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Political equilibria with electoral uncertainty

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Abstract After an election, when party positions and strengths are known, there may be a centrally located large party at the core position. Theory suggests that such a core party is able to form a minority government and control policy. In the absence of a core party, theory suggests that the outcome be a lottery associated with coalition risk. Stochastic models of elections typically indicate that all parties, in equilibrium, will adopt positions at the electoral center. This paper first presents an existence theorem for local Nash equilibrium (LNE) under vote maximization, and then constructs a more general model using the notion of coalition risk. The model allows for the balancing of office and policy motivations. Empirical analyses of elections in the Netherlands and Israel are used as illustrations of the model and of the concept of a structurally stable LNE.

1 Valence and political equilibria

A standard model of political competition is one where party leaders adopt positions to maximize vote share in the context of an electorate whose preferences exhibit a stochastic element. For example, Hinich (1977) and Enelow and Hinich (1984, 1989) examined those circumstances under which vote maximizing candidates would adopt a position at the mean of the voter distribution.

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The model of two-party competition has been extended by Coughlin (1992) and most recently by McKelvey and Patty (2005) and Banks and Duggan (2005). Lin et al. (1999) have also obtained a "mean voter theorem," for the general case of many candidates.

Applying a stochastic model of voting is the standard technique for estimating voter response in empirical analyses (Alvarez and Nagler 1998; Alvarez et al. 2000). In an early application it was noted by Poole and Rosenthal (1984) that there was no evidence of convergence to the electoral mean in US presidential elections. Recently, empirical analyses of party competition in the United States, Britain, Germany, the Netherlands, and Israel based on the stochastic electoral model have found divergence of party positions away from the electoral mean[1].

These empirical models have all entailed the addition of heterogeneous intercept terms for each party. One interpretation of these intercepts or constant terms is that they are valences or party biases. "Valence" refers to voters' judgements about positively or negatively evaluated aspects of candidates, or party leaders, which cannot be ascribed to the policy choice of the party or candidate (Stokes 1992). Valence may be conceived of as an average electoral judgement of the candidate's quality or competence. This idea of valence has been utilized in a number of recent formal models of voting (Ansolabehere and Snyder 2000; Groseclose 2001; Aragones and Palfrey 2002, 2005).

The next section of this paper presents a characterization of equilibrium in terms of the Hessian of the vote share function of the party leader or candidate who has the lowest valence. For the case when the stochastic errors have the Type I extreme value (or log Weibull) distribution, Ψ, Theorem 1 shows that there exists a "convergence coefficient" which is a function of all the parameters of the model. A sufficient condition for the existence of a "local pure strategy Nash equilibrium" (LNE) at the electoral mean is that this coefficient is bounded above by 1. When the policy space is of dimension w, then the necessary condition for existence of a PNE at the electoral mean, and thus for the validity of the "mean voter theorem," is that the coefficient is bounded above by w. In the two-dimensional case, the eigenvalues of the Hessian can be computed. It is shown that the convergence coefficient is (1) an increasing function of the maximum valence difference and (2) an increasing function of the electoral variance of the voter preferred points.

When the necessary "convergence condition" fails, then the origin will be a saddlepoint or minimum of the vote share function for the lowest valence party. By changing position in the major electoral axis (or eigenspace of the vote function) this party will increase vote share. It follows that in equilibrium, all parties will adopt positions on this principal axis, with the lowest valence parties the furthest from the origin. No party will adopt a position at the electoral mean. Section 3 of this paper shows that the convergence condition fails for an electoral model for the election of 1992 in Israel, but is satisfied for the model the Netherlands in 1979. Simulation of the empirical model for Israel found that

[1] See Schofield et al. 1998a, b, 2000; Quinn et al. 1999; Quinn and Martin 2002; Schofield and Sened 2002, 2005a, b, 2006; Miller and Schofield 2003; Schofield 2004; Schofield 2005a, b.

the vote maximizing positions of the parties were indeed not at the electoral mean. Indeed the theorem and the simulation found that the low valence parties should be located along a "principal electoral axis" in order to maximize votes. Although there was a close correspondence between the estimated actual positions of the parties and the equilibrium positions obtained by simulation, a number of parties were found to be located off the principal electoral axis. This discrepancy between the formal model and estimated positions suggests that the model be extended to include policy preferences of the parties.

Since the stochastic model assumes that the party leaders are motivated simply to maximize expected vote shares in order to gain office, it ignores the possibility of uncertainty in electoral response. One way to introduce uncertainty, at least in the two-party model, is to focus instead on the "probability of victory" (Duggan 2000; Patty 2002, 2005, 2006). Implicitly, such a model acknowledges the vote share functions are stochastic variables. The model can then be further extended to the multiparty case by introducing uncertainty over policy outcomes through the probabilities associated with different collections of decisive coalitions. Policy decisions may be made by party principals who have policy preferences. Section 4 models such policy-motivated choice using the idea of coalition risk and suggests that the deviation of party position from vote maximizing equilibria in the Netherlands and Israel can be accounted for by this notion.

Section 5 attempts to combine the notion of "local Nash equilibrium" with the concepts of the core and heart derived from social choice theory. This allows for a model where party principals balance the two different maximands – vote share and the ability to influence government coalition. These resulting equilibria will depend on the beliefs that voters have (as described by valence) and the beliefs that party principals have over coalition risk. The concluding section briefly considers local equilibria that are *structurally stable* (*ss*), in the sense of being insensitive to small changes in these beliefs.

2 Local Nash equilibrium with electoral certainty and office-motivated parties

The purpose of this section is to construct a model of positioning of parties in electoral competition so as to account for the generally observed phenomenon of nonconvergence. The model adopted is an extension of the multiparty stochastic model of Lin et al. (1999) constructed by inducing asymmetries in terms of valence. The basis for this extension is the extensive empirical evidence that valence is a significant component of the judgements made by voters of party leaders. There are a number of possible choices for the appropriate game form for multiparty competition. The simplest one, which is used here, is that the utility function for agent j is proportional to the vote share, V_j, of the agent. With this assumption, I shall examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE) for this particular game form. Because the vote share functions are differentiable, calculus techniques are used to estimate optimal positions. As usual with this form of analysis, we can obtain sufficient conditions for the existence of local optima, which I shall term as local pure strategy Nash equilibria (LNE). Clearly, any PNE will be a LNE, but not conversely. Additional conditions of concavity or quasi-concavity are sufficient to guarantee existence of PNE. However, in the models considered here, it is evident that these sufficient conditions will fail, leading to the inference that PNE are typically non-existent. Existence of mixed strategy Nash equilibria is an open question in such games. It is of course true that the real utility functions of party leaders are unknown. However, comparison of LNE, obtained by simulation of empirical models, with the estimated positions of parties in the various polities that have been studied, can provide insight into the true nature of the game form of political competition.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of party declarations, or manifestos, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that party leaders cannot predict vote response precisely. In the model with "exogenous" valence, the stochastic element is associated with the weight given by each voter, i, to the average perceived quality or valence of the party leader.

Definition 1 The stochastic vote model.

The data of the spatial model is a distribution, $\{x_i \in X\}_{i\in N}$, of voter ideal points for the members of the electorate, N, of size n. As usual, X is assumed to be a compact convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Each of the parties, or agents, in the set $P = \{1, \dots, j, \dots, p\}$ chooses a policy, $z_j \in X$, to declare. Let $\mathbf{z} = (z_1, \dots, z_p) \in X^p$ be a typical vector of agent policy positions. Given $\mathbf{z}$, each voter, i, is described by a vector $\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \dots, u_{ip}(x_i, z_p))$, where

$$u_{ij}(x_i, z_j) = \lambda_j - \beta||x_i - z_j||^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j \quad (1)$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The term, λ_j is the "exogenous" valence of agent j, β is a positive constant, and $||\cdot||$ is the usual Euclidean norm on X. The terms $\{\epsilon_j\}$ are the stochastic errors, whose cumulative distribution will be denoted by Ψ. In empirical analyses and in this paper it is assumed that Ψ is the "extreme value Type I distribution" (sometimes called log Weibull). It is natural to suppose that the valence of party j, as perceived by voter i is the stochastic variate $\lambda_{ij} = \lambda_j + \epsilon_j$, where λ_j is the expectation $\mathcal{E}xp(\lambda_{ij})$ of λ_{ij}. I assume that the valence vector

$$\lambda = (\lambda_1, \lambda_2, .., \lambda_p) \textit{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \dots \geq \lambda_2 \geq \lambda_1.$$

Because of the stochastic assumption, voter behavior is modeled by a probability vector. The probability that a voter i chooses party j is

$$\rho_{ij}(\mathbf{z}) = \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \textit{ for all } l \neq j]. \quad (2)$$

$$= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \textit{ for all } l \neq j] \quad (3)$$

Here Pr *stands for the probability operator generated by the distribution assumption on ϵ. The expected vote share of agent j is*

$$V_j(\mathbf{z}) = (1/n)\sum_{i\in N}\rho_{ij}(\mathbf{z}) \tag{4}$$

The function $V: X^p \to \mathbb{R}^p$ is called the party profile function.

