Modeling the interaction of parties, activists and voters: Why is the political center so empty?

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Abstract. The formal stochastic model of voting should be the theoretical benchmark against which empirical models can be gauged. A standard result in the formal model is the 'mean voter theorem' stating that parties converge to the electoral center. Empirical analysis based on the vote-maximizing premise, however, invalidates this convergence result. We consider both empirical and formal models that incorporate exogeneous valence terms for the parties. Valence can be regarded as an electorally perceived attribute of each party leader that is independent of the policy position of the party. We show that the mean voter theorem is valid for empirical multinomial logit and probit models of a number of elections in the Netherlands and Britain. To account for the non-centrist policy positions of parties, we consider a more general formal model where valence is also affected by the behavior of party activists. The results suggest that non-convergent policy choice by party leaders can be understood as rational, vote-maximizing calculation by leaders in response to electoral and activist motivations.

Introduction

How do political leaders choose policies to present to the electorate? Is the political center necessarily empty, as Duverger (1954: 215) argued? On these questions, theoretical models and empirical analysis seem to contradict one another. Models of two-party competition (Calvert 1985; Banks et al. 2002; Banks & Duggan 2004) suggest that parties will converge to an electoral center in order to maximize their plurality over the opposition. For multiparty competition, the 'mean voter theorem' (Hinich 1977; Lin et al. 1999) also suggests that vote-maximizing parties will adopt the mean electoral position.

In multiparty polities using proportional electoral rules, coalition governments typically require the cooperation of several parties. Under the assumption that party positions and strengths are given, models of coalition bargaining indicate that a large, centrally located party, at a position known as the 'core', will be predominant. Such a core party can, if it chooses, form a minority government by itself and control policy outcomes (Schofield et al. 1989; Schofield 1993, 1995, 1997; Sened 1995, 1996; Laver & Schofield 1998;

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Banks & Duggan 2000). If party leaders are aware of the fact that they can control policy from the core, then this 'centripetal' pressure should lead parties to position themselves at the center.

Yet, contrary to this intuition, there is ample empirical evidence, for electoral systems based on both proportional representation and plurality rule (or 'first past the post'), that party leaders or political contenders do not necessarily adopt centrist positions. Budge et al. (1987) and Laver and Budge (1992), in their study of European party manifestos, found no evidence of a strong centripetal tendency. More recent electoral models for Israel (Schofield, Sened & Nixon 1998), Germany and the Netherlands (Schofield et al. 1998; Quinn et al. 1999; Quinn & Martin 2002) and Norway (Adams & Merrill 1999) have estimated party positions in various ways, and concluded that there is no indication of policy convergence by parties. For the United States, earlier empirical evidence for non-convergence (Poole & Rosenthal 1984) has been substantiated by more recent studies (Miller & Schofield 2003; Schofield et al. 2003).

The conflict between theory and evidence suggests that the models be modified to provide a better explanation of party policy choice. This can be done either by changing the model of voter choice (e.g., Adams 1999, 2001; Adams & Merrill 1999; Merrill & Grofman 1999) or by considering more complex versions of the rational calculations of politicians. The purpose of this article is to determine the degree to which there is indeed a centripetal or centrifugal tendency in multiparty polities. As far as electoral models are concerned, we show that, for reasons of empirical predictive power, it is necessary to add to each voter's comparative evaluation of the parties a term we call 'valence'. In the simplest model we consider, a party's valence is a measure of the average evaluation of the party leader by the voters that cannot be accounted for in terms of policy differences. The notion of valence was introduced many years ago (Stokes 1963), but has only recently been introduced into formal models. So far, however, the formal results have been limited to two-party competition (Ansolabehere & Snyder 2000; Groseclose 2001).

We discuss empirical analyses of the Netherlands and Britain to show why these valence terms are required to improve the predictive power of the electoral model. We then cite a result that we call the 'first electoral theorem' (Schofield 2004a, 2004b, 2005a) for an underlying formal stochastic model. This result gives necessary and sufficient conditions, when valence terms are included, under which the electoral mean is a vote-maximizing position for all parties. When the necessary condition fails, then the electoral mean cannot satisfy the conditions for a 'local optimum', and therefore cannot satisfy the condition for a Nash equilibrium. More generally, the theorem also implies that non-centrist equilibrium positions can occur. Our empirical results then allow us to determine whether convergence should be expected on theoretical grounds for the various electoral competitions we consider.

We first examine multinomial logit and probit models of elections in the Netherlands in 1977 and 1981, and in Britain in 1979 (Quinn et al. 1999). Simulation of these models showed that all parties could have increased vote share by moving to the electoral center (Schofield et al. 1998). We show that the estimated coefficients of these models imply that the sufficient condition is satisfied. In such a case, the electoral mean is the only possible vote-maximizing equilibrium. Since there is no indication that the parties in these elections did indeed cluster at the electoral center, we infer that the simple vote-maximizing model, even with valence, is inadequate to explain party positioning.

We then constructed a multinomial logit model for the election of 1997 in Britain based on a single economic dimension and average voter perceptions of party locations. Again, the estimated coefficients implied that the vote-maximizing positions of all parties were at the electoral center – although the electorally perceived position of the Liberal Democrats was centrist, the Conservative Party was perceived to be very far from the center (see also Alvarez et al. 2000). To account for this discrepancy between prediction and the empirical estimation, we extended the model to include a second dimension based on attitudes to European Union. The formal model was unable to account for the estimated extreme position of the Conservative Party on both axes. We then considered a more general valence model based on activist support for the parties (Aldrich & McGinnis 1989). This activist valence model (Schofield 2003) presupposes that party activists donate time and other resources to their party. These resources allow the party to present itself more effectively to the electorate, thus increasing its valence. We show formally in the Appendix to this article that choosing an optimal position for the party requires the party leader to balance the more ‘radical’ preferences of activists against the attraction of the electoral center. We suggest that the valence of the Labour Party under Tony Blair increased in the period up to 1997, thus changing the importance of activists for the party. As a consequence of the relative decline of the Conservative Party leader’s valence, activist support for the party became more important, forcing it to adopt a more radical position, particularly with regard to the European issue. The conclusion argues that the two electoral theorems, based on the model of exogeneous and activist valences, have the potential to explain the great variation in political configurations observed in representative democracies.

The spatial model with vote-maximizing politicians

Since empirical analyses of voting must deal with the uncertainty of estimation, it is natural to assume that voter choice is not completely deterministic, but incorporates stochastic errors. Depending on the estimation procedures used, various assumptions can be made about the distribution of these errors. It is also useful to have a formal model of voting that can be used as a benchmark to the empirical work. For reasons of tractability, an attractive formal model is one that assumes that the errors are normally distributed. The stochastic spatial, formal model of voting is based on this assumption. It assumes that a voter chooses one of a number of parties with a probability that is derived from the difference between the voter's most preferred point and the declared positions of the parties. This stochastic model was first applied to two-party competition as part of an extensive research program (Riker & Ordeshook 1973). One result of this formal model was that the mean voter position was an 'attractor' for vote-maximizing parties (Hinich 1977). We shall refer to this result as the 'mean voter theorem'. Poole and Rosenthal (1984) observed that this formal result seems to conflict with empirical evidence from American elections (cf. Enelow & Hinich 1989). Recent empirical studies of the United States, Britain and a number of polities with proportional electoral systems suggests that there is no strong tendency for parties to approach the electoral mean. In spite of this evidence, Lin et al. (1999) derived a formal stochastic model suggesting that, in fact, the convergent point at the mean of the voter distribution would be a pure strategy Nash equilibrium (PSNE) for vote-maximizing parties as long as the variance of the errors was sufficiently large.

Whether or not this 'mean voter theorem' is valid in various electoral systems is an important theoretical issue. If the theorem is valid and parties do not converge, then it is necessary to modify either the vote-maximizing assumption by parties or the model of voter choice. The latter strategy is the one adopted by Adams and Merrill (1999) who propose a model of 'directional voting'. However, empirical analyses based on the stochastic voter model with the usual spatial assumptions do give statistically significant log likelihoods. With regard to the vote-maximizing assumption, while there are many possible reasons why parties may adopt pre-election policy positions that are not intended to maximize their vote shares, parties seem intent on successfully contesting elections.

This article is based on the premise that the fundamental motivation of each party is to do as well as it can at election time, subject to the constraint imposed on the party by the party elite and activists. Under this premise, we examine the validity of the mean voter theorem. We argue that, for a general version of the model incorporating what we call 'valence', there is no general reason to expect existence of a PSNE where all parties adopt the same position at the electoral mean. Since the valence terms are statistically significant in empirical analyses, the first electoral theorem presented here gives the necessary and sufficient conditions under which vote-maximizing parties will adopt convergent positions.

Our theoretical analysis leads us to distinguish between global and local equilibria. In a *global equilibrium*, no agent may gainfully and feasibly deviate. A *local Nash equilibrium* (LNE) is an equilibrium only with respect to a neighborhood of the point (or strategy). Since a PSNE must be a global equilibrium with respect to any feasible strategy change by any party, it must be a local equilibrium as well. Thus, the conditions under which a LNE exists are weaker than for existence of a PSNE. Indeed, a LNE may exist when there is no PSNE. The local equilibrium concept has two theoretical advantages. First, in games of the kind considered here, the payoff functions of the protagonists are smooth. This feature has been used to show that LNE exist for almost all possible parameters (Schofield & Sened 2002). Second, LNE can generally be readily computed by simulation techniques (Schofield & Parks 2000; Schofield & Sened 2005).

One feature of Lin et al.'s (1999) formal model is that it does not include valence terms. In empirical analysis, valence terms associated with each party are crucial for the validity of the electoral model. Such valence terms can be an exogenous feature of the election characterizing each party by an average electoral evaluation of the competence (or innate attractiveness) of the party leader. If we use j to denote a party, and let $P = \{1, \ldots, j, \ldots, p\}$ be the set of parties, then the *exogenous* valence of party j is denoted λ_j. In the formal stochastic electoral model with valence, whether the convergent point will be an equilibrium depends on a 'domain constraint' defined in terms of the variance of the distribution of voter ideal points. Because the errors of the formal model are assumed to be normally distributed, 'concavity' of the vote share functions depends on concavity of the cumulative normal distribution. However, this cumulative distribution function has a point of inflection at a point sufficiently distant from the origin so that below this point, concavity fails. Consequently, it can be shown that necessary and sufficient conditions for equilibrium can be expressed as a domain constraint involving the standard deviation of the errors and the spatial parameter, β. As Lin et al. (1999) argued, if the standard deviation is 'sufficiently' large, this constraint is generally satisfied so that concavity, and thus existence of a convergent PSNE, may be possible. When the constraint fails, however, then the convergent equilibrium cannot be guaranteed.

