can dramatically change the probability functions, so in some sense political choice under plurality is riskier than under proportionality. Although it has not been formally derived in any way here, it seems likely that there are circumstances under which the political elite in a plurality system will be more willing to take political risk than the elite in a political system based on proportional representation. The next section will offer a number of speculations about the degree to which such differences in electoral institutions may lead to different political and economic choices.

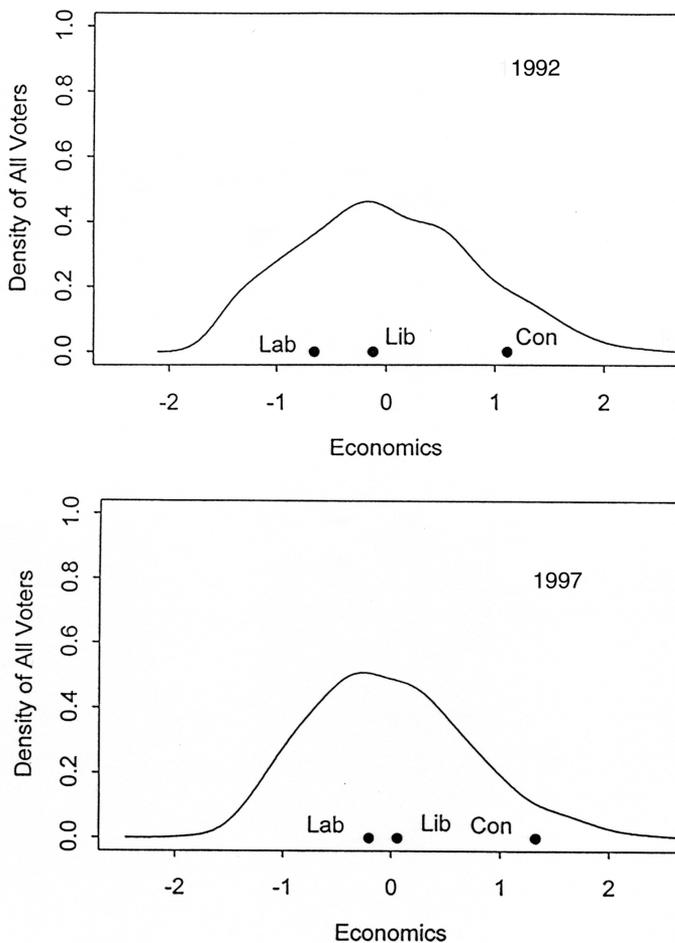


Fig. 6. Estimated density function of voter ideal points and the party positions of Labour (Lab), Liberal Democrats (Lib) and Conservatives (Con) in a one-dimensional policy space, for 1992 and 1997.

5. Electoral risk-taking and economic or political quandaries

As noted in the Introduction, it is well understood that plurality electoral systems tend to give rise to two-party systems. Indeed, the use of a plurality electoral system would seem to reduce the degree of fragmentation of political support [Rae (1967)]. For example, if we use the “effective number” (the inverse of the Herfindahl index of concentration) as a measure of fragmentation, then the effective number based on vote shares in Britain was 2.8 in 1979 and 3.12 in 1997. Using seat shares, the effective numbers were 2.15 in 1979 and 2.12 in 1997. (See Tables 1 and 3). In contrast, the

effective number (in both vote and seat shares) in Israel increased from 4.62 in 1988 to 9.61 in 1999.

There is a “correlation” between effective number of seat shares and government duration in European polities based on proportional representation. The correlation is quite weak however. In Italy, for example, the average effective number was 3.5 (for the period 1945 to 1987) while government duration averaged 13 months. Nonetheless the Christian Democrat party was in every government. In the Netherlands, with an average effective number of 4.5, average duration was 27 months [Schofield (1993)]. With plurality rule, and single-party majorities, Britain has had relatively long-lived governments. Duverger (1954) has argued that it is not the fragmentation of polities based on proportional representation that is of concern. Rather, under “proportionality”, governments must necessarily be based on coalitions, and such coalitions require compromise. In contrast, under plurality rule voters are given a clear choice. The resulting majority party can thus obtain a mandate to pursue its declared economic or political objectives.