In the vote model it is assumed that each agent j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, ..z_{j-1}, z_{j+1}.., z_p)$.

Definition 2 Equilibrium concepts.

(i) *A strategy vector $\mathbf{z}^*{=}(z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \in X^p$ is a* strict Nash equilibrium *(PSNE) for the profile function $V: X^p \to \mathbb{R}^p$ iff, for each agent $j \in P$,*

$$V_j(z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) > V_j(z_1^*, \ldots, z_j..z_p^*) \text{ for all } z_j \in X - \{z_j^*\}.$$

(ii) *A strategy vector $\mathbf{z}^*{=}(z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a* weak Nash equilibrium *(PNE) iff, for each agent j,*

$$V_j(z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^* \ldots, z_p^*) \geq V_j(z_1^*, \ldots z_{j-1}^*, z_j, z_{j+1}^*, .., z_p^*) \text{ for all } z_j \in X.$$

Now let $x^* = (1/n)\Sigma_i x_i$. Then the mean voter theorem for the stochastic model asserts that the "joint mean vector" $\mathbf{z} = (x^*, \ldots, x^*)$ is a PNE. Because of the differentiability of the cumulative distribution function, the individual probability functions $\{\rho_{ij}\}$ are C^2 -differentiable in the strategies $\{z_j\}$. Thus, the vote share functions will also be C^2-differentiable. Lin et al. (1999) used C^2-differentiability of the expected vote share functions, in the situation with zero valence, to show that the validity of the theorem depended on the concavity of the vote share functions. Because concavity cannot in general be assured, I utilize a weaker equilibrium concept, that of *local weak Nash Equilibrium* (LNE). A strategy vector $\mathbf{z}^*$ is a LNE if, for each j, z_j^* is a critical point of the vote function $V_j(z_1^*, ..z_{j-1}^*, z_j, z_{j+1}^*, ..z_p^*)$ and the eigenvalues of the Hessian of this function (with respect to z_j), at $\mathbf{z}^*$ are nonpositive, but not all are zero. A strategy vector $\mathbf{z}^*$ is said to be a *local strict Nash Equilibrium*(LSNE) if it is an LNE and all eigenvalues are negative. Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. I use the notion of LSNE to avoid problems with the degenerate situation of zero eigenvalues to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}^* = (x^*, \ldots, x^*)$ to be a LNE and thus a PNE, without having to invoke concavity. In dimension w, the theorem can be used to show that, for $\mathbf{z}^*$ to be an LSNE, the necessary condition is that a "convergence coefficient," defined in terms of the parameters of the model, must be strictly bounded above by w.

Similarly, for $\mathbf{z}^*$ to be an LNE, then the convergence coefficient must be weakly bounded above by w. When this condition fails, then the joint mean vector $\mathbf{z}^*$ cannot be an LNE and therefore cannot be a PNE. In the two-dimensional case we can explicitly determine the eigenvalues, and if one of these is shown to be positive, then $\mathbf{z}^*$ must fail to be a PNE. To state the theorem, we first transform coordinates so that in the new coordinates, $x^* = 0$. I shall refer to $\mathbf{z}_0 = (0, \ldots, 0)$ as the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends on the distribution of voter ideal points. These are encoded in the voter covariation matrix. I first define this, and then use it to characterize the vote share Hessians.

Definition 3 The electoral covariance matrix, ∇^*.

The variation in voter preferences is represented in a simple form by the covariation matrix, ∇, *of the distribution of voter ideal points. Let X have dimension w and be endowed with a system of coordinate axes* $(1, \ldots, s, \ldots, t, \ldots, w)$. *For each coordinate axis, let* $\xi_t = (x_{1t}, x_{2t}, \ldots, x_{nt}) \in \mathbb{R}^n$ *be the vector of the* tth *coordinates of the set of n voter ideal points. I use* $(\xi_s, \xi_t) \in \mathbb{R}$ *to denote scalar product.*

The symmetric $w \times w$ *electoral variation data matrix* ∇ *is then defined to be* $[(\xi_s, \xi_t)]_{t=1,..w}^{s=1,..w}$.

The electoral covariance matrix is $\nabla^* = 1/n\nabla$. *The covariance between the* sth *and* tth *axes is denoted* $(v_s, v_t) = 1/n(\xi_s, \xi_t)$ *and* $v_s^2 = 1/n(\xi_s, \xi_s)$ *is the electoral variance on the* s^{th} *axis. The total electoral variance is*

$$v^2 = \sum_{r=1}^{w} v_r^2 = \frac{1}{n} \sum_{r=1}^{w} (\xi_r, \xi_r) = \text{trace}\left(\nabla^*\right).$$

Definition 4 The extreme value distribution, Ψ.

The cumulative distribution has the closed form

$$\Psi(h) = \exp[-\exp[-h]],$$

with probability density function

$$\varphi(h) = \exp[-h] \exp\left[-\exp[-h]\right],$$

and variance $1/6\pi^2$.

With this distribution it follows from Definition 1, for each voter i, and party, j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u_{ij}^*(x_i, z_j)]}{\sum_{k=1}^{p} \exp u_{ik}^*(x_i, z_k)}. \tag{5}$$

This implies that the model satisfies the independence of irrelevant alternative property (II A): for each individual i, and each pair, j,k, the ratio

$$\frac{\rho_{ij}(\mathbf{z})}{\rho_{ik}(\mathbf{z})}$$

is independent of a third party l (see Train 2003 p. 79).

The formal model just presented, and based on Ψ is denoted as $M(\lambda,\beta;\Psi,\nabla^*)$, though I shall usually suppress the reference to ∇^*.

Definition 5 The convergence coefficient of the model $M(\lambda,\beta;\Psi)$.

(i) *For agent j, define*

$$\rho_j = \left[1 + \sum_{k \neq j} \exp\left[\lambda_k - \lambda_j\right]\right]^{-1} \tag{6}$$

(ii) *The coefficient A_j for party j is*

$$A_j = \beta(1 - 2\rho_j)$$

(iii) *The characteristic matrix for party j at $\mathbf{z}_0$ is*

$$C_j = \left[2A_j\nabla^* - I\right]$$

where I is the w by w identity matrix.

(iv) *The convergence coefficient of the model $M(\lambda,\beta;\Psi)$ is*

$$c(\lambda,\beta;\Psi) = 2\beta\left[1 - 2\rho_1\right]v^2 = 2A_1v^2. \tag{7}$$

It is readily shown that at $\mathbf{z}_0 = (0,\ldots,0)$, the probability, $\rho_{ij}(\mathbf{z}_0)$, that i votes for party j is independent of i. Indeed, $\rho_{ij}(\mathbf{z}_0) = \rho_j$, where ρ_j is given by Definition 5(i).

I now examine the first-and second-order conditions at $\mathbf{z}_0$ for each vote share function, which are necessary and sufficient for local equilibria. The first-order condition is obtained by setting $\mathrm{d}V_j/\mathrm{d}z_j = 0$ (where I use this notation for full differentiation, keeping $z_1,\ldots,z_{j-1},z_{j+1},\ldots,z_p$ constant). This allows us to show that $\mathbf{z}_0$ satisfies the first-order condition. The second-order condition for an LSNE is that the Hessians $\{\mathrm{d}^2V_j/\mathrm{d}z_j^2 : j = 1,\ldots,p\}$ are negative definite at the joint origin. It can be shown that this condition only depends on the characteristic matrix, C_1, of the lowest valence party. This gives the following theorem and corollaries(Schofield 2006a,b).

Theorem 1 *The necessary condition for the joint origin to be a LSNE in the model $M(\lambda,\beta;\Psi)$ is that the characteristic matrix*

$$C_1 = \left[2A_1\nabla^* - I\right] \tag{8}$$

of party 1, with lowest valence, has negative eigenvalues.

The theorem immediately gives the following corollaries.

Corollary 1 *In the case that X is w-dimensional then the necessary condition for the joint origin to be a LNE for the model $M(\lambda, \beta; \Psi)$is that $c(\lambda, \beta; \Psi) \leq w$. Ceteris paribus, a LNE at the joint origin is "less likely" the greater are the parameters β, $\lambda_p - \lambda_1$, and v^2.*

Corollary 2 *In the two-dimensional case, a sufficient condition for the joint origin to be a LSNE for the model $M(\lambda, \beta; \Psi)$ is that $c(\lambda, \beta; \Psi) < 1$. Moreover, the two eigenvalues of C_1 are given by the real numbers*

$$A_1 \left\{ \left[v_1^2 + v_2^2\right] \pm \left[\left[v_1^2 - v_2^2\right]^2 + 4(v_1, v_2)^2\right]^{1/2} \right\} - 1.$$

Even when the sufficient condition is satisfied, so the joint origin is an LSNE, the concavity condition (equivalent to the negative semidefiniteness of all Hessians *everywhere*) is so strong that there is no good reason to expect it to hold.