When valence is involved, the conditions for existence of a centrist equilibrium also depend on the magnitude of the differences in the terms $\{\lambda_j\}$. We present necessary and sufficient conditions for a local Nash equilibrium where all parties adopt the mean position. These conditions are given in terms of a 'convergence coefficient' that is defined by the valence differences, the spatial parameter, and the stochastic and electoral variances. When the necessary condition fails, then a local equilibrium at the mean is impossible. It follows that a PSNE at the mean is also impossible. In general, when exogeneous valence terms $\{\lambda_j\}$ differ sufficiently among parties, the theoretical analysis shows that there can exist multiple non-convergent LNE. Unlike the presumed PSNE at the origin, these local equilibria will be 'strategically interdependent': each party's equilibrium position will be highly dependent on the positions of other parties. The formal model implies that vote-maximizing positions of high-valence parties will be symmetrically located about the electoral mean, but the vote-maximizing positions of low-valence parties will be much more radical at some considerable distance from the electoral mean. Moreover, all parties will be located on a principal policy axis displaying the greatest electoral variance.

Simulations of an empirical model for Israel (Schofield, Sened & Nixon 1998; Schofield & Sened 2005) suggest that this phenomenon is particularly pronounced when the electoral system is based on proportional representation and the political configuration is highly fragmented. This seemingly counter-intuitive feature is a fundamental property of the formal electoral model. We interpret it to imply that many equilibrium configurations are possible, and that simple convergence to an electoral center is unlikely. In the case of the Netherlands and Britain, the contradiction between the observed positions of the parties and the estimated vote-maximizing positions leads us to consider an extension of the model to include activists.

In their analysis of Israel, Schofield and Sened (2005) found that the valences, although regarded as constant at each election, did vary considerably *across* elections. They inferred that the *exogeneous* valence terms correspond to average perceived party competence, and these terms change from one election to another in response to the changing political environment. Later in this article we propose a model where valence is not simply exogeneously determined, but is a function of the party's policy choice. We conjecture that this is an indirect effect due to the contributions of activists. By representing a coalition of activists, the party obtains resources. These contributions allow it to advertise its effectiveness and thus gain electoral support (Aldrich & McGinnis 1989). Since the members of an activist coalition will tend to be more radical than the average voter, all parties are faced with a complicated electoral calculus. By accommodating the political demands of its activists, the party gains resources that it can use to enhance its valence. However, by adopting radical policies in support of its activists, it loses electoral support due to the effect of more radical policies.

Instead of assuming that valence is fixed at the time of the election, it is possible to construct a more general model where the valence of a party is affected by activist support and thus, indirectly, by the policy position that it adopts at the election (Schofield 2003). This more general framework is developed in Theorem 2, which asserts that the party leader must balance the purely electoral effect, determined by the party position and the voter distribution, against the activist valence effect.

One crucial difference emerges when valence is interpreted in this more general fashion. When valence is affected by activist support, it will depend on activist contributions of time and money. The effect of these contributions on the electoral support of the party is bound to exhibit 'decreasing returns to scale', so the party's valence function will be a concave function of its position. The Appendix shows that when concavity of this activist valence function is sufficiently pronounced for all parties, a non-centrist PSNE will exist (Schofield 2004a). To determine the precise nature of such a PSNE is difficult since the model requires data not just on voter preferred positions, but also on the motivations of party activists. We contend that extending the standard spatial model in this fashion could provide an understanding of the rich diversity of political configurations that are possible under representative democracy.

In the next section, we present the empirical stochastic voting model and illustrate it first with an application to electoral politics in the Netherlands. The empirical results indicate the relevance of what we term 'exogeneous valence'. We cite the electoral theorem to argue that convergence to the electoral mean should be expected. Instead, the actual location of parties is clearly non-centrist. We then introduce the hypothesis that the selection of a particular local equilibrium is due to the interaction of the party activists, policy preferences of the electorate and the vote-maximizing motivations of party leaders. This more complex model is discussed in the section that follows the next one in the context of empirical analyses of the British general elections of 1992 and 1997.

The empirical electoral model with valence

In the 'stochastic' model of voting, each voter is presented with a choice between p parties. Given the pre-election policy declarations of the parties described by the vector $\mathrm{z} = (z_1, \ldots, z_p)$, voter i chooses party $j \in P$ with some probability ρ_{ij}, that is a function of z. In this article we focus on the spatial model of voting – that is, the declaration of each party j is identified with a policy point, z_j, in a policy space W, of some dimension, w. Each voter is identified with an 'ideal' policy point x_i in W, together with a vector of individual characteristics, I_i. The variate X_i describes i's choice (e.g., if voter i actually chooses j, then $X_{ij} = 1$; otherwise, $X_{ij} = 0$). The probability that $X_{ij} = 1$ is denoted ρ_{ij} (z). We write this as ρ_{ij} for convenience. By definition $\Sigma_{j\in P}\,\rho_{ij} = 1$. Since X_{ij} is a binary variable, the expectation $E(X_{ij})$ is ρ_{ij}. Thus, V_j, the vote share of party j, can be estimated by taking the average $av_j(\rho_{ij})$ of ρ_{ij} across the population. We estimate ρ_{ij} for i in a N of size n and obtain V_j by some estimation procedure on the sample. Clearly, V_j is a random variable with expectation $E(V_j)$ that can thus be estimated from the sample by taking $av_j(\rho_{ij})$. The empirical variance of V_j can also be estimated (Schofield et al. 1998). It is possible to derive formal models where the electoral risk (or variance in π_j) is relevant (Schofield 2002); here we shall simply focus on the expectations $\{E(V_j)\}$: we refer to these expectations as 'vote shares'.

Since the empirical model is contrasted with a formal model in order to establish conditions for existence of Nash equilibria, it is important to ensure continuity of the 'payoff functions' of the parties. We therefore require that the voter response, ρ, be a continuous function of the ideal points and party positions. Write $\rho = \rho(x;z) = \rho(x;z_1, \ldots, z_p)$ for the $(n \times p)$ matrix (ρ_{ij}), Here x is the vector of ideal points derived from sample of size n. Spatial stochastic voting models generally assume that ρ is derived from the $(n \times p)$ matrix of distances $\delta(x;z) = (\delta_{ij}) = \delta(x - z_1, \ldots, x - z_p)$. Here δ is some appropriate metric, usually the 'quadratic' or Euclidean metric, on W.

If we let $\in\, = (\in_1, \ldots, \in_p)$ be the vector of errors with a cumulative multivariate distribution function, Φ, then ρ_{ij}, the probability that i chooses j, can be determined by

$$\text{Prob}(\Phi)\left(\in_j - \beta\delta_{ij}^{\ 2} > \in_k - \beta\delta_{ik}^{\ 2} \quad \text{for all } k \neq j\right) \tag{1}$$

Here β is the positive spatial coefficient and Prob(Φ) is the appropriate probability determined by Φ. The derivation of this term from assumptions made on voter utility is found in the Appendix.

In multinomial logit (MNL) models based on independent and identically distributed (iid) errors the distribution function is log-Weibull (essentially a normal distribution with truncated tails). The ratio ρ_{ij}/ρ_{ik} (for parties j, k) is $[exp(-\beta\delta_{ij}^2]/[exp(-\beta\delta_{ik}^2)]$, which is independent of other choices (Quinn & Martin 2002; Adams 2001). For reasons explained in Alvarez and Nagler (1998) and Alvarez et al. (2000), models of this kind must satisfy the 'Independence of Irrelevant Alternatives' property (IIA), which may be overly restrictive for analyzing multiple-choice problems. Most formal models with 'stochastic' voters assume that the errors are independently and identically normally distributed (iind). This implies that Φ is normal, with a diagonal covariance matrix (Coughlin 1992; Enelow & Hinich 1984; Hinich 1977). While these assumptions may be overly restrictive, they nonetheless allow for formal analysis that can be used as a benchmark against which to evaluate empirical analyses. We shall obtain results both for iind errors and for the more general case where the errors are multivariate normal.

Under some conditions we can empirically estimate ρ by a multinomial probit (MNP) model. Such a model does not require the assumption of independent errors; instead the error covariance matrix will have non-zero off diagonal terms. The probability matrix $\rho(x;z) = (\rho_{ij})$ is determined by a $(p-1)$ dimensional vector of error differences

$e^j = (\in_1 - \in_j, \ldots, \in_{j-1} - \in_j, \in_{j+1} - \in_j, \ldots, \in_p - \in_j)$ characterized by a cumulative distribution function (Φ) and probability density function, h. Let

$$\Delta i^j = \left(\beta\delta_{i1}{}^2 - \beta\delta_{ij}{}^2, \ldots, \beta\delta_{ip}{}^2 - \beta\delta_{ij}{}^2\right) \tag{2}$$

Assuming $\rho_{ij} = \text{Prob}(\Phi)(\in_j - \beta\delta_{ij}{}^2 > \in_k - \beta\delta_{ik}{}^2$ for all $k \neq j$), we find $\rho_{ij} = \int h(e^j)de^j$ with bounds from $-\infty$ to Δi^j; in other words, the probability that voter i picks party j is a function of the differences between the ideal point of the voter i and the positions of the various parties.

An important inference from the point of view of this article is that the explanatory power of the empirical model is increased by adding valence terms (Stokes 1963, 1992). In this modified model, the exogeneous valence for party j can be treated as a stochastic variable with expectation λ_j. This expectation is the additional 'utility' allocated to a typical voter because of a non-policy attribute of party j. The error term $\in_j$ for party j describes the variation in this term within the electoral sample. This changes the bounds of the estimation problem for ρ_{ij} since these valence terms change Equation 2 to Equation 3, so that

$$\Delta i^j = \left(\beta\delta_{i1}{}^2 - \beta\delta_{ij}{}^2 - \lambda_1 + \lambda_j, \ldots\right) \tag{3}$$

Notice that as the valence of party j increases, the probability that a voter chooses party j over party 1, say, also increases. Also note that it is not the absolute values of the valences that are relevant, but the pair-wise differences in the valences. The computation problem is to estimate h, and the matrix $\rho(x;z)$ without assuming independence of errors. This involves estimating the variance/covariance matrices of the $(p-1)$ dimensional error difference vectors: $e^1, e^2, e^3, \ldots, e^p$. It suffices to estimate a single variance/covariance matrix $\Theta = (\theta_{jk})$ for the $(p-1)$ dimensional error difference vector

$e^p = (\in_1 - \in_p, \ldots, \in_j - \in_p, \ldots, \in_{p-1} - \in_p)$.

Recent developments in Bayesian estimation (Chib & Greenberg 1996) permit an estimation of the multinomial probit (MNP) model using Markov chain Monte Carlo techniques (details of this technique can be found in Quinn et al. (1999); applications to the study of Dutch and German electoral behavior can be found in Schofield et al. (1998) and Quinn & Martin (2002)). All empirical models depend on estimation of the $(p - 1)$ by $(p - 1)$ variance/covariance matrix $\Theta = (\theta_{jk})$. As we show in the next section, the conclusion of the formal model depends on Θ as well as the other parameters of the model.