In the 1970s it seemed entirely possible that electoral systems based on plurality rule were subject to the factional turbulence feared by Madison. Many authors observed that attempting to adjust inflation–unemployment levels to electorally advantageous values could induce inflationary expectations. In particular, since a plurality electoral system magnifies small swings in voter preferences, governments in Britain and the USA could be held hostage by small, but powerful groups [Beer (1982)]. In contrast, under proportional electoral rule, it was suggested that different factions in the polity would be represented by different parties and that these could bargain more effectively with each other so as to attain economically efficient outcomes [Crouch (1985)].

A second line of argument, based perhaps more on economic reasoning, suggested that both Britain and the USA were undergoing relative economic decline vis à vis the economies of western Europe and Japan. Figure 7 presents estimates of GDP per capita (in constant 1985 dollars) of six OECD countries. It is obvious enough that the gap of approximately 2000 dollars (in 1985 values) between Britain and Germany as of 1950 has disappeared, and indeed been reversed. Moreover, the gap of over \$5000 between the USA and Germany (as of 1950) has decreased to about \$3200 (in 1985 values). However, Figure 7 suggests that GDP/capita in the West-European countries is flattening out, or reaching an asymptote, at a lower level than in the USA.

Indeed, relatively high levels of unemployment in Germany, Italy and France have persisted for some time. (As of February 2002 these levels were approximately 9.6%, 9.3% and 9.0%, respectively). In Japan a low average level of unemployment of 2.5% (for the 1980s) has increased to 5.6%. In the USA and Britain unemployment is currently 5.6% and 5.2% (respectively), much lower than the average levels of 7.1% and 9.5% for the 1980s [see Garrett (1998), and the discussion in Schofield (2000)]. It would seem plausible that the nature of the political system in these various countries can induce macro-economic effects of this order of magnitude. I shall offer

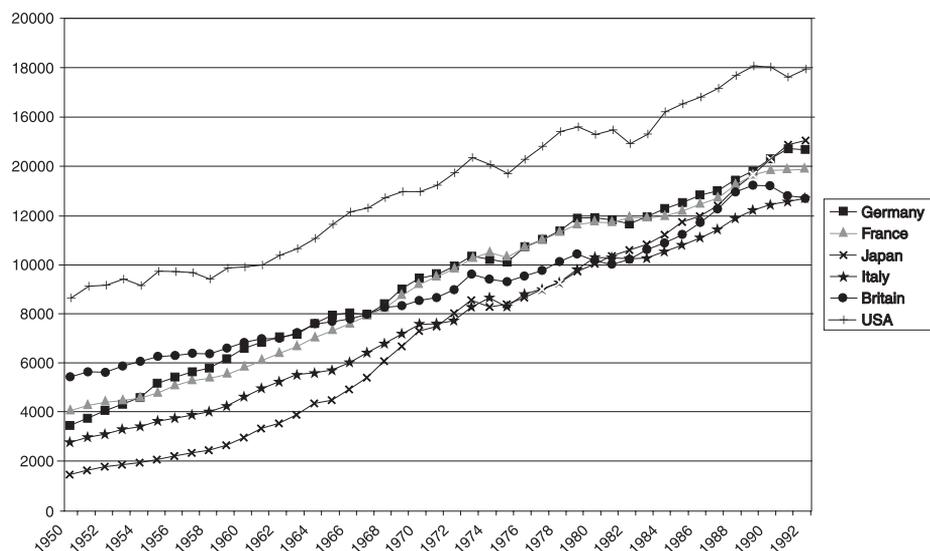


Fig. 7. Estimates of GDP/capita (in 1985 dollars) for six OECD countries, 1950–1992. (Figure drawn by Tsvetan Tsvetkov.)

a number of speculative hypotheses, that derive from Duverger's discussion, in an attempt to account for these differences.

Firstly, consider polities based on proportional representation.