In the next section, Corollaries 1 and 2 are applied to determine the eigenvalues of the appropriate Hessians for empirical models of elections in the Netherlands and Israel. The empirical analyses of Israel show that the necessary condition fails. In this polity, a PNE, even if it exists, will generally not occur at the origin.

3 Empirical analyses

3.1 The vote maximizing model in the Netherlands

I consider a logit model for the elections of 1977 and 1981 in the Netherlands (Schofield et al. 1998a, Quinn et al. 1999). There are four main parties: Party for Labour (PvdA), Christian Democratic Appeal (CDA), Liberals (VVD), and Democrats (D'66), with approximately 40, 35, 20, and 5% of the popular vote.

Figure 1 gives the estimated positions of the parties and the electoral distribution ca 1980, while Table 1 gives data on the elections of 1977, 1979 and 1981.

For empirical analysis we include sociodemographic variables (SD) such as education, religion, etc. The characteristics of individual i are given by the vector η_i, while the effect of these is given by the transposed vector θ_j^{T}. Thus, it is necessary to change the voter utility from Eq.1 to

$$u_{ij}(x_i, z_j) = \lambda_{ij} - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i. \tag{9}$$

The MNL model with SD is denoted as $M(\lambda,\beta,\theta; \Psi)$.

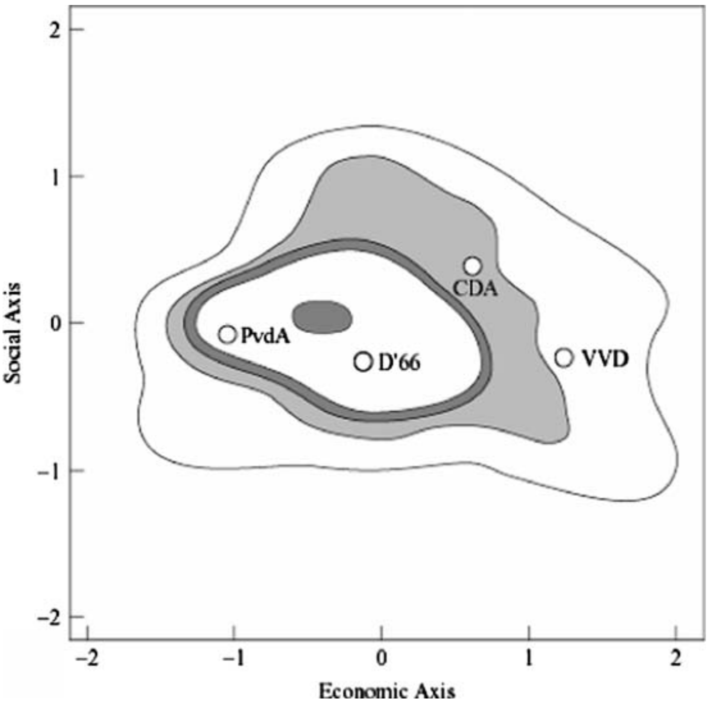


Fig. 1 Party positions and electoral distribution (at the 95, 75, 50, and 10% levels) in the Netherlands, based on 1979 data

Table 1 Election results in the Netherlands, 1977–1981

Party (acronym)	Seats	
	1977	1981
Labour (PvdA)	53	44
Democrats'66(D'66)	8	17
Liberals (VVD)	28	26
Christian Dem Appeal (CDA)	49	48
Subtotal	138	135
Communists (CPN)	2	3
Dem'70 (D70)	1	0
Radicals (PPR)	3	3
Pacific Socialists (PSP)	–	3
Reform Federation (RPF)	–	2
Reform Pol Ass (GDV)	1	1
Farmers Party (BP)	1	0
State Reform Party (SGP)	3	3
Subtotal	11	15
Total	149	150

Table 2 Log likelihoods and eigenvalues in the Dutch electoral model

Model	Coefficient	Eigenvalue 1	Eigenvalue 2	LML
MNL without valence or SD	NA	NA	NA	−606
MNL without valence, with SD	NA	NA	NA	−565
MNL with valence, no SD	1.19	−0.18	−0.64	−531
MNL model with valence and SD	1.38	−0.04	−0.58	−464

Since the SD are presumed unaffected by party movement, we can use the estimated *electoral variance matrix*, ∇^*, coefficient β, and valence vector, λ, to compute the convergence coefficients for the MNL empirical model. These convergence coefficients, empirical parameters, and log marginal likelihoods (LML) are given in Table 2.

The Bayes' factor (or difference in LML) is clearly significant when valence terms are added. Thus, adding valence to the MNL model without SD has a Bayes factor of $75 = [606 - 531]$, while the Bayes factor for adding valence to the MNL with SD is $101 = [565 - 464]$.

I can illustrate the computation of these parameters as follows. The electoral variance on the first axis is $v_1^2 = 0.658$, while on the second it is $v_2^2 = 0.289$. The covariance is negligible.

Using Corollary 2 we can compute the eigenvalues for the MNL estimation for the basic model $M(\lambda,\beta;\Psi)$ without SD. The lowest valence is $\lambda_{\text{d66}} = 0$. The other valances are now $\lambda_{\text{vvd}} = 1.105$, $\lambda_{\text{pvda}} = 2.112$, and $\lambda_{\text{cda}} = 1.140$, while $\beta = 0.737$. Using this valence model we find the probability of voting for the D66, when it is at the origin, to be

$$\rho_{\text{d66}} = \frac{1}{1 + e^{1.596} + e^{1.403} + e^{1.015}} = 0.074.$$

Thus,

$$A_{\text{d66}} = 0.737(0.852) = 0.627,$$
$$C_{\text{d66}} = (1.25)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I = \begin{pmatrix} -0.18 & 0 \\ 0 & -0.64 \end{pmatrix}$$

and

$$c(\Psi) = 1.187.$$

Using the model $M(\lambda,\beta,\theta;\Psi)$ with SD, the valence is changed and we find that the CDA is the lowest valence party, $\lambda_{\text{cda}} = -0.784$. The other valances are now $\lambda_{\text{vvd}} = 0.313$, $\lambda_{\text{pvda}} = 2.112$, and $\lambda_{\text{d66}} = 0$. Using this exogenous valence model we find $\beta = 0.665$. Thus

$$\rho_{\text{cda}} = \frac{1}{1 + e^{2.896} + e^{1.097} + e^{0.784}} = 0.04,$$
$$A_{\text{cda}} = 0.665(0.99) = 0.664,$$
$$C_{\text{cda}} = (1.33)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I = \begin{pmatrix} -0.12 & 0 \\ 0 & -0.61 \end{pmatrix},$$

and

$$c(\Psi) = 1.25.$$

For both models, the two eigenvalues for the lowest valence party are negative. By Theorem 1, the origin is a local equilibrium for the vote maximizing game.

This conflict between the convergence implied by the theorem, and the divergent positions seen in Fig. 1 suggests that the CDA positioned located itself off the first economic or principal electoral axis, in order so as to be better positioned with regard to coalition outcomes. I can illustrate this by the following example of "coalition risk."

3.2 Coalition risk in the Netherlands

In this section, I consider a model where the party principals regard the electoral outcome as uncertain. Instead of basing their policy decisions simply on vote shares, they consider the differing possibilities associated with different coalition structures. As Table 1 indicates, with uncertainty about the elections, there are two essentially different coalition structures in the polity of the Netherlands in the period 1977–1981:

$$\mathcal{D}_0 = \{\text{PvdA,CDA}\}, \{\text{PvdA,VVD}\}, \{\text{CDA,VVD}\}$$
$$\mathcal{D}_1 = \{\text{PvdA,CDA}\}, \{\text{PvdA,VVD,D66}\}, \{\text{CDA,VVD,D66}\}.$$

For purposes of exposition let us consider the simpler case given by the coalition structure $\mathcal{D}_0$. Suppose further that the positions of the three party leaders are given, as in Fig. 2 by $\mathbf{z} = (z_{\text{PvdA}}, z_{\text{VVD}}, z_{\text{CDA}}) = ((-\surd 3, 0), (\surd 3, 0), (0, 1))$. Assume further that the outcome associated with any vector $\mathbf{z}$ of party positions and the coalition structure $\mathcal{D}_0$ will lie within the convex hull of the party positions. For purposes of illustration, let us assume that the party leaders believe that coalition outcomes resulting from $\mathbf{z}$ and $\mathcal{D}_0$ are given by a lottery, $\tilde{g}_0(\mathbf{z})$ that specifies the uniform distribution across the convex hull of the positions given by $\mathbf{z}$. To show why the best response of the CDA may be "radical" suppose the positions of PvdA and VVD are given by $(z_{\text{PvdA}}, z_{\text{VVD}})$ as in Fig. 2. and let us compare the utilities for the CDA at the positions $z^*_{\text{CDA}} = (0, 3)$ and $z_{\text{CDA}} = (0, 1)$. From the symmetry of the figure it follows that the von Neumann–Morgenstern utility function U_{CDA} satisfies the equation