To estimate ρ, we need a sample of $\{x_i, I_i\}$, a choice vector z and actual voter choices. This allows us to estimate $\rho(x;z)$ from the data. Exploratory factor analysis can be performed on the voter response profile to estimate the nature of the underlying policy space, W. For example, in the Netherlands, two dimensions are significant: the usual left-right economic dimension and a second concerned with scope of government (see Quinn et al. (1999) and Schofield et al. (1998) for estimation details based on survey data obtained by Rabier and Inglehart (1981)). The response of individual i to the survey allows us to locate the individual's ideal point in the country-specific policy space. For each party j, we let $\{x_t^j : t \in C_j\}$ represent the ideal points of the members of the set, C_j, the elite of party j. For the Dutch example, we used the data on party delegate preferences from ISEIUM (1983) to estimate elite preferences. Since W is two-dimensional, we chose the position $z_j \in W$, for party j, by taking the two-dimensional median of the delegate positions to represent the 'sincere' ideal point of party j. We call a 'representative' delegate of party j whose ideal point is z_j, the *principal* of party j.

Figure 1 gives the sincere or principals' policy positions of four parties in the Netherlands: Labor (PvdA), Christian Democratic Appeal (CDA), Liberals (VVD) and Democrats '66 (D'66). These positions are almost identical to those independently obtained by De Vries (1999) on the basis of analysis of party manifestos. Figure 1 also presents an estimate of the distribution of voter ideal points based on factor analysis of the Rabier-Inglehart survey data for 1979. Factor analysis of these data for many European countries also showed that the underlying policy space is typically two-dimensional. Here, and in the figures that follow, the origin is the mean of this electoral distribution. The ISEIUM elite survey was based on similar questions to the Rabier-Inglehart survey and so the estimated voter ideal points and party principal points were comparable. We constructed MNL models with and without sociodemographic variables (SD), and with and without the valences terms, and compared these to the MNP models discussed in Schofield et al. (1998) and Quinn et al. (1999).

Table 1 reports sample vote shares and national vote shares in the elections of 1977 and 1981, and estimated vote shares from the MNL models with

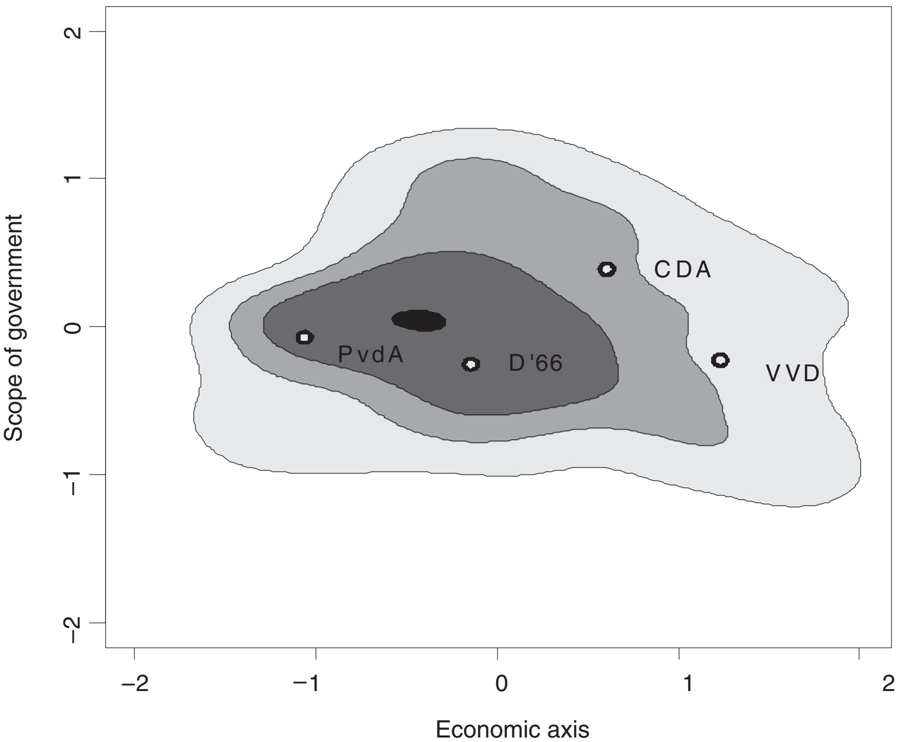


Figure 1. Party positions in the Netherlands based on survey data for 1979 (estimated by Schofield et al. 1998) showing highest density plots of the voter sample distribution at the 95, 75, 50 and 10 per cent levels.

and without exogeneous valence. The vote share of each party is given as a percentage of the total vote for these four parties. For example, the vote share of the PvdA declined from 38 per cent in 1977 to 32.4 per cent in 1981. Its sample share was 36.9 per cent in 1979. Its estimated expectation (without the valence terms) was 35.3 per cent with a 95 per cent confidence interval of (30.9, 39.7). With the vector of sincere party positions, z, fixed, and the voter ideal points, x, given, we estimated the voter probabilistic response ρ(x;z). In principle, this allows us to determine how voter response varies with z for the different electoral models we consider.

Table 2 lists the log likelihoods associated with the eight different models that were constructed. The sociodemographic (SD) models estimate the probability matrix as a function ρ(x,*I*) simply in terms of the vector *I* of individual characteristics (e.g., status as a manual worker would be expected to increase the probability of voting for the PvdA.). The models with both valence and SD estimate the matrix as a function ρ(x,*I*,z). The difference in log likelihoods

Table 1. Elections in the Netherlands, 1977–1981

Party	National vote share (%)		Sample vote share (%)	Estimated vote share (%)		λ	95% confidence interval
	1977	1981	1979	($\lambda = 0$)	($\lambda \neq 0$)		
D'66	6.1	12.6	10.4	10.6	10.4	0.000	–
PvdA	38.0	32.4	36.9	35.3	36.9	1.596	[1.28, 1.90]
CDA	35.9	35.2	33.8	29.9	33.8	1.403	[1.09, 1.71]
VVD	20.0	19.8	18.9	24.2	18.9	1.015	[0.65, 1.37]
MNL spatial coefficient ($\lambda \neq 0$)				$\beta = 0.737$			[0.62, 0.85]
MNL spatial coefficient ($\lambda = 0$)				$\beta = 0.678$			[0.57, 0.78]

Table 2. Eigenvalues for D'66 and CDA and the log likelihoods for the Dutch model

Model	Eigenvalues	Coefficient	Log likelihood
MNL without valence or sociodemographic variables (SD)	n.a.	0.00	–606
MNL without valence, with SD	n.a.	0.00	–565
MNL with valence, without SD, 1st axis	–0.40	0.85	–531
For D'66, 2nd axis	–0.75		
MNL with valence and SD, 1st axis	–0.36	0.92	–464
For CDA, 2nd axis	–0.72		
MNP without valence or SD	n.a.	0.00	–626
MNP without valence, with SD	n.a.	0.00	–596
MNP with valence, without SD, 1st axis	–0.74	0.37	–545
For D'66, 2nd axis	0.88		
MNP with valence, with SD, 1st axis	–0.30	1.00	–427
For CDA, 2nd axis	–0.70		

of the various models gives the Bayes Factors (Kass & Raftery 1995), which can be computed from Table 2.

Adding SD to the simplest MNL model gives a Bayes' Factor of 41, which suggests that individual sociodemographic characteristics have some influence on voting propensity. The natural inference is that the causal model is of the form: $I_i \rightarrow x_i \rightarrow \rho_i \rightarrow X_i$; that is, individual characteristics, I_i, influence beliefs, represented by the ideal point, x_i. This, in turn, determines the probability vector, ρ_i, used to estimate the voter's political choice.

In the model with exogeneous valence, only the differences in valence can be estimated. We therefore set the valence of one party to zero. Thus, setting $\lambda_{D66} = 0$ (as in Table 1), the estimation for the MNL spatial model without SD obtained $\lambda_{PvdA} = 1.596$, $\lambda_{VVD} = 1.015$ and $\lambda_{CDA} = 1.403$ (see Table 1 for the 95 per cent confidence intervals). Notice, however, that the estimation of the valence terms and spatial coefficient depends on the 'scale' of the model, and this scaling depends on two 'normalizations'. The first is a stochastic normalization. In the MNP models, this normalization is determined by setting the first variance term (θ_{11}) of the error difference vector

$e^p = (\in_1 - \in_p, \ldots, \in_j - \in_p, \ldots, \in_{p-1} - \in_p)$ in the covariance matrix Θ equal to 1. In MNL models, the error distribution is log Weibull with variance $\pi^2/6$. The second electoral variance normalization fits the factor model to the voter ideal points.

With the exogeneous valence terms added to the spatial MNL model without SD, the estimated vote shares, as shown in Table 1, were much improved and identical to the sample vote shares. Approximately 45 per cent of the voter choices were correctly predicted, and the log marginal likelihood increased from –606 to –531, giving a Bayes' Factor of 75. Adding valence to the MNL model with SD has a Bayes' Factor of 101, and this model accurately predicts over 55 per cent of voter choice. This, together with the 95 per cent confidence intervals for the valences, suggest that the valence terms significantly differ between the parties. Note also that an even superior model is the joint MNP model with both valence and SD. This model accurately predicts 56 per cent of the voter choice. However, the difference between the two joint models is trivial and the MNL model has the advantage of simplicity.

We can now compare the results of the *empirical* analysis with the conclusions of analysis of the benchmark *formal* model to determine whether there is cause to believe that vote maximizing parties in the Dutch elections would have converged to an electoral mean. The conclusions depend on the eigenvalues and coefficients given in Table 2, and these are explained below.

The formal electoral model with valence

The formal model typically assumes that parties are 'Downsian' expected vote-maximizers (Downs 1957). In this case, the individual probability functions can be obtained from an equation analogous to Equation 1, but derived from the assumption that the errors are multivariate normal. In precisely the same fashion as before, each party's expected vote share function can be obtained by taking the average of the probability functions. If we write the random vector of vote shares as $V(z) = (V_1(z), \ldots, V_p(z))$, the Downsian assumption is that each party, j, chooses z_j to maximize the expectation $E(V_j(z))$, given the $(p-1)$ vector $z_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p)$ – that is, z_j is chosen as a 'weak best response' to the vector z_{-j}.

> *Definition 1.* A *pure strategy Nash equilibrium* (PSNE) is a vector $\mathbf{z}^* = (z_1^*, z_j^*, \ldots, z_p^*)$ such that
> $E(V_j(z_1^*, \ldots, z_j^*, \ldots, z_p^*)) \geq E(V_j(z_1^*, \ldots, z_j, \ldots, z_p^*))$ for any $z_j \in W$, for each $j \in P$.

Lin et al. (1999) show existence of a PSNE in such a formal model of stochastic voters with quadratic utility functions and Downsian parties. If the errors are iid with 'sufficiently large' variance, var($\in$), there exists a 'convergent' Nash equilibrium z* where $z_j^* = z^*$ for all j and $z^* = av_i(x_i)$, the mean of the voter ideal points. We termed this result the 'mean voter theorem' above. Yet, this Downsian 'convergence' does not appear to occur in *any* multiparty polity.