In a country such as Israel, with a low electoral threshold, it is relatively easy for new parties to form and enter the political arena. As this occurs and political fragmentation increases, it may become increasingly difficult to construct and maintain coalitions. Although an equilibrium model for party choice was proposed in Section 4, this does not necessarily imply political stability. In fact, the game form, denoted \tilde{g} , may become much more complicated as the number of possible coalition structures increases. The complexity of coalition bargaining suggests why Netanyahu was eventually defeated, and an election required in 1999.

In both Germany and France the electoral system is approximately proportional, although the effective thresholds are quite high. In a sense this imposes a bound on the level of political fragmentation. Nonetheless coalition governments are the norm. For example, in Germany after the last election, Gerhard Schröder of the Social Democrat Party has had to depend on the Greens for a majority. Similarly, Jospin, leader of the Socialists in France, depends on support from the far left. It seems plausible that the necessity for coalition restricts the ability of political leaders to implement policies that are deemed risky by significant portions of the electorate. Attempting to deal with high unemployment might necessitate restricting the intervention of the state, or dealing with budget deficits. Both Schröder and Jospin have found these strategies difficult to deploy.

Now consider the USA and Britain. Obviously their political institutions are different from each other. However, both presidential elections in the USA and parliamentary elections in Britain are based on plurality electoral systems. This chapter has argued that all elections are risky, in the sense that they involve significant variance. Moreover, the degree of risk (or variance) under plurality rule is, in a sense, much greater than under proportional rule. As we have seen, a party in Britain may gain an overwhelming Parliamentary majority with approximately 40% of the vote. Indeed, an election for president in the USA, or government in Britain, may in fact be won by offering what may indeed be a risky strategy. Both Britain in 1979 and the USA in 1980 appeared to be in an economic quandary, a situation of extreme uncertainty over appropriate policy to pursue in an attempt at reducing inflationary expectations. Reagan and Thatcher both offered relatively untested and risky policies that they believed would eventually lead to a significant transformation in economic behavior. In a sense, they invited the electorates of the two countries to act as juries, to perform a judgement on the relative merits of different kinds of policies. This does not mean to say that the Condorcet Jury Theorem necessarily applies to the elections of 1979 and 1980. However, it is clear that the judgements of the two electorates brought about what can be viewed as constitutional change in these two polities. Both economies had to undergo a period of high interest rates, and unemployment. Eventually their labor markets and their economic structures were transformed.

Clearly, presidential choices do not entirely determine US politics. Since political choice in Congress necessarily involves compromise, the committee-based model of decision-making would suggest that congressional choices will tend to generate a greater degree of risk-aversion than that displayed by US presidents. Efforts by Franklin Roosevelt, Harry Truman, John Kennedy, Lyndon Johnson and Ronald Reagan to address fundamental economic or political quandaries were often blocked or made more difficult by Congressional decisions. Part of the genius of the US Constitutional design may have been to balance presidential risk-taking with Congressional risk-aversion.

This remark is not, of course, intended as unqualified praise for the US system of representative government. The presidential election of 2000 obviously left much to be desired.

European political systems also display undesirable features. Corruption scandals have plagued France, Germany and Italy. The move towards European Union has also created possibilities for corruption and bureaucratic incompetence. On the one hand, qualified (or weighted) rule in the Council of Ministers, and a weak European Parliament suggests European-wide decision-making will be unambitious in any attempt at making their markets more competitive [Schofield (2000)]. On the other hand, there is continuing risk-preferring pressure to deepen the Union, and to extend it to the countries of Eastern Europe. How Europe will respond to these constitutional quandaries is unclear.

6. Concluding remarks

Societies often face constitutional quandaries: should a Declaration of Independence be made? should slavery be resisted? should Hitler be opposed? should states federate? It would seem that the beliefs of the members of the society are called into question in such situations. Both Condorcet and James Madison raised the question about how a “fit choice” could be made by the representatives of the society. It is this question that this chapter pursues, in an attempt to see whether methods of social choice theory can contribute to our understanding of representative democracy.

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