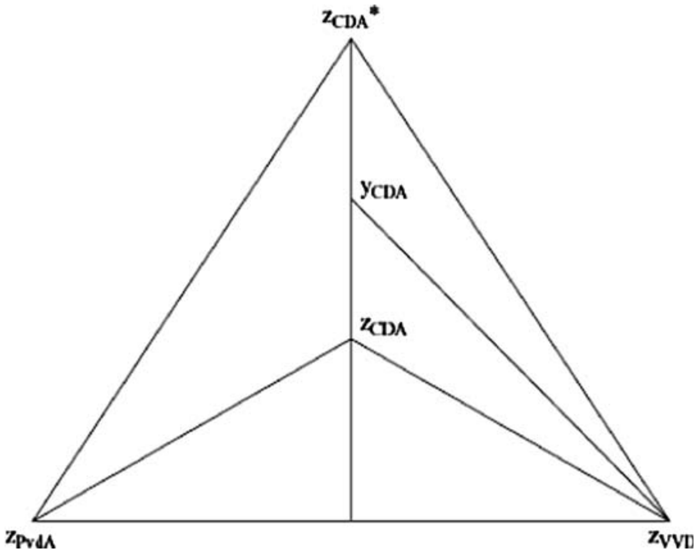


Fig. 2 Coalition risk in the Netherlands at the 1981 election

$$
\begin{aligned}
U_{\mathrm{CDA}}(\tilde{g}_0^0(z_{\mathrm{PvdA}}, z^*_{\mathrm{CDA}}, z_{\mathrm{VVD}}) &= \frac{1}{3} U_{\mathrm{CDA}}(\tilde{g}_0^0(z_{\mathrm{PvdA}}, z_{\mathrm{CDA}}, z_{\mathrm{VVD}}) + \\
&\quad \frac{1}{3} U_{\mathrm{CDA}}(\tilde{g}_0^0(z_{\mathrm{PvdA}}, z_{\mathrm{CDA}}, z^*_{\mathrm{CDA}}) + \\
&\quad \frac{1}{3} U_{\mathrm{CDA}}(\tilde{g}_0^0(z^*_{\mathrm{CDA}}, z_{\mathrm{CDA}}, z_{\mathrm{VVD}}) \\
&= U_{\mathrm{CDA}}(\tilde{g}_0^0(z_{\mathrm{PvdA}}, z_{\mathrm{CDA}}, z_{\mathrm{VVD}})
\end{aligned}
$$

By continuity, there is a position denoted y_{CDA} on the arc $[(0,1),(0,3)]$ which gives the best response of the CDA to $(z_{\mathrm{PvdA}}, z_{\mathrm{VVD}})$. The analysis of the general case of coalition risk is developed further in Schofield and Parks (2000). Assuming that policy leaders have utility functions that involve a Euclidean metric on policy distance they show that non centrist local equilibrium can occur for the profile $(U_{\mathrm{CDA}}, U_{\mathrm{VVD}}, U_{\mathrm{PvdA}})$.

This example suggests that members of a party may choose positions for their leaders in order to obtain more advantageous outcomes from coalition bargaining. I call this phenomenon *the centrifugal effect of coalition risk.*

The next subsection develops this idea to model party positioning in Israel.

3.3 The Israel Knesset in 1992

To further illustrate Theorem 1, consider the case of Israel in 1992. Figure 3 shows the estimated positions of the parties at the time of this election while Table 3 presents election results for 1988–2003.

Using the empirical stochastic model, we can readily show that one of the eigenvalues of the lowest valence part is positive. Indeed it is obvious that

Table 3 Elections in Israel 1988–2003

Party	Knesset seats				
	1988	1992	1996	1999	2003
Left					
Labour (LAB)	39	44	34	28	19[a]
Democrat (ADL)	1	2	4	5	2[a]
Meretz (MZ)	–	12	9	10	6
Arab parties	9	–	–	–	3
Communist (HS)	4	3	5	3	3
Balad	–	–	–	2	3
Subtotal	53	61	52	48	36
Centre					
Olim	–	–	7	6	2[b]
III Way	–	–	4	–	–
Centre	–	–	–	6	–
Shinui (S)	2	–	–	6	15
Subtotal	2		11	18	17[b]
Right					
Likud (LIK)	40	32	30	19	38[b]
Gesher	–	–	2	–	–
Tzomet (TZ)	2	8	–	–	–
Israel Beiteinu	–	–	–	4	7
Subtotal	42	40	32	23	45
Religious					
Shas (SHAS)	6	6	10	17	11
Yahadut (AI)	7	4	4	5	5
NRP (Mafdal)	5	6	9	5	6
Moledet (MO)	2	3	2	4	–
Techiya (TY)	3	–	–	–	–
Subtotal	23	19	25	31	22
Total	120	120	120	120	120

[a] Am Ehad or ADL, under Peretz, combined with Labour, to give the party $19 + 2 = 21$ seats
[b] Olim joined Likud to form one party giving Likud $38 + 2 = 40$ seats, and the right $40 + 7 = 47$ seats

there is a principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. The formal analysis indicates that low valence parties should position themselves on this eigenspace as illustrated in the simulation given below in Fig. 4. The fact that Shas is not located on this axis in Fig. 3 suggests that it is responding to coalition risk.

Using survey data taken from Arian and Shamir (1995), the empirical model gives

$$\lambda_{\text{shas}} = -4.67, \lambda_{\text{likud}} = 2.73, \lambda_{\text{labour}} = 0.91, \beta = 1.25.$$

The (normalized) electoral variance on the first axis is 1.0, while on the second axis it is 0.435, with covariance between the two axes of 0.453. When all parties are at the origin, then we can compute the probability that a voter chooses

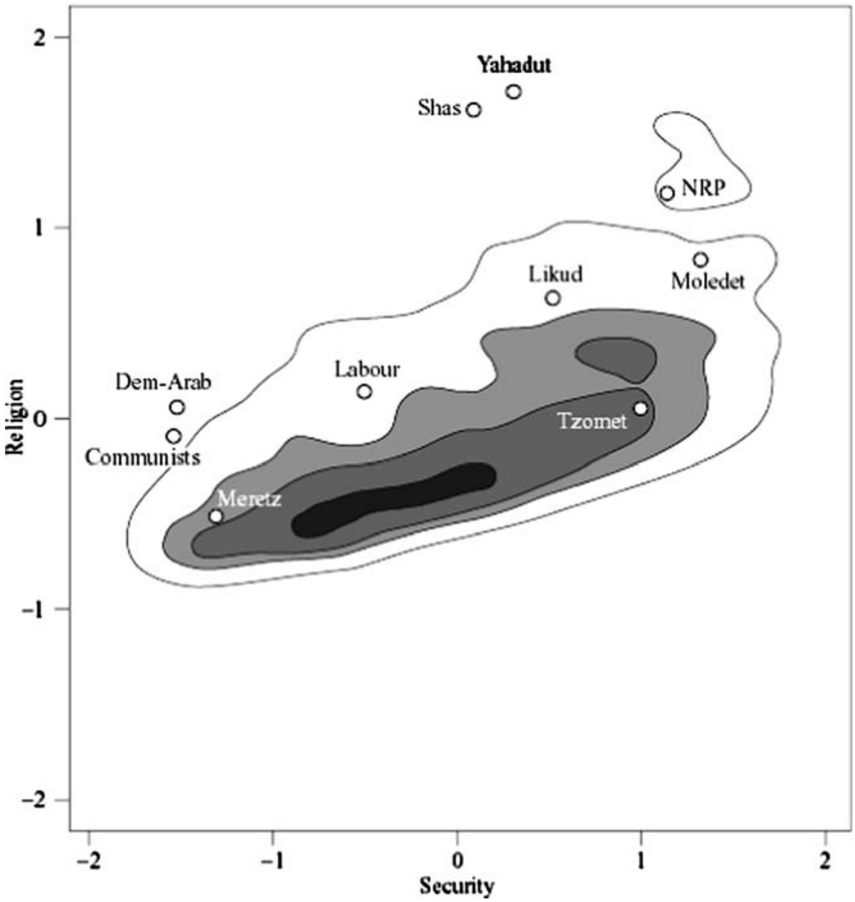
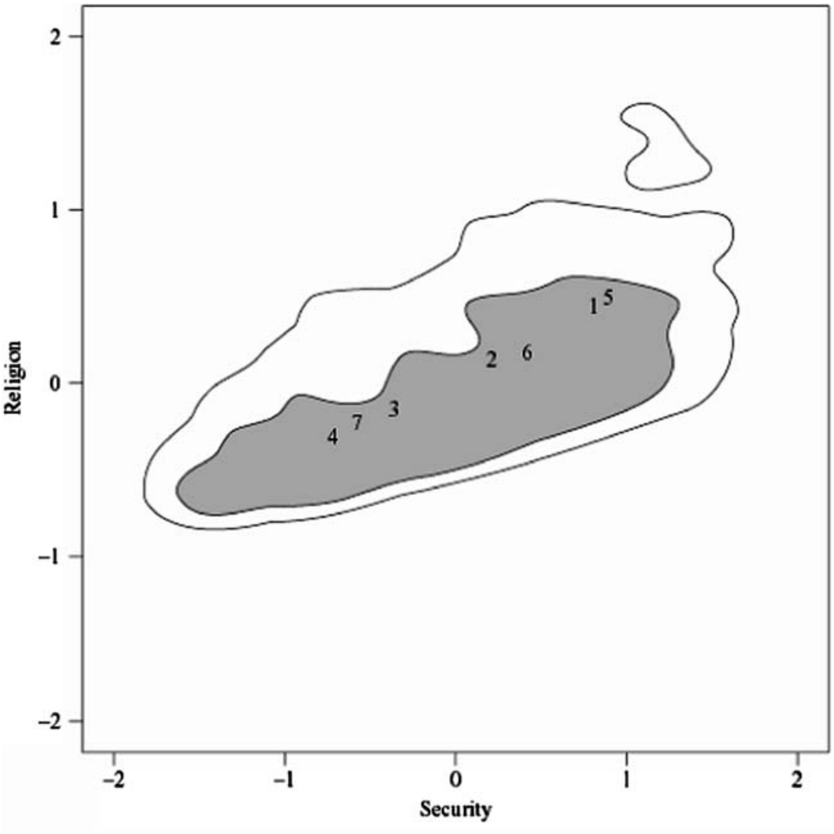