One way to examine the applicability of this formal result is to compare the results of an empirical analysis with the analogous formal model. Thus, given the estimation of the probability matrix $\rho = \rho(x;z)$ and expectation vector $E(V(x;z) = (\ldots, E(V_j(x;z)), \ldots)$ we can estimate the effect of changing the vector of party positions. We are led to the notion of an empirical PSNE, defined just as in *Definition 1*. The computation of the parameters of the empirical model then permit the construction of what we shall term the 'benchmark formal model'. The characteristics of equilibria in the empirical and formal models need not be identical. Nonetheless, the features of the formal model will give insight into the structure of the empirical model. By this method, we may draw some inferences about vote-maximizing behavior. A first step in this exercise is to examine the equilibrium properties of the benchmark formal model. The key theoretical idea underlying proof of existence of PSNE is a condition on the vote-share functions known as 'concavity'.

Definition 2. A real-valued function V is *concave* if
$V(ax + by) \geq aV(x) + b\pi(y)$, where a and b are any real numbers.

Proof of existence of the PSNE then follows from assuming or proving that the condition of concavity is satisfied by each vote-share function, $E(V_j)$, regarded as a function of z_j. A condition sufficient to guarantee that all vote-share functions are concave is that the probability functions $\{\rho_{ij}\}$ of all voters are everywhere concave. This is equivalent to the condition that these functions have Hessian, or second differential, which is *everywhere* negative semi-definite. Obviously, this is a very strong condition. Moreover, the probability functions in the formal model are derived from the cumulative normal distribution. Since the cumulative normal distribution fails concavity at a value of the variate far below zero, the probability functions may also fail concavity if the perceived differences between parties are sufficient in magnitude. While this comment is valid for the formal model involving normal errors, it also holds true for empirical models involving different distribution assumptions on the errors. For example, the MNP model of the Netherlands, discussed above, can be used to estimate the probability functions of any voter, for any given vector, z, of party positions. The illustration in Schofield et al. (1998) makes it clear that these individual probability functions fail concavity when the distance between the voter ideal point, x_i, and the party position is sufficiently large. Nonetheless, the simulation exercise on the model showed that the joint origin was an attractor for vote-maximizing parties.

To illustrate, consider the situation where high- and low-valence parties occupy the same position; any voter deciding between the two can only compare valences. If the valence difference is sufficiently high and the voter's ideal point is distant from both parties, then the voter response to the low-valence party is not concave in that party's position. If there are sufficient voters with distant ideal points, the low-valence party may be able to increase its vote share by vacating the electoral center. Indeed, simulation of vote-maximizing party behavior in the 1992 and 1996 elections in Israel, with many parties and wide variation in party valences, has shown that the electoral mean is a *vote-minimizing position for low-valence parties* (Schofield & Sened 2005). Vote-maximizing positions for such parties were then at the electoral periphery. In the case of the Netherlands, the valence differences are less pronounced than in Israel. We shall show that the estimated parameters of the empirical models just discussed imply that concavity of the vote-share functions is satisfied near the origin.

We now define the notion of a local equilibrium and present the necessary and sufficient conditions for existence at the mean. Informally, a local equilibrium is a vector z* such that for all j, $E(\mathrm{V}_j(\mathrm{z}^*))$ is maximized in a neighborhood W_j of each z_j*.

> *Definition 3*. A vector $\mathrm{z}^* = (z_1^*, \ldots, z_p^*)$ is a *local Nash equilibrium* (LNE) if, for each j, there exists a neighborhood W_j of z_j* in the policy space, W, such that z_j* is a best response in W_j with respect to the vote-share function $E(\mathrm{V}_j)$.

For the formal model, this local equilibrium property can be ensured by imposing the first-order conditions $\mathrm{d}E(\mathrm{V_j})/\mathrm{d}z_j = 0$, for all j, and then verifying the second-order Hessian condition at the vector z* of positions. As in the case of concavity, the second-order condition is that the Hessian be negative semi-definite. This Hessian condition is only required at the particular point and it is convenient to refer to it as 'local concavity'. This is a much weaker condition than 'global' concavity. Consequently, any PSNE must also be a LNE. Because the spatial component involving β is quadratic, this local condition gives a quadratic expression and thus necessary and sufficient conditions for local concavity. (The Appendix shows the computation of the eigenvalues of the Hessian) The formal analysis shows that though the first-order condition is always satisfied at the joint mean position, there are many possible non-centrist solutions. Consequently, when the local concavity condition fails at the origin, the formal model implies that, with exogeneous valence, there can exist many different non-convergent LNE.

To state the theorem, we first chose a system of orthogonal axes indexed by $t = 1, \ldots, w$. On each axis, let $v_t^2 = 1/n\Sigma_i(x_{it})^2$ be the variance of the voter ideal points about the origin on the t axis, and let $v^2 = \Sigma_t v_t^2$ be the *total voter* *variance*. The Appendix examines the conditions under which the eigenvalues of the Hessians are negative and shows that the election is classified by a dimensionless coefficient, denoted c (see also Schofield 2004a, 2004b, 2005a for technical details). First, we need to define the relevant stochastic variance measure used for both MNP and MNL models. Given the stochastic error difference covariance matrix $\Theta = (\theta_{jk})$, let τ^2 be the sum of all terms in the matrix Θ, and let $\kappa^2 = \tau^2/[p - 1]^2$. For example, in the case where the errors are iind, with variance σ^2, then the matrix Θ has diagonal entries $2\sigma^2$ and off diagonal entries σ^2. Thus $\tau^2 = p(p - 1)\sigma^2$ and $\kappa^2 = \sigma^2 p/(p - 1)$. Call κ^2 the *corrected stochastic variance*.

> *Definition 4*. Consider the formal stochastic model on a closed bounded domain, W, in Euclidean space of dimension w. Let v^2 be the total empirical variance of the voter ideal points (defined as above) and let β be the spatial parameter of the model. Suppose the errors are multivariate normal with difference covariance matrix Θ and corrected variance κ^2. Suppose further that the exogeneous valence terms are ranked $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$. Let $\lambda_{av} = [1/(p - 1)][\lambda_1 + \lambda_2 + \cdots + \lambda_{p-1}]$ and define the valence difference to be $\Lambda = (\lambda_{av} - \lambda_p)$. The convergence coefficient, c, is defined to be
>
> $$c = 2\beta\Lambda[v/\kappa]^2 \tag{4}$$

To illustrate, in the case with just two parties, so $p = 2$, the convergence coefficient takes the simpler form $c = \beta(\lambda_1 - \lambda_p)(v/\sigma)^2$.

> **Electoral Theorem 1.** *Existence of a local equilibrium at the joint mean in the formal stochastic model with exogeneous valence*: suppose that the policy space, W, is a closed bounded domain in Euclidean space of dimension, w. Then, $z_0^* = (0, \ldots, 0)$ is a LNE if $c < 1$, and only if $c \leq w$.

To apply this result, we may note from Table 1 for the Netherlands that, for the MNL model without SD, β is estimated to be 0.737. Taking D'66 to have zero valence ($\lambda_4 = 0$) gives $\lambda_{av} = 1.338$. Thus $\Lambda = (\lambda_{av} - \lambda_4) = 1.338$. The electoral normalization followed by taking the variance of the elite ideal points on the first axis to be 1.0. As a consequence, the estimated electoral v_1^2 on the first axis is 0.658, while on the second $v_2^2 = 0.289$. Thus we estimate $v^2 = 0.944$. Although the MNL model is based on the assumption that the errors are distributed log Weibull, we can insert the parameter values for the MNL model in the equation for the convergence coefficient for the benchmark formal model with iid normal errors $\kappa^2 = \sigma^2 p/(p - 1)$. Because of the error variance is $\sigma^2 = 1.649$, we find $\kappa^2 = 2.19$, so the convergence coefficient is $c = 0.85$. We are led to infer that both the necessary and sufficient conditions for the vector z_0^* to be a LNE are satisfied. Because the parameters are estimated, there will necessarily be some uncertainty with regard to the estimated value of c. Nonetheless, the confidence intervals on the valences imply that there is a high probability that the vector z_0* is a LNE. Simulation of the model suggested that z_0* was a PSNE.

A more general point is that the dimensionless convergence coefficient, c, is a measure of the degree of convergence in the electoral model in the sense that the larger c, then the greater the centrifugal tendency towards divergence, while the smaller c, then the greater the centripetal tendency towards convergence. Notice also, other things being equal, that a LNE at the origin is less likely the larger Λ, β, v^2 and p, while a large κ^2, or error variance, σ^2, makes the existence of a LNE at the origin more likely. Thus the coefficient provides a means of classifying electoral models. This point can be illustrated by using Table 2 to compare the results of the MNL models with those of the MNP. As Table 2 indicates, the convergence coefficient was below 1.0 for the two MNL models with valence. We can infer that a PSNE at the origin is highly likely. Table 2 shows that the eigenvalues for the D'66 on both axes were negative, implying that the origin is a local maximum. When the sociodemographic variables are added to the MNL model, then the valences of the parties will change. Because the coefficient for religion for the CDA is large and positive, the valence for this party drops to –0.784, while the convergence coefficient changes slightly to 0.92.

In the case of the MNP model, given in Table 2 with valence and with SD, the convergence coefficient is 1.0. Although the sufficient condition for a LNE at the origin is not satisfied, both the eigenvalues for the CDA are negative. Indeed, the same conclusion holds for the D'66 and VVD. This model has the highest log likelihood of –427. In the MNP model with valence and without SD, the convergence coefficient is certainly bounded above by 1. For this model, the lowest valence party is the D'66. We can assert that, for all these models, there is a LNE at the origin and then check whether concavity holds, giving a centrist PSNE.

Earlier work on MNP estimation without valence for the Netherlands noted that simulation indicated that parties could move position to the electoral mean and increase vote shares (see Schofield et al. 1998; Quinn & Martin 2002). This contradiction between the estimated positions of the parties, as given in Figure 1, and their vote-maximizing positions, provided the motivation for the inclusion of exogeneous valence in an attempt to account for the discrepancy. What we have demonstrated is that the prediction of the formal model, at least in the case of the Netherlands, does not depend on the particular modeling assumptions.

Using the locations of the party principals in Figure 1, we have seen that the log likelihoods of the models built on these positions are very high. Thus, we can infer that these positions approximate the actual positions of the parties. Even though the valence terms are statistically significant, their inclusion does not change the basic inference from both the MNL and the MNP models that the origin is an equilibrium. However, it follows that the formal model does not accurately predict the location of the parties.