Fig. 3 Electoral distribution (showing contour lines at the 95, 75, 50, and 10% levels) and party positions in the Knesset at the election of 1992

Shas, ρ_{shas}, and the Shas Hessian as follows:

$$\rho_{\text{shas}} \simeq \frac{1}{1 + e^{2.73+4.67} + e^{0.91+4.67}} \simeq 0.$$
$$A_{\text{shas}} = \beta = 1.25.$$
$$C_{\text{shas}} = 2(1.25)\begin{pmatrix} 1.0 & 0.453 \\ 0.453 & 0.435 \end{pmatrix} - I = \begin{pmatrix} 1.5 & 1.13 \\ 1.13 & 0.08 \end{pmatrix}.$$

Then the two eigenvalues for Shas can be calculated to be +2.12 and −0.52 with a convergence coefficient for the model of 3.6. We find that the origin is a saddlepoint for the Shas Hessian. The eigenvector for the positive eigenvalue is the vector (1.0, 0.55). Note that this vector coincides with the principal electoral axis. The eigenvector for the negative eigenvalue is perpendicular to the



Key: 1=Shas, 2=Likud, 3=Labour, 4=Meretz, 5=NRP, 6=Moledet, 7=Tzomet

Fig. 4 A representative local Nash equilibrium of the vote maximizing game in the Knesset for the 1992 election

principal axis. To maximize vote share, Shas should adjust its position but only on the principal axis. Figure 4 presents a simulation of the empirical model to illustrate this inference.

Notice that the two high valence parties, Labour and Likud, were located in Fig. 3 at positions relatively close to the simulated vote maximizing equilibrium positions given in Fig. 4.

It is important to note, however, that the position of Shas in Fig. 3 is not at all similar to its estimated vote maximizing equilibrium position in Fig. 4. This suggests that Shas adopted a position off the principal axis so as to be able to pivot between the coalitions led by either Labour or Likud. In fact, Shas was a coalition ally of Likud until very recently. This strongly suggests that an appropriate model of party positioning assumes that parties are concerned with policy, and adopt positions with a view towards the coalitions that may form after the election. To examine this possibility, I now consider in more detail the nature of coalition formation in the Knesset.

3.4 An example of coalition risk in Israel

Note that, after the 1992 election in Israel, the coalition M_1 ={ Labour, Meretz, ADL, Communist Party} controlled 61 seats while the coalition, M_2 of the remaining parties, including Likud, controlled only 59 seats out of 120. The set of decisive or winning coalitions is called the decisive structure and I write the 1992 decisive structure as

$$\mathcal{D}_{1992} = \{M_1, M_2 \cup \text{Labour}, M_2 \cup \text{Meretz}\}.$$

Since the Labour position z_{labour} in Fig. 3 obviously lies inside the convex hull of the set of positions of parties in any winning coalition, I call z_{labour} the core position, styled $\mathbb{C}_{1992}(\mathbf{z})$, given $\mathcal{D}_{1992}$ and $\mathbf{z}$ (McKelvey and Schofield 1987). Moreover, since this property is insensitive to small perturbations in the positions of the parties I shall say that z_{labour} is structurally stable (Schofield 1985) and denote this by writing $\mathbb{SC}_{1992}(\mathbf{z})$ for the structurally stable core. Now it is possible to find a profile $\mathbf{z}$ with z_{likud} lying inside the convex hull of the positions of the parties in M_1. Such a profile can be regarded as politically infeasible at that time. It therefore follows that Labour would be the uniquely feasible core party under $\mathcal{D}_{1992}$. Say Labour is *dominant* under $\mathcal{D}_{1992}$ with the party positions similar to those given in Fig. 3. To see this, Fig. 5 gives a schematic diagram showing the median lines, drawn through the positions of pairs of parties. (A median line has a majority in both closed half spaces created by the line.) All the median lines in the figure intersect at z_{labour}, which is one way to characterize the core. I use $\mathbb{D}_1$ to denote the set of decisive structures that permit a dominant party. Laver and Schofield (1998) argue that a dominant party is able to form a minority government, even when it fails to have a majority. Indeed, Yitzchak Rabin, leader of Labour in 1992, formed a coalition with Shas so as to create a majority, but shortly after reconstructed a minority government coalition without Shas.

As Table 3 indicates, after the 1996 election the coalition, M_2 controlled 68 seats and so belonged to $\mathcal{D}_{1996}$. Figure 6 gives the profile of estimated party positions in 1996. For this profile, it is obvious that z_{likud} does not lie inside the convex hull of the positions of the parties in M_1 . I therefore assert that, for any politically feasible $\mathbf{z}$, the core $\mathbb{C}_{1996}(\mathbf{z})$ is empty. I use $\mathbb{D}_0$ to denote the set of decisive structures with empty core and infer that $\mathcal{D}_{1996} \in \mathbb{D}_0$.

Prior to the 1996 election there were therefore two qualitatively distinct possible outcomes, namely $\{\mathbb{D}_0, \mathbb{D}_1\}$. Optimal party positions prior to the election of 1996 depend on estimates of the probabilities associated with these outcomes. Under $\mathbb{D}_1$ the outcome will be $\mathbb{SC}_1(\mathbf{z}) = z_{\text{labour.}}$ Since I assume that party principals have policy preferences, the principal of Likud should choose a position

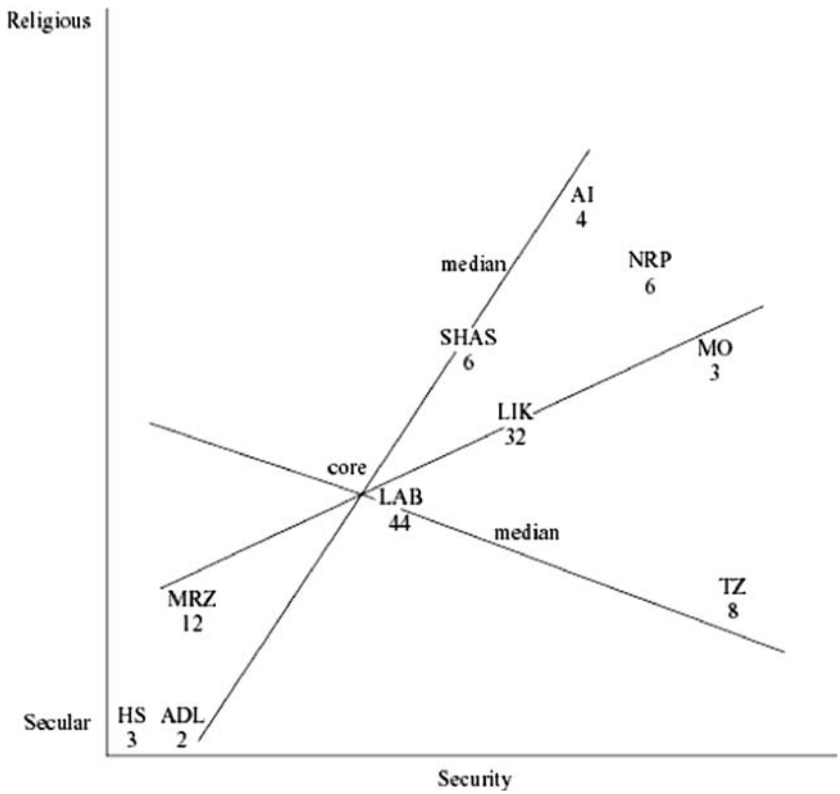


Fig. 5 A schematic representation of the estimated median lines and core in the Knesset after the 1992 election

to minimize $\Pr[\mathbb{D}_1]$, the probability that $\mathbb{D}_1$ ocurs. One obvious way to do this is to choose z_{likud} as a best response in order to maximize its expected vote share.

Now consider the situation under $\mathbb{D}_0$. The model proposed below suggests that the government policy position lies inside the "heart", namely a subset of convex hull of the positions in the coalition M_3 = {Likud, Labour, Shas}. This follows because there are essentially three different possible governments: Labour with Shas, supported by Meretz; Likud with Shas and the other religious parties; and Labour and Likud. The heart is the set bounded by these three medians. As in the previous example from the Netherlands, this suggests that Shas adopt a "radical" position in order to influence coalition outcomes.

To summarize, Labour should adopt a position as a best response in order to maximize $\Pr[\mathbb{D}_1]$, where while Likud should attempt to minimize $\Pr[\mathbb{D}_1]$. As a first approximation, these strategies can be interpreted as maximizing the vote share functions V_{labour}, V_{likud} respectively. For Shas, the optimal position will depend on its estimates of these probabilities. The positions of Shas in 1992 and 1996 appear to be compatible with this interpretation of the motivations of the leader of the party.

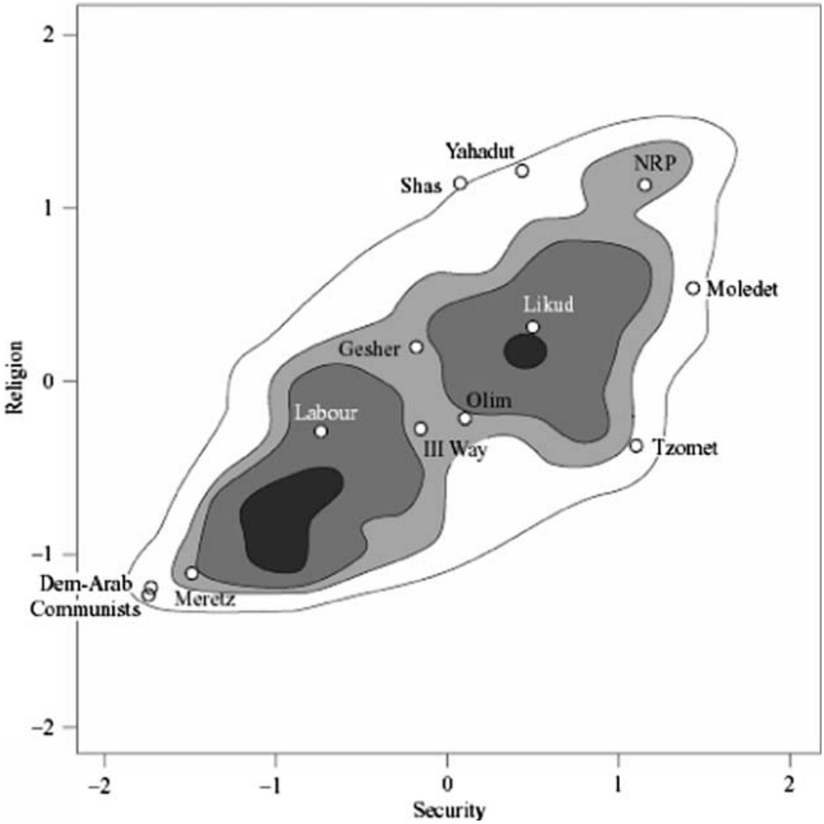


Fig. 6 Electoral distribution (showing contour lines at the 95, 75, 50, and 10% levels) and party positions in the Knesset at the election of 1996

3.5 Recent party positioning in the Knesset

I shall also make some comments with regard to recent changes in the political configuration in Israel to illustrate the logic of this model. After the election of 1996, Netanyahu of Likud formed a governing coalition with the support of Shas. After the election of 1999, this gave way to a Labour led coalition under Ehud Barak, and then a Likud led coalition under Ariel Sharon in 2001. After both elections, the vertices of the heart included the Labour and Shas positions. In 2003, Likud under Sharon gained 38 seats, with the further support of the two seats of Olim. Figure 7 provides a schematic representation of the Knesset after 2003. The median lines in the figure do not intersect, indicating that the policy core is empty. Notice that the heart has changed its configuration, since it based on the convex hull of Shas, Shinui, and Likud. The heart in this figure represents the coalition possibilities open to Sharon.

The figure can be used to understand the consequences after Sharon seemingly changed his policy on the security issue in August 2005, by pulling out

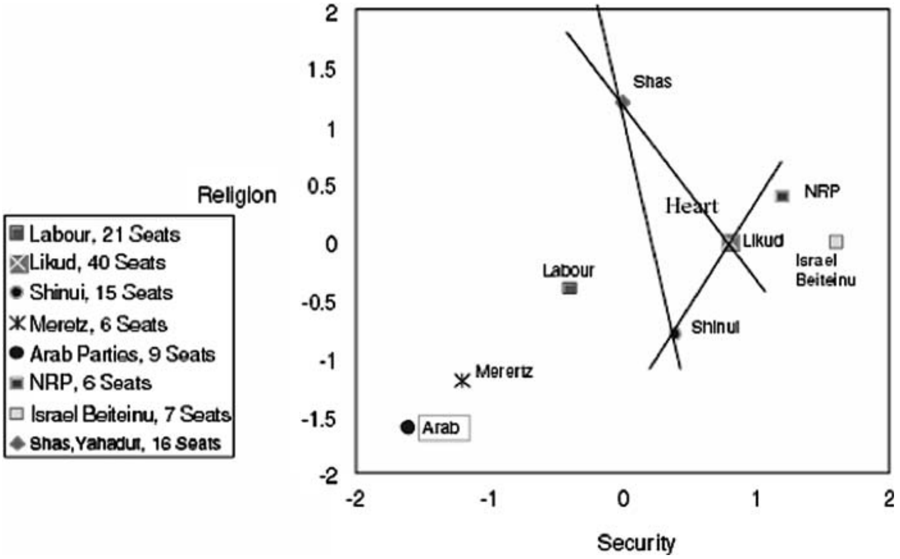


Fig. 7 A schematic representation of party positions in the Knesset after 2003

of the Gaza Strip. First, the Likud party elite and the leaders of the religious parties reacted strongly against this change in policy. On 10 November 2005, Amir Peretz won the election over Peres for leadership of the Labour Party, and this signified a move by the party to the left, down the principal electoral axis. This change may have caused Sharon to leave Likud to set up a new centrist party, Kadima (Forward) in late November. On 1 December 2005, Peres joined Kadima.

Consistent with the model presented here, we can infer that Sharon's intention was to position Kadima near the electoral center on both dimensions, to take advantage of his relatively high valence among the electorate. Sharon's hospitalization in January 2006 had an adverse effect on the valence of Kadima, under its new leader, Ehud Olmert. Even so, in the election of 28 March 2006, Kadima took 29 seats, against 19 seats for Labour, and only 12 for Likud. Because Kadima with Labour and the other parties of the left had 70 seats, Olmert was able to put together a majority coalition on 28 April 2006, including Labour, the pensioners party and Shas. I infer that Shas was still centrist on the security dimension (as in Fig. 8) indicating that this was the key issue of the election. The pensioner's party then joined Kadima, giving it 36 seats. The median lines in Fig. 8 all intersect, indicating that Kadima is at the structurally core position. These changes in party positions appear to have brought back the coalition structure, $\mathbb{D}_1$, this time under a new dominant party.

This brief sketch of coalition politics in Israel suggests that while the major parties are concerned to gain electoral votes, the party leaders are also concerned about policy, and to judge how to choose party positions they must

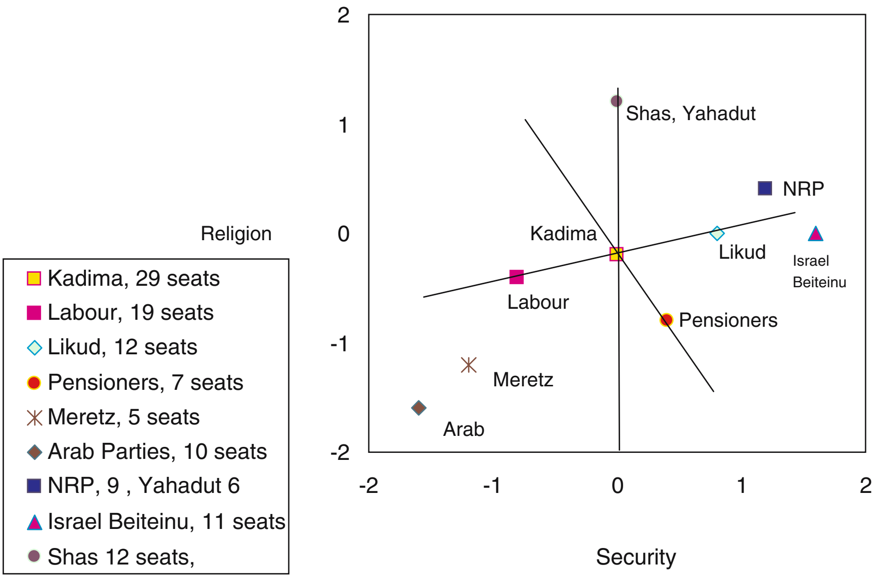


Fig. 8 A schematic representation of party positions in the Knesset after the election of March 28, 2006

estimate the probabilities associated with different electoral outcomes. The next section presents such a model based on the notion of electoral uncertainty.