The analysis has assumed that the valence terms for the parties are independent of the particular position adopted by the party. In an attempt to account for divergence between the theoretical equilibria and estimated positions, we now examine the situation when valence is not exogeneously determined, but is affected by the contributions of party activists. In this case, valence will be a function of the location of the party. One way to conceive of the trade-off involved is to model the role of the party leader. Although we may assume that the party's total activist contribution is maximized at the principal's position, we are led to consider the optimization problem facing the party leader over the choice of a position to declare to the electorate in order to maximize the vote share. Indeed, we shall show that if the valence function of each party is highly concave in the party position, then there can exist a non-centrist PSNE.

Activist valence politics in Britain

To examine intra-party decision making, we turn to an application of the model to Britain, where the electoral system is based on plurality rather than proportional electoral rule. Again, the locations of elite party members are used to determine the position of maximum activist support for each party. This, in turn, will determine the precise equilibrium location of each party. Activists contribute time and money, and affect overall political support. Since activist preferred positions tend to be more radical than the average voter, this presents the party leader with a complex 'optimization problem'. We use the valence model to offer a conjecture about how party leaders may deal with this problem.

Figure 2 presents estimates of the three principals' positions for the Labour, Conservative and Liberal parties, as inferred from the ISEIUM (1983) data set, just as in the Dutch example. (Figure 2 is from Schofield (2002) and is presented here with permission of Elsevier Science). Details of the factor model can be found in Quinn et al. (1999).

For the MNL model, the convergence coefficient can be calculated to be 0.08, with negative eigenvalues of –0.95 on the first axis and –0.97 on the second. For the MNP model, the coefficient is 0.04, with similar eigenvalues. Using these principals' positions, both models correctly predict approximately

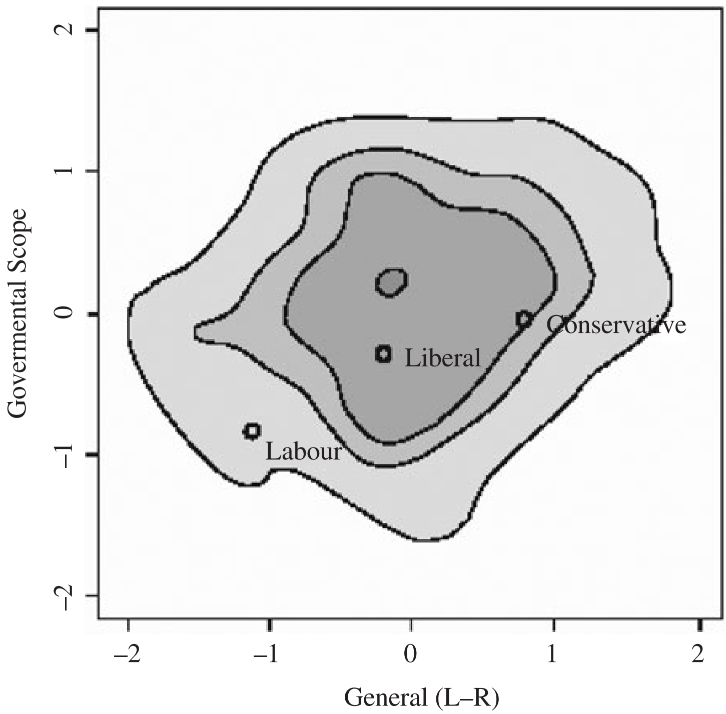


Figure 2. Estimated party positions in the British Parliament in 1979 for a two-dimensional model (based on the European Elections Study) showing the estimated density function of the voter ideal points.

50 per cent of the voter choice. According to the electoral theorem, all parties should have converged to the origin to increase vote share. However, the estimated positions of the parties in Figure 2 accord well with general perceptions of the party locations.

To address this contradiction between the predictions of the formal and empirical models, we modeled the 1997 election in Britain (see Table 3 for the election results in Britain for 1992–2001). First, we performed a factor analysis of response data obtained from the 1997 British Election Study. Table 4 gives the factor scores from the survey. The analysis showed that, to a large degree, a single economic dimension captured much of the political variation in Britain. In Scotland, the issue of Scottish Nationalism was relevant, and was incorporated into the first factor.

Table 3. Elections in Britain in 2001, 1997 and 1992

Party	Seats	Seats (%)	Vote (%)
2001			
Labour Party (LAB)	413	62.6	41.0
Conservative Party (CON)	166	25.1	31.9
Liberal Democrats Party (LIB)	52	7.8	18.4
Scottish National Party (SNP)	5	0.8	1.8
Plaid Cymru (PC)	4	0.6	0.7
Independent	1		
Northern Ireland	18		6.2
Ulster Unionists (UU)	6		
Democratic Union	5		
SDLP	3		
Sinn Fein	4		
1997			
Labour Party (LAB)	419	63.6	44.4
Conservative Party (CON)	165	25.0	31.4
Liberal Democrats Party (LIB)	46	6.9	17.2
Scottish National Party (SNP)	6	0.9	2.0
Plaid Cymru (PC)	4	0.6	0.5
Independent	1		
Northern Ireland	18	2.7	4.5
Ulster Unionists (UU)	10		
UK Unionists	1		
Democratic Union	2		
SDLP	3		
Sinn Fein	2		
1992			
Labour Party (LAB)	271	41.6	34.5
Conservative Party (CON)	336	51.6	41.9
Liberal Democrats Party (LIB)	20	3.1	17.9
Scottish National Party (SNP)	3	0.5	1.9
Plaid Cymru (PC)	4	0.6	0.5
Northern Ireland	17	2.6	3.3

As Table 4 indicates, attitudes to the European Union were also relevant. This factor (termed 'nationalism') is included in the estimation of the distribution of voter ideal points given in Figure 3, where a 'northern' position on the vertical axis represents an anti-European Union position, while 'southern'

Table 4. Question wordings from the British National Election Surveys for 1997, together with the factor scores for the questions (Britain without Scotland, and Scotland)

Britain (without Scotland) 1997

Issue	Factor scores
1. Unemployment and Inflation	0.265
2. Taxation and services	0.223
3. Nationalization	0.225
4. Redistribution	0.318
5. European Community	0.087
6. Women's rights	0.149

Scotland 1997

Issue	Factor scores
1. Unemployment and inflation	0.127
2. Taxation and services	0.104
3. Nationalization	0.156
4. Redistribution	0.580
5. European Community	0.008
6. Women's rights	0.137
7. Scottish nationalism	0.101

Questions:

1. Do you feel that the government's top priority should be getting people back to work, keeping prices down, or somewhere in between?
2. Do you feel the government should raise taxes and spend more money on health and social services, or do you feel they should cut taxes and spend less on these services?
3. Do you feel the government should nationalise or privatise more industries?
4. Do you feel the government should be more concerned with equalising people's incomes, or less concerned?
5. Do you feel Britain should unite with the European Union or protect its independence from the European Union?
6. Do you feel women should share an equal role in business, industry and government, or do you feel a women's place is in the home?
7. Do you feel Scotland should (a) become independent, separate from the UK and the European Union, (b) become independent, separate from the UK, but a part of the European Union, (c) remain part of the UK, with its own elected Assembly, with taxation and spending powers, or (d) remain as it is?

points represent pro-European Union attitudes. We argue that this second dimension was important for the vote-maximizing game between the parties. To estimate the location of the party principals, we obtained responses by party MPs to the British National Election Survey. This elite survey consisted

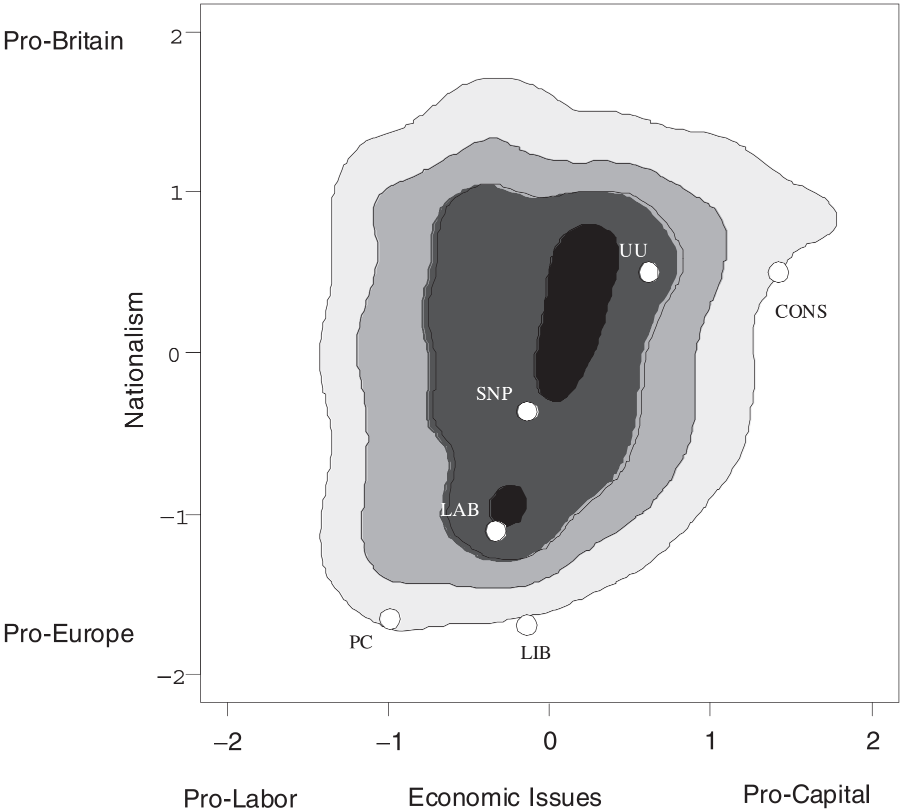


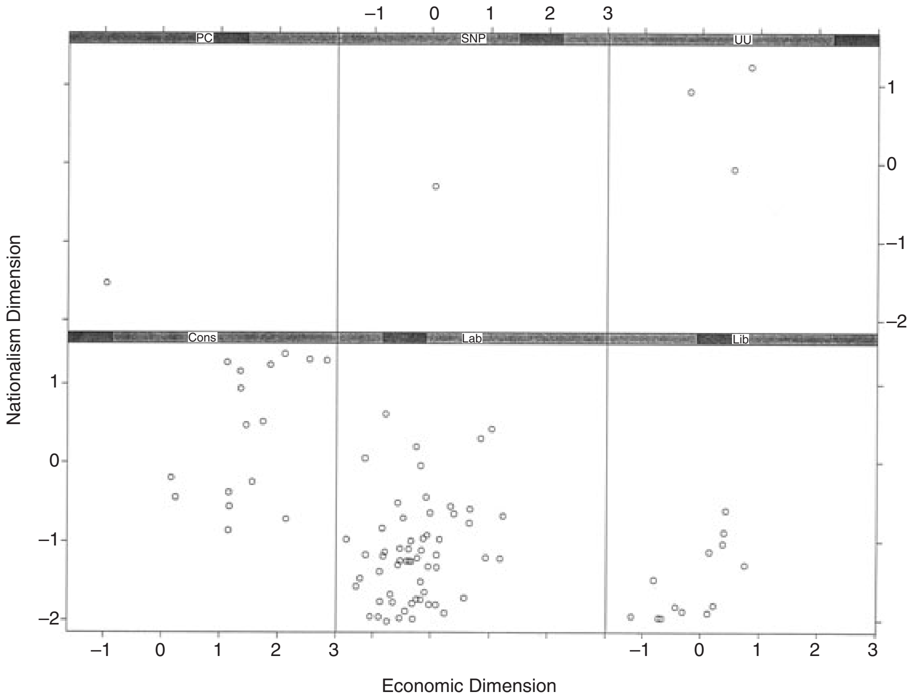
Figure 3. Estimated party positions in the British Parliament in 1997 (based on MP survey data and a National Election Survey) showing highest density plots of the voter sample distribution at the 95, 75, 50 and 10 per cent levels.

of 50 Labour (LAB) MPs, 16 Conservative (CON) MPs, 14 Liberal Democrat (LIB) MPs, 3 Ulster Unionist (UU) and single members of Plaid Cymru (PC) and the Scottish Nationalist Party (SNP). We processed these responses using the electoral factor analysis. The two-dimensional median for the set of responding MPs for each party was then used as the estimate of the principal's position. We hypothesize that total activist contributions to each party are maximized at the party principal position.

Figure 3 includes these estimated positions of party principals. It is noteworthy that when the two-dimensional principals' locations in Figure 3 are projected onto the single economic dimension they coincided with the average positions of the parties as determined by voter perceptions from the Election Survey. The estimated locations of the smaller groupings, as given by Figure 3, may appear counter-intuitive, but can in fact be rationalized. The distribution of Conservative MP positions occupied almost the full top-right quadrant of the figure. Thus the Ulster Unionists can be seen as merely economically centrist MPs, generally opposed to the European Union, but not dissimilar from a number of Conservative MPs. The SNP member is centrist with regard to both economic and European axes. The member of Plaid Cymru is fairly typical of left-wing members of the Labour Party from industrial areas of Britain. An MP from such a poor region of the country might expect advantages from closer European Union ties. Finally, Liberal Democrat members are traditionally economically centrist and generally pro-European Union. Figure 4 presents the distribution of the estimated positions of the MPs of the various parties who responded to our survey.

It is obvious that the electoral variance on the second 'European' axis is much greater than on the economic axis. Since the electoral distributions on the two axes are uncorrelated, we were able to use the MNL estimates of the valence terms to compute party eigenvalues. (The analysis is discussed in detail in Schofield (2004a, 2005b).) On the single economic axis, the eigenvalue for the Liberal Democrat Party is –0.56. On this axis, the position of the Liberal Democrat Party is correctly predicted. Given the higher electoral variance on the European axis, the position of the party 'south' of the origin in Figure 3 is compatible with the model. However, the MNL model suggests that the Labour and Conservative positions given in Figure 3 cannot be those that locally maximize vote shares. We conjecture that the non-centrist locations of these two parties are the consequence of activist influence. (The Appendix presents a formal model of voter and party choice when both exogenous and activist valences are involved.) We assume, as before, that the leader of party j is personally characterized by the valence term λ_j. In addition, however, activist support, $\mu_j(z_j)$, for the party depends on the party leader position, z_j, and it is this position that is declared to the electorate.

When activist valence is incorporated into the model, the first-order condition for vote-share maximization is not satisfied at the electoral mean. The Appendix shows that the first order equation can be interpreted as requiring a balance for each party between 'marginal electoral pull' and 'marginal activist pull'. The marginal electoral pull for a party is zero at a weighted electoral mean (which depends on all the parties' exogeneous valence terms). The marginal activist pull, in contrast, is zero at the party principal position. Figure 5 gives an illustration of this required balancing. In this formal model, the LNE can be determined by imposing this first-order balance condition and then verifying the second-order local concavity condition. The conclusions of the formal model can be summarized in the following theorem (see Schofield (2003) for the proof):



Key

PC: Plaid Cymru (Welsh Nationalist Party)
SNP: Scottish Nationalist Party
UU: Ulster Unionists
Cons: Conservative Party
LAB: Labour Party
LIB: Liberal Democrat Party

Figure 4. Estimated MP positions in the British Parliament in 1997 (based on MP survey data and a two-dimensional factor model derived from the National Election Survey).

Electoral Theorem 2. *The stochastic model with exogeneous and activist valences*. Consider a formal vote maximization game with both exogenous valences $\{\lambda_j\}$ and activist valences $\{\mu_j\}$. The first-order condition for z* to be an equilibrium is that, for each *j*, the electoral and activist pulls must be balanced. Other things being equal, the position z_j* will be closer to a weighted electoral mean the greater the exogenous valence λ_j. Conversely, if the activist valence function μ_j is increased (due to the greater willingness of activists to contribute to the party), then z_j* will be nearer to the activist preferred position. If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, the solution given by the first-order condition will be a PSNE.

As Table 3 indicates, popular vote for the Conservative Party dropped between 1992 and 1997, and empirical analysis indicated that the overall Conservative valence dropped from 1992 to 1997 while the Labour valence increased. The estimated valences include both exogenous popularity terms for the party leaders and the activist component. Clarke et al. (1998, 2004) suggest that Labour exogenous valence (λ_{LAB}), due to Blair, rose in this period. In contrast, the exogenous valence term, λ_{CON}, for the Conservatives fell. Since the coefficients in the equation for the electoral pull for the Conservative Party depend on ($\lambda_{CON} - \lambda_{LAB}$), these must all fall in this period (see the Appendix for the demonstration of this effect). The consequence is that the marginal effect of activism for the Conservative Party is increased. The empirical analysis of Conservative Party activism given in Richardson et al. (1995) is compatible with this observation.

It is possible to extend the analysis to include intra-party bargaining by modeling the conflict between activist groups within each of the major parties. For example, given the dispersion of preferred points within the Conservative Party, we may model the resulting potential conflict by considering two opposed groups who compete for influence over the Conservative Party – one 'pro-British' group (centered at the position marked B in Figure 5) and one 'pro-Capital' group (marked C in the figure). The optimal Conservative position will be determined by a balancing that equates the 'electoral pull' against the two 'activist pulls'. Since the electoral pull fell between the elections, the optimal position, z_{CON}*, will be one where z_{CON}* is closer to the locus of points where the marginal activist pull is zero (i.e., where $d\mu_{CON}/dz_{CON} = 0$). We can refer to this locus of points as the 'activist contract curve' for the Conservative Party.

Figure 5 describes the indifference curves of representative activists for various groups within the parties by ellipses to indicate that preferences of different activists on the two dimensions may accord different saliences to the policy axes. The 'activist contract curve' given in the figure for the Labour Party will be the locus of points satisfying the equation $d\mu_{LAB}/dz_{LAB} = 0$. With the assumption of differing saliences, this curve will have the catenary form as shown in Figure 5 (see Miller and Schofield (2003) for a derivation of the form of this curve). The Labour activist curve represents the balance between Labour supporters most concerned with economic issues (centered at L in the figure) and those more interested in Europe (centered at E). The optimal

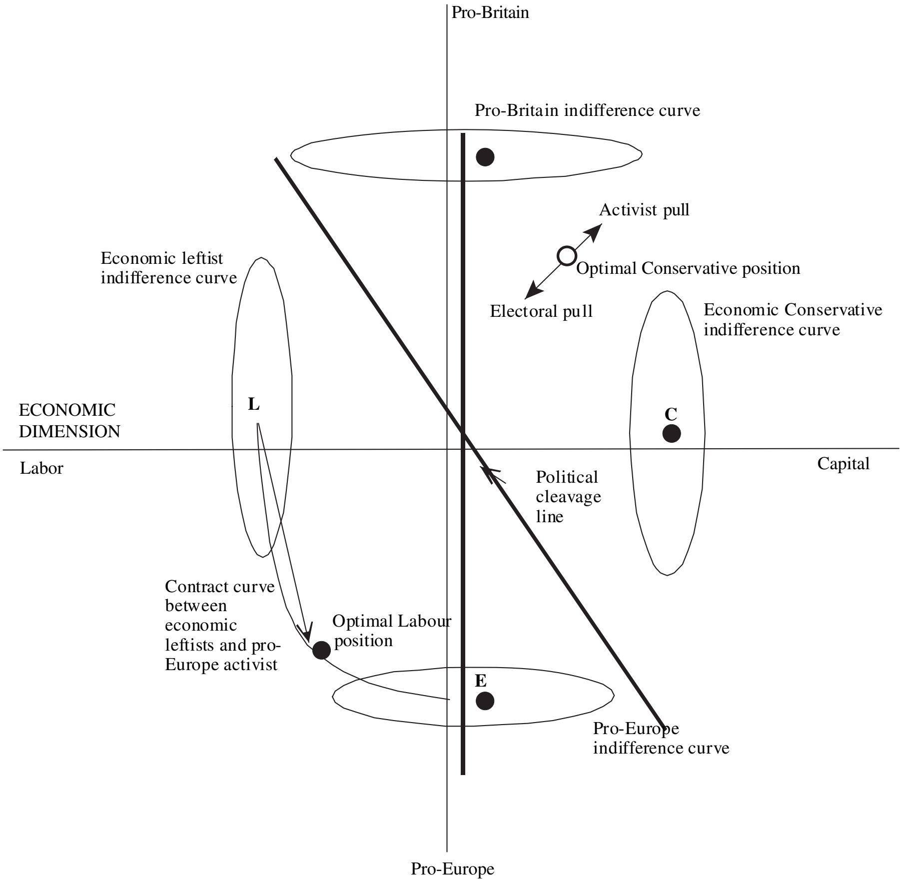


Figure 5. Illustration of vote-maximizing positions of Conservative and Labour party leaders in a two-dimensional policy space.

positions for the two parties will be at appropriate points on the locus between the respective activist contract curves and a point near the origin where the electoral pull is zero. The political cleavage line in Figure 5 represents the electoral dividing line if there were only the two parties in the election.

Recall that if the relative exogenous valence for a party falls, the optimal party position approaches the activist contract curve. The optimal position on this contract curve depends on the relative intensity of political preferences of the activists of each party. For example, if grassroots pro-British Conservative Party activists have intense preferences on the nationalism dimension, it will be reflected in the activist contract curve and in the optimal Conservative position. Thus, the activist model can, in principle, provide an account of the reason why the Conservative Party emphasized an anti-European Union position during the last few elections. For the Labour Party, two effects are apparent. Blair's high exogenous popularity gave an optimal Labour Party position closer to the electoral center than the optimal position of the Conservative Party. This affected the balance between Labour's 'old left' activists in the party, and 'New Labour' activists concerned to modernize the party through a European-style 'social democratic' perspective. This inference, based on our theoretical model, is compatible with Blair's successful attempts to bring New Labour members into the party (Seyd & Whiteley 2002).