4 Local equilibria under electoral uncertainty

Using the expected vote share functions as the maximand for the electoral game has its attraction. As we have seen, the expected vote share functions can be readily computed because they are linear functions of the entries in the voter probability matrix $\big(\rho_{ij}(\mathbf{z})\big)$. At least for two-party competition, more natural payoff functions to use are the parties' *probability of victory*. To develop this idea, I introduce the idea of the stochastic vote share functions $\{V_j^*(\mathbf{z}) : j = 1,\ldots,p\}$. Then the expected vote share functions used above are simply the expectations $\{\mathrm{Exp}(V_j^*(\mathbf{z})\}$ of these stochastic variables. In the two-party case, the probability of victory for agents 1 and 2 can be written as

$$\pi_1(\mathbf{z}) = \Pr[V_1^*(\mathbf{z}) > V_2^*(\mathbf{z})] \text{ and } \pi_2(\mathbf{z}) = \Pr[V_2^*(\mathbf{z}) > V_1^*(\mathbf{z})].$$

As Patty (2005) has commented, an agent's probability of victory is a complicated nonlinear expression of the voters' behavior as described by the vote matrix $\big(\rho_{ij}(\mathbf{z})\big)$. Just as it is possible to define LNE and PNE for the game given by the profile function $V : X^p \to \mathbb{R}^p$, we can also define LNE and PNE for the two party profile function $\pi = (\pi_1, \pi_2) : X^2 \to \mathbb{R}^2$. Duggan (2000) and Patty (2005) have explored those conditions under which equilibria for expected vote share functions and probability of victory are identical. As might be expected these equilibria are generically different (Patty 2006).

I shall now develop a model based on electoral uncertainty, which can be considered to be a generalization of the Duggan/Patty models of two-party competition. To do this I introduce the idea of a party principal.

The strategy, z_j, of party j corresponds to the position of the party leader and is chosen by the *party principal*, j, whose preferred position is x_j. I shall develop the model first with only two parties. If party j wins the election with a leader at position $z_j \in X$, while party j receives a non-policy perquisite δ_j, then the payoff to the principal, j, is

$$U_j^{\alpha_j}(z_j, \delta_j) = - \parallel z_j - x_j \parallel^2 + \alpha_j \delta_j$$

Thus, the profile function $U = (U_1, U_2) : X^2 \to \mathbb{R}^2$ can be taken to be given by the expected payoffs

$$\begin{aligned} U_1(z_1, z_2) &= \pi_1(z_1, z_2) U_1^{\alpha_1}(z_1, \delta_1) + \pi_2(z_1, z_2) U_1^{\alpha_1}(z_2, 0) \\ U_2(z_1, z_2) &= \pi_2(z_1, z_2) U_2^{\alpha_2}(z_2, \delta_2) + \pi_1(z_1, z_2) U_2^{\alpha_2}(z_1, 0). \end{aligned}$$

This expression ignores the probability of a draw. In the case of a draw, the outcome can be assumed to be lottery between the party positions z_1 and z_2. As before, I can examine conditions sufficient for existence of LNE or PNE for such a two party profile function. To extend this to a model of *multiparty* competition (with $p \geq 3$) it is necessary to deal with the fact that it is possible for no party to gain a majority of the parliamentary seats (or in the case of US Presidential elections, a majority of the electoral college). I shall argue that in multiparty competition the possible outcomes of the election correspond to the family of all decisive coalition structures

$$\mathbb{D} = \{\mathcal{D}_1, \ldots \mathcal{D}_t, ..\mathcal{D}_T\}$$

which can be obtained from the set P of parties. For convenience, I assume that the subfamily $\mathcal{D}_1, \ldots \mathcal{D}_p$, with $p < T$, corresponds to the subfamily of coalition structures where the parties $1, ..p$, respectively, win the election with a majority of the seats in the parliament. Notice that the outcomes $\{\mathcal{D}_1, \ldots \mathcal{D}_T\}$ are defined in terms of the distribution of seat shares $(S_1, S_2, \ldots S_p)$ in the parliament, and not simply vote shares. The more interesting cases are given by $t > p$, and for convenience I shall assume that for such a t, the coalition structure is $\mathcal{D}_t = \{M \subset N : \Sigma_{j \in M} S_j > 1/2\}$. Decisive coalition structures can of course be defined in more complex ways. Since there is an intrinsic uncertainty in the way votes are translated into seats, it makes sense to focus on the probabilities associated with these decisive structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability that $\mathcal{D}_t$ occurs is denoted by $\pi_t(\mathbf{z})$. I shall assume that the vector

$$(\pi_1(\mathbf{z}), \ldots \pi_p(\mathbf{z}))$$

corresponds to the probabilities that parties 1,..,p, respectively, win the election. When party j wins then the outcome, of course, is the situation $(z_j, 1)$. That is party j implements the position z_j of its party leader and takes a share 1 of non policy perquisites. When no party wins, but a decisive coalition $\mathcal{D}_t$ occurs, for $t \geq p+1$, then the outcome is a lottery which I denote by $\tilde{g}_t^{\alpha}(\mathbf{z})$. I assume $\tilde{g}_t^{\alpha}(\mathbf{z}) \in \tilde{W} = \text{Bor}(X \times \Delta_P)$. Here Δ_P is the set of possible distributions of government perquisites among the parties, and $W = (X \times \Delta_P)$ while $(\text{Bor}(X \times \Delta_P)$ is the space of Borel probability measures over $X \times \Delta_P$ endowed with the weak topology (Parthasarathy 1967). Thus, $\tilde{g}_t^{\alpha}(\mathbf{z})$ specifies a finite lottery of points in X coupled with a lottery of distributions of perquisites among the parties belonging to the decisive structure $\mathcal{D}_t$. (see Banks and Duggan 2000 for a method of deriving this lottery). I implicitly assume that the utility function of the principal of party j, given by the expression $U_j(z_j, \delta_j)$ above, can be regarded as a function $U_j \colon (X \times \Delta_P) \to \mathbb{R}$ and can be extended to a function $\mathbf{U}_j \colon (\text{Bor}(X \times \Delta_P) \to \mathbb{R}$, measurable with respect to the *sigma- algebra* on $\text{Bor}(X \times \Delta_P)$. Note that if $g \in \tilde{W}$, then it is a measure on the Borel *sigma-algebra* of W. Since $U_j : W \to \mathbb{R}$ is assumed measurable the integral $\int \mathbf{U}_j \, dg$ is well defined and can be identified with $\mathbf{U}_j(g) \in \mathbb{R}$. In the weak topology a sequence $\{g_k\}$ of measures converges to g if and only if $\int \mathbf{U} dg_k$ converges to $\int \mathbf{U} dg$ for *every* bounded, continuous utility function $\mathbf{U}$ with domain W. I further assume that $\tilde{g}_t^{\alpha} : X^P \to \tilde{W}$ is C^2-differentiable as well as continuous. This means that for all j the induced function $U_j^t \colon X^P \to \mathbb{R}$, given by $U_j^t(\mathbf{z}) = \mathbf{U}_j(\tilde{g}_t^{\alpha}(\mathbf{z}))$, is also C^2-differentiable, so its Hessian with respect to z_j is everywhere defined and continuous. Observe that $\tilde{g}_t^{\alpha}$ is used to model the common beliefs of the principals concerning the outcome of political bargaining in the postelection situation given by $\mathcal{D}_t$. The common beliefs of the principals concerning electoral outcomes are given by a C^2-differentiable function $\pi : X^p \to \Delta_T$ from X^p to the simplex Δ_T (of dimension $T-1$) where T is the cardinality of the set of all possible coalition structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability is $\pi_t(\mathbf{z})$ that the distribution of parliamentary seats among the parties gives the decisive structure $\mathcal{D}_t$. The *electoral probability function* π models the uncertainty associated with the election. Note that this uncertainty also includes the uncertainty over the valences of the various party leaders. I now provide the formal definitions for the multiparty political game.

Definition 6 The game form derived from policy preferences.

(i) *The electoral probability function* $\pi = (\pi_1, .., \pi_T) : X^p \to \Delta_T$ *is a smooth function from* X^p *to the simplex* Δ_T *(of dimension* $T-1$*) where* $\mathbb{D} = \{\mathcal{D}_1, \ldots, \mathcal{D}_T\}$ *is the set of all possible decisive coalition structures. This function captures the notion of electoral risk.*

(ii) *For fixed* $\mathcal{D}_t$*, the outcome of bargaining at the parameter* $\alpha = (\alpha_1, .., \alpha_p)$ *and at the strategy vector* $\mathbf{z}$ *is a lottery* $\tilde{g}_t^{\alpha}(\mathbf{z}) \in (\text{Bor}(X \times \Delta_P)$. *This captures the notion of coalition risk at* $\mathcal{D}_t$.