We suggest that the Conservative Party did not adopt a position near the electoral mean during this period because of the need to balance the declining exogeneous valence of party leaders and activist valence. This resulted in the increased importance of the party activists, particularly the anti-Europe group. In contrast, Blair's increasing exogeneous valence in the period up to 1997 resulted in a decrease in the importance of the activists in the party, particularly the 'old left'. This led to a vote-maximizing position by Blair, more centrist on the economic axis and very pro-European Union on the second, giving Labour an electoral dominance over the Conservative Party. As Table 3 indicates, the plurality electoral system in Britain magnifies the effect of changing electoral support for parties, and increases the degree of political concentration. This is reflected in the decrease of the effective number of parties when computed in terms of votes against seats (Taagepera & Shugart 1989). Schofield (2005b) suggests that the plurality electoral system of Britain also magnifies the importance of activist support for the party. In the formal model, this has the effect of increasing the degree of concavity of the valence functions. This feature enhances the tendency of a party with a low exogeneous valence to adopt a more radical position and leads, as a result, to more dramatic shifts in party position in response to changes in the political environment.

Conclusion

The theoretical puzzle we have attempted to address is the disjunction between the predictions of the formal vote model and estimates of party position. Other work (Schofield & Sened 2005) has indicated that, in a polity such as Israel with many parties and differing party valences, a vote-maximization model with exogeneous valence can account for divergence. Our presentation of the spatial or policy maps for the Netherlands and Britain, together with the MNP and MNL models, has shown that such valence models cannot account for party position. Instead, we have been led to introduce the notion of 'activist valence'. We have argued that party positioning will depend on balancing these two kinds of valence against the electoral response. Such a model could give an indication of the differing political configurations of party positions that are possible in polities whose electoral systems are based either on proportional representation or on plurality rule. The analysis suggests that party activism is an essential component of any electoral model, especially of polities such as Britain's that use plurality rule.

It has been argued that proportional rule and plurality lead to very different political patterns (Duverger 1954; Riker 1953, 1982; Taagepera & Shugart 1989). Although the electoral theorems we have presented have been based on the simple assumption of vote maximization, it should be possible to extend it to deal with seat maximization under different electoral rules. This could provide a theoretical explanation for the quite different configurations observed in different polities.

The various spatial maps presented here and for Israel (Schofield & Sened 2005) and Germany (Schofield et al. 1998; Schofield 2002) demonstrate considerable variety. One conclusion that can be drawn from the electoral theorems on the extended spatial model is that, in general, a single large party will be unable to control the central electoral domain. This follows because activist coalitions will typically be located far from the center. An argument to this effect can be seen as the basis for Duverger's contention that the 'centre does not exist in politics' (Duverger 1954: 215; Daalder 1984). In line with this assertion, the valence model suggests, contrary to the mean voter theorem, that a crowded political center is highly unlikely since low-valence parties will tend to flee to the electoral periphery. Moreover, high-valence parties will also position themselves at some distance from the center. In such multiparty systems, based on proportional rule, coalition formation becomes very difficult since the larger parties must seek allies from among the smaller, more radical parties.

The electoral theorems do not imply that all parties will avoid the electoral center. As we have seen, the Liberal Democrat Party is low valence and moderately centrist. Yet, although Britain has a plurality electoral system and there is a tendency for one of the two high-valence parties to predominate, it is also apparent that smaller, 'regional' parties like the SNP can survive. However, due to the effect of plurality rule, the seat share of the Liberal Democrats in 1997 was well below its vote share. It is plausible that the realization of this effect among the British electorate is the reason why the valence of the Liberal Democrat leader has tended to remain below that of the other two main party leaders. As a consequence, this smaller centrist party has been unable to offer itself as a credible candidate for government. In the recent local elections, the fall in Blair's valence due to the international situation resulted in the Liberal

Democrats gaining 30 per cent of the popular vote. Under plurality rule, it seems likely that small changes in the exogeneous valences of the party leaders could lead to a transformation of the political configuration. At a more general level, the spatial theory offered here could be used to construct a theory to account for the various patterns of political configuration that are possible. The method adopted here, of combining empirical analyses with model building, may lead to a better understanding of the nature of representative government under different institutional arrangements and electoral rules.

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Appendix

In this Appendix, we briefly sketch the procedure for determination of the first-order condition for local equilibrium when both exogeneous and activist valence are involved. To be more precise about voter preferences, let

z = $(z_1, \ldots, z_p)$ be a typical vector of party policy positions. Given z, each voter, i, is described by a utility vector $\mathrm{u}_i\,(x_i,\mathrm{z}) = (u_{i1}(x_i,z_1), \ldots, u_{ip}(x_i,z_p))$, where

$$u_{ij}(x_i, z_i) = \lambda_j + \mu_j(z_j) - \beta \| x_i - z_j \|^2 + \epsilon_j \tag{5}$$

Here, λ_j is the 'exogenous' valence of party j, $\mu_j(z_j)$ is the activist valence, β is a positive constant and $\| \; \|$ is an appropriate norm on W (used to represent the importance of the distance between the preferred point of the voter, and the party). The term ϵ_j is the stochastic error. In the full abstract model, voters are assumed to have the usual Euclidean norm, but activists may have different 'ellipsoidal' norms depending on the saliences they have for different policy dimensions. This feature is used by Miller and Schofield (2003) to account for the willingness of activists to contribute to their party.

Proof of the first electoral theorem

In the case of the Euclidean norm, the probability that i votes for party j is given by

$$\text{Prob}(\Phi)\left(\lambda_j + \mu_j(z_j) + \in_j - \beta\delta_{ij}^{\;2} > \lambda_k + \mu_k(z_k) + \in_k - \beta\delta_{ik}^{\;2} \text{ for all } k \neq j\right) \quad (6)$$

Prob(Φ) denotes the probability given by the bounds under the cumulative distribution function Φ, and $\delta_{ij}^2, \delta_{ik}^2$ to denote the respective quadratic Euclidean distances. When activist valence is set to zero, Equation 6 gives Equation 1 in the text. The expected vote share $E(\pi_j(\text{x;z})) = (1/n)\Sigma_i\rho_{ij}$ (x;z) where ρ_{ij}(x;z) is the probability i chooses party j. Collecting the stochastic terms in Equation 6 on the right, Schofield (2004a) shows that the probability can be expressed as

$$\rho_{ij}(\text{x;z}) = \Phi(g_{ij}(\text{x;z})) \quad (7)$$

Here $g_{ij}(\text{x;z}) = \lambda_j + \mu_j(z_j) - \beta\delta_{ij}^2 - \lambda_{av} - \mu_{av}(\text{z}_{-j}) + \beta\delta_{iav}^{\;2}$, while Φ is the cumulative distribution function associated with the stochastic variate $\in_{av} - \in_j$. The variate $\in_{av} = [1/(p-1)]\Sigma_k \in_k$, while $\lambda_{av} = [1/(p-1)]\Sigma_k\lambda_k$ and $\mu_{av}(\text{z}_{-j}) = [1/(p-1)]\Sigma_k\mu_k(\text{z}_{-j})$. All three summations are taken over the indices k different from j, while z_{-j} is the vector of positions other than j.

Under the assumption that the errors are multivariate normal, then it can be shown that Φ can be taken to be the cumulative univariate normal distribution with variance κ^2, and expectation 0. The term g_{ij}(x;z) is a 'comparison' function for voter i with regard to party j against a composite of the other parties. This function involves $\{\lambda_j, \mu_j\}$, for all j, and (z,x). The higher the utility for i from j's position in comparison with other parties, the greater is the value of this comparison function. The first-order condition for maximizing the vote-share function π_j can be shown to be:

$$\Sigma_i\, h(g_{ij})dg_{ij}/dz_j = 0 \quad (8)$$

Here, h is the normal probability density function. In the case all $\mu_j \equiv 0$ and all λ_j are fixed, this equation is satisfied at the voter mean (to see this, note that at the mean all g_{ij} are identical). The equation $\Sigma_i\, dg_{ij}/dz_j = 0$ is then independent of z_j and has the solution $z_j^* = (1/n)\Sigma_i\, x_i$, for all j. As before, we can redefine the coordinate system so that the mean position is the origin. However, this is not the only solution to Equation 8. If the party positions are different, then not all g_{ij} are identical and the solution becomes

$$z_j{}^* = \Sigma_i \alpha_{ij} x_I \tag{9}$$

In this equation, the coefficients $\{\alpha_{ij}\}$ involve z, x and the exogenous valence terms $\{\lambda_j\}$. It is important to note that the solution to Equation 9 is then interdependent in the sense that $z_j{}^*$ depends on $\{z_k{}^*\}$. Moreover, the coefficient α_{ij} is an increasing function of λ_j, and a decreasing function of $\{\lambda_k{:}k \neq j\}$. It follows that there may exist many non-centrist LNE. The second-order condition for LNE involves the second differential of the terms $\{\rho_{ij}(\mathrm{x;z})\}$. The Hessian for party j is:

$$\mathrm{H}_j = [1/n]\Sigma_i[h(g_{ij})]\left[[-g_{ij}][dg_{ij}/dz_j \cdot dg_{ij}/dz_j]\big/[\kappa^2] + d^2g_{ij}\big/dz_j{}^2\right] \tag{10}$$

This summation is taken over all voters. When all parties are at the origin, then $g_{ij} = (\lambda_j - \lambda_{\mathrm{av}})$ is independent of x_i. Moreover the first term is $[h(g_{ij})] = h(\lambda_j - \lambda_{\mathrm{av}})$ is necessarily positive. The term, $[dg_{ij}/dz_j \cdot dg_{ij}/dz_j]$ can be written as $4\beta^2\mathrm{D}(x_i{}^2)$, where D is a quadratic form. (In the one-dimensional case, this quadratic form is simply x_i^2.) In the w-dimensional case, D is a symmetric matrix whose diagonal entries are $\{x_{it}{}^2{:}t = 1,w\}$. Finally, the second differential $d^2g_{ij}/dz_j{}^2$ is $-2\beta\mathrm{I}$. We defined $v_t{}^2 = 1/n\ \Sigma_i(x_{it})^2$ to be the empirical variance of the voter ideal points on the t-coordinate axis. To determine necessary and sufficient conditions for the Hessian of Equation 10 to have negative eigenvalues, we examine the matrix

$$\mathrm{C} = \Sigma_i(\lambda_{\mathrm{av}} - \lambda_j)2\left[\beta\mathrm{D}\left(x_i{}^2\right)\right] - \kappa^2\mathrm{I} \tag{11}$$

Here I is the w by w identity matrix. We can illustrate the logic in the two-dimensional case. The trace of C (the sum of the diagonal terms) must be non-positive. Thus we obtain $2(\lambda_{\mathrm{av}} - \lambda_j)\beta(v_1{}^2 + v_2{}^2) \leq 2\kappa^2$. If we let $v^2 = (v_1{}^2 + v_2{}^2)$ and $j = p$, then the required condition is that $2\beta\mathrm{I}v^2 \leq 2\kappa^2$. The w-dimensional case follows in parallel fashion to give the necessary condition

$$2\beta\Lambda v^2 \leq w\kappa^2 \tag{12}$$

If Equation 12 fails for $j = p$, then one of the eigenvalues of the Hessian H_p must be positive. This implies that $z_p = 0$ cannot be a local best response to $(0, \ldots, 0)$. Consequently, $\mathrm{z}_0{}^*$ cannot be a LNE. The sufficient condition that $z_j = 0$ be a best response to $(0, \ldots, 0)$ depends on the determinant of C, and this condition can be shown to be that $2(\lambda_{\mathrm{av(j)}} - \lambda_j)\beta v^2 < \kappa^2$. Since we require this condition for all j, we may let $j = p$. Thus we obtain the sufficient condition

$$2\beta\Lambda v^2 < \kappa^2 \tag{13}$$

If this condition is satisfied, then the analogous conditions for $j = 1, \ldots, p - 1$ will be satisfied. Thus Equation 13 is a sufficient condition for $\mathrm{z}_0{}^*$ to be a LNE. This proves the theorem.

Proof of the second electoral theorem

With non-zero activist valence, the first-order solution for party j takes the more general form $z_j^* = d\mu_j^*/dz_j + \Sigma_i\, \alpha_{ij}\, x_i$. In this equation, the coefficients $\{\alpha_{ij}\}$ depend on the functions $\{\mu_j\}$. The function μ_j^* is renormalized and satisfies $2\beta\mu_j^* = \mu_j$. Let us denote the vector $\Sigma_i\, \alpha_{ij}\, x_i$ by $d\pi_j^*/dz_j$ and call it the 'marginal electoral pull' of party j, due to exogenous valence. The first-order condition can then be written

$$d\pi_j^*/dz_j + d\mu_j^*/dz_j - z_j^* = 0 \tag{14}$$

The first term in this expression, the 'marginal electoral pull', is a gradient vector pointing towards the 'weighted electoral mean'. As λ_j is exogenously increased, this vector increases in magnitude. The vector $d\mu_j^*/dz_j$ 'points towards' the position at which the total of activist 'contributions' for the party is maximized. We may term this vector the '(marginal) activist pull'. Moreover, if the activist functions of all parties are sufficiently concave, then the vector z* given by the solution of Equation 14 for all j will be a LNE. This follows because the second-order term $[d^2\mu_j^*/dz_j^2]$ will have negative eigenvalues of large modulus.

References

Adams, J. (1999). Policy divergence in multicandidate probabilistic spatial voting. *Public Choice* 100: 103–122.

Adams, J. (2001). *Party competition and responsible party government*. Ann Arbor, MI: University of Michigan Press.

Adams, J. & Merrill, III, S. (1999). Modelling party strategies and policy representation in multiparty elections: Why are strategies so extreme? *American Journal of Political Science* 43: 765–781.

Aldrich, J. & McGinnis, M. (1989). A model of party constraints on optimal candidate positions. *Mathematical and Computer Modelling* 12: 437–450.

Alvarez, M. & Nagler, J. (1998). When politics and models collide: Estimating models of multi-candidate elections. *American Journal of Political Science* 42: 55–96.

Alvarez, M., Nagler, J. & Bowler, S. (2000). Issues, economics and the dynamics of multiparty elections: The 1997 British general election. *American Political Science Review* 94: 131–150.

Ansolabehere, S. & Snyder, J. (2000). Valence politics and equilibrium in spatial election models. *Public Choice* 103: 327–336.

Banks, J. & Duggan, J. (2000). A bargaining model of collective choice. *American Political Science Review* 94: 73–88.

Banks, J. & Duggan, J. (2004). Probabilistic voting in the spatial model of elections: The theory of office-motivated candidates. In D. Austen-Smith & J. Duggan (eds), *Social choice and strategic decisions*. Heidelberg: Springer.

Banks, J., Duggan, J. & Le Breton, M. (2002). Bounds for mixed strategy equilibria and the spatial model of elections. *Journal of Economic Theory* 103: 88–105.

Budge, I., Robertson, D. & Hearl, D. (eds) (1987). *Ideology, strategy and party change: A spatial analysis of post-war election programmes in nineteen democracies*. Cambridge: Cambridge University Press.

Calvert, R. (1985). Robustness of the multidimensional voting model: Candidates, motivations, uncertainty and convergence. *American Journal of Political Science* 29: 69–85.

Chib, S. & Greenberg E. (1996). Markov chain Monte Carlo simulation methods in econometrics. *Econometric Theory* 12: 409–431.

Clarke, H., Stewart, M. & Whiteley, P. (1997). Tory trends, party identification and the dynamics of Conservative support since 1992. *British Journal of Political Science* 26: 299–318.

Clarke, H., Stewart, M. & Whiteley, P. (1998). New models for New Labour: The political economy of Labour support, January 1992–April 1997. *American Political Science Review* 92: 559–575.

Clarke, H. et al. (2004). *Political choice in Britain*. Oxford: Oxford University Press.

Coughlin, P. (1992). *Probabilistic voting theory*. Cambridge: Cambridge University Press.

Daalder, H. (1984). In search of the center of European party systems. *American Political Science Review* 78: 92–109.

De Vries, M. (1999). *Governing with your closest neighbor*. Nijmegen: Ipskamp.

Downs, A. (1957). *An economic theory of democracy*. New York: Harper & Row.

Duverger, M. (1954). *Political parties: Their organization and activity in the modern state*. New York: Wiley.

Enelow, J. & Hinich, M. (1984). *The spatial theory of voting*. Cambridge, Cambridge University Press.

Enelow, J. & Hinich, M. (1989). The location of American presidential candidates. *Mathematical and Computer Modelling* 12: 461–470.

Groseclose, T. (2001). A model of candidate location when one candidate has a valence advantage. *American Journal of Political Science* 45: 862–886.

Hinich, M. (1977). Equilibrium in spatial voting: The median voter theorem is an artifact. *Journal of Economic Theory* 16: 208–219.

ISEIUM (1983). *European elections study: European political parties' middle-level elites*. Mannheim: Europa Institute.

Kass, R. & Raftery, A. (1995). Bayes factors. *Journal of the American Statistical Association* 91: 773–795.

Laver, M. & Budge, I. (eds) (1992). *Party policy and government coalitions*. London: Macmillan.

Laver, M. & Schofield, N. (1998). *Multiparty government: The politics of coalition in Europe*. Ann Arbor, MI: University of Michigan Press.

Lin, T.-M., Enelow, J. & Dorussen, H. (1999). Equilibrium in multicandidate probabilistic spatial voting. *Public Choice* 98: 59–82.

Merrill, III, S. & Grofman, B. (1999). *A unified theory of voting*. Cambridge: Cambridge University Press.

Miller, G. & Schofield, N. (2003). Activists and partisan realignment in the US. *American Political Science Review* 97: 245–260.

Poole, K. & Rosenthal, H. (1984). US presidential elections, 1968–1980: A spatial analysis. *American Journal of Political Science* 28: 283–312.

Quinn, K. & Martin, A. (2002). An integrated computational model of multiparty electoral competition. *Statistical Science* 17: 405–419.

Quinn, K., Martin, A. & Whitford, A. (1999). Voter choice in multiparty democracies. *American Journal of Political Science* 43: 1231–1247.

Rabier, J.-R. & Inglehart, R. (1981). *Eurobarometer II, April 1979: The year of the child in Europe*. Ann Arbor, MI: Inter-University Consortium for Political and Social Research.

Richardson, J., Seyd, P. & Whiteley, P. (1995). *True blues*. Oxford: Oxford University Press.

Riker, W. (1953). *Democracy in the US*. New York: Macmillan.

Riker, W. (1982). The two-party system and Duverger's Law: An essay on the history of political science. *American Political Science Review* 76: 753–766.

Riker, W. & Ordeshook, P. (1973). *Positive political theory*. Englewood Cliffs, NJ: Prentice-Hall.

Schofield, N. (1993). Political competition and multiparty coalition governments. *European Journal of Political Research* 23: 1–33.

Schofield, N. (1995). Coalition politics: A formal model and empirical analysis. *Journal of Theoretical Politics* 7: 245–281.

Schofield, N. (1997). Coalition politics and representative democracy. *European Journal of Political Research* 31: 183–192.

Schofield, N. (2002). Representative democracy as social choice. In K. Arrow, A. Sen & K. Suzumura (eds), *The handbook of social choice and welfare*. New York: North Holland.

Schofield, N. (2003). Valence competition in the spatial stochastic model. *Journal of Theoretical Politics* 15: 371–383.

Schofield, N. (2004a). Equilibrium in the spatial valence model of politics. *Journal of Theoretical Politics* 16: 447–481.

Schofield, N. (2004b). Local political equilibria. In D. Austen-Smith & J. Duggan (eds), *Social choice and strategic decisions*. Heidelberg: Springer.

Schofield, N. (2005a). Social choice and elections. In J. Alt, J. Aldrich & A. Lupia (eds), *A positive change in political science*. Ann Arbor, MI: University of Michigan Press.

Schofield, N. (2005b). A Valence Model of Political Competition in Britain, 1992–1997. *Electoral Studies*, forthcoming.

Schofield, N. & Parks, R. (2000). Nash equilibrium in a spatial model of coalition bargaining. *Mathematical Social Sciences* 39: 133–174.

Schofield, N. & Sened, I. (2002). Local Nash equilibrium in multiparty politics. *Annals of Operations Research* 109: 193–210.

Schofield, N. & Sened, I. (2005). Multiparty competition in Israel, 1988–1996. *British Journal of Political Science*: forthcoming.

Schofield, N., Grofman, B. & Feld, S. (1989). The core and stability in spatial voting games. *American Political Science Review* 82: 195–211.

Schofield, N. et al. (1998). Multiparty electoral competition in the Netherlands and Germany: A model based on multinomial probit. *Public Choice* 97: 257–293.

Schofield, N., Miller, G. & A. Martin. (2003). Critical elections and political realignment in the US, 1860–2000. *Political Studies* 51: 217–240.

Schofield, N., Sened, I. & Nixon, D. (1998). Nash equilibrium in multiparty competition with 'stochastic' voters. *Annals of Operations Research* 84: 3–27.

Sened, I. (1995). Equilibria in weighted voting games with side-payments. *Journal of Theoretical Politics* 7: 283–300.

Sened, I. (1996). A model of coalition formation: Theory and evidence. *Journal of Politics* 58: 350–372.

Seyd, P. & Whiteley, P. (2002). *New Labour's grassroots*. Basingstoke: Macmillan.

Stokes, D. (1963). Spatial models and party competition. *American Political Science Review* 57: 368–377.

Stokes, D. (1992). Valence politics. In D. Kavanagh (ed.), *Electoral politics*. Oxford: Clarendon Press.

Taagepera, R. & Shugart, M.S. (1989). *Seats and voters: The effects and determinants of electoral systems*. New Haven, CT: Yale University Press.

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