(iii) *At the fixed decisive structure, $\mathcal{D}_t$, and strategy vector* $\mathbf{z}$, *the payoff to the principal of party j is*

$$U_j^t(\mathbf{z}) \ = \ \mathbf{U}_j(\tilde{g}_t^{\alpha}(\mathbf{z})).$$

(iv) *The game form* $\{\tilde{g}_t^{\alpha}, \pi_t\}$ *at the parameter* α *is denoted* $\tilde{g}^{\alpha}$. *At the strategy vector* $\mathbf{z}$, *the payoff to the principal j is given by the von Neumann–Morgenstern utility function*

$$U_j^g(\mathbf{z}) = \mathbf{U}_j(\tilde{g}^{\alpha}(\mathbf{z})) = \sum_{t=1,..T} \pi_t(\mathbf{z}) U_j^t(\mathbf{z}).$$

(v) *The game profile derived from the game form* $\tilde{g}^{\alpha}$ *at the utility profile* $\{U_j\}$ *is denoted as*

$$U^g = (\mathbf{U}_1 \circ \tilde{g}^{\alpha}, \ldots, \mathbf{U}_p \circ \tilde{g}^{\alpha}) = (.., U_j^g, ..) \colon X^p \to \mathbb{R}^p.$$

(vi) *The game form* $\tilde{g}^{\alpha}$ *is smooth iff the function* $U^g : X^p \to \mathbb{R}^p$ *is* C^2-*differentiable. Let* $\mathbb{U}(X^p, \mathbb{R}^p)$ *be the set of* C^2-*differentiable utility profiles* $\{U \colon X^p \to \mathbb{R}^p\}$ *endowed with the* C^2 *topology. (Roughly speaking, two profiles are close in this topology if all values and first and second derivatives of each* U_j *are close).*

(vii) *A generic property in* $\mathbb{U}(X^p, \mathbb{R}^p)$ *is one that is true for a set of profiles which is open dense in the* C^2 *topology (See Hirsch 1976; Schofield 2003 for the definition of the* C^2- *topology and the notion of generic property).*

(viii) *For the fixed smooth game form* $\tilde{g}^{\alpha}$, *let* $\{U^g : X^p \to \mathbb{R}^p\} \subset \mathbb{U}(X^p, \mathbb{R}^p)$ *be the set of utility profiles induced as the parameters of voter ideal points and electoral beliefs are allowed to vary.*

(ix) *Let* $\mathbb{G}$ *be the set of smooth game forms. The transformation* $\tilde{g} \to U^g : \mathbb{G} \to \mathbb{U}(X^p, \mathbb{R}^p)$ *induces a topology on the set* $\mathbb{G}$, *where this topology is obtained by taking the coarsest topology such that this transformation is continuous.*

(x) *The vector* $\mathbf{z}^* = (z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^* .. z_p^*) \in X^p$ *is a local strict Nash equilibrium (LSNE) for the profile* $U \in \mathbb{U}(X^p, \mathbb{R}^p)$ *iff for each j there is a neighborhood* X_j *of* z_j^* *in X, with the property that*

$$U_j(z_1^*, \ldots, z_j^*, z_{j+1}^*, \ldots, z_p^*) > U_j(z_1^*, \ldots, z_j, z_{i+1}^*, \ldots, z_p^*)$$

for all $z_j \in X_j - \{z_j^*\}$.

(xi) $\mathbf{z}^* \in X^p$ *is a critical Nash equilibrium (CNE) for the profile U iff, for each j, the first order condition* $\mathrm{d}U_j/\mathrm{d}z_j = 0$ *is satisfied at* $\mathbf{z}^*$.

(xii) *A strict Nash Equilibrium (PSNE) for U is a LSNE for U with the additional requirement that each* X_j *is in fact X.*

(xiii) *For a fixed profile $x \in X^n$ of voter ideal points, fixed electoral beliefs π, and fixed game form g, the vector $\mathbf{z}^*$ is called the LSNE, PSNE, or CNE if it satisfies the appropriate condition for the game profile $U^g : X^p \to \mathbb{R}^p$.*

(xiv) *An LSNE $\mathbf{z}^* \in X^p$ for the profile U is locally isolated iff there is a neighborhood Z^* of $\mathbf{z}^*$ in X^p which contains no LSNE for U other than $\mathbf{z}^*$.*

Schofield (2001) shows that, for each parameter, α, there is an open dense set of smooth game forms, with the property that each form $\tilde{g}^\alpha$ in the set exhibits a CNE. In principle, this result suggests that if the electoral function is smooth, and if the outcome of coalition bargaining is differentiable in the location of parties, then there will exist local equilibria which can be used to deduce party positions. Of course, this model is very much more complex than the vote maximizing version presented in the previous section.

For the result to be valid, I require that the strategy space X^p is compact convex subset of a finite dimensional topological vector space. Such a space I shall call a *Fan space* (Fan 1964) I also require the following boundary condition on the profile. Say a profile $U \in \mathbb{U}(X^p, \mathbb{R}^p)$ satisfies the *boundary condition* iff for every point $\mathbf{z}$ on the boundary of the Fan space, X^p, the induced gradient $(\mathrm{d}U_1/\mathrm{d}z_1, ..\mathrm{d}U_p/\mathrm{d}z_p)$ points towards the interior of X^p. Let $\mathbb{U}_b(X^p, \mathbb{R}^p)$ be the subspace of profiles obtained from varying the parameters and from different sets of voter ideal points, but all satisfying this boundary condition.

Theorem 2 *Assume X is a Fan space and p is finite. Then the property that the LSNE exists and is locally isolated is generic in the topological space $U_b(X^p, R^p)$.*

This theorem suggests that if we consider any fixed game form $\tilde{g}$, then existence of locally isolated LSNE is a generic property in the space $\{U^g : X^p \to \mathbb{R}^p\} \subset \mathbb{U}_b(X^p, \mathbb{R}^p)$. Moreover, if the transformation $\mathbb{G} \to \mathbb{U}(X^p, \mathbb{R}^p)$ (from game forms to utility profiles) is well behaved, in the sense that open sets are transformed to open sets, then continuity of the transformation would imply that existence of LSNE is a generic property in the space $\mathbb{G}$.

5 The core and the heart

In the previous section, I assumed that the outcome of bargaining between the party leaders could be described by a lottery $\tilde{g}_t^\alpha(\mathbf{z})$, determined by the vector $\mathbf{z}$ of positions of party leaders. The analysis of Banks and Duggan (2000) indicated that in general this outcome would coincide with the *core* of the coalition game determined by the postelection decisive structure $\mathcal{D}_t$ and the vector $\mathbf{z}$. To develop this idea further I now give the formal definitions of the core and other solution concepts based on social choice theory.

Definition 7 Concepts of Social Choice Theory

(i) *A (strict) preference Q on a set, or space, W is a correspondence $Q\colon W \to 2(W)$ where $2(W)$ stands for the family of all subsets of W (including the empty set ϕ). Again, I assume W is a Fan space.*

(ii) *Let* $Q : W \to 2(W)$ *be a preference correspondence on the space* W. *The choice of* Q *is*

$$\mathbb{C}(Q) = \{x \in W : Q(x) = \phi\}.$$

(iii) *The covering correspondence* Q^* *of* Q *is defined by* $y \in Q^*(x)$ *iff* $y \in Q(x)$ *and* $Q(y) \subset Q(x)$. *Say* y *covers* x. *The uncovered set,* $\mathbb{C}^*(Q)$ *of* Q, *is*

$$\mathbb{C}^*(Q) = \mathbb{C}(Q^*) = \{x \in W : Q^*(x) = \phi\}.$$

(iv) *If* W *is a topological space, then* $x \in W$ *is locally covered (under* Q*) iff for any neighborhood* Y *of* x *in* W, *there exists* $y \in Y$ *such that*

$$y \in Q(x) \text{ and } Y \cap Q(y) \subset Y \cap Q(x).$$

If x *is not locally covered, then write* $Q^{**}(x) = \phi$.

(v) *The heart of* Q, *written* $\mathcal{H}(Q)$, *is defined by*

$$\mathcal{H}(Q) = \{x \in W : Q^{**}(x) = \phi\}.$$

A preference Q is convex iff for all x, the preferred set $Q(x)$ of x is strictly convex. In general if $\mathbb{C}(Q)$ is non-empty, then it is contained in both $\mathbb{C}^*(Q)$ and $\mathcal{H}(Q)$. It can be shown that if $\mathbb{C}(Q) \neq \phi$ and $Q' \to Q$ in an appropriate topological sense, then it is possible to find a sequence $\{z^s \in \mathcal{H}(Q')\}$ such that $\{z^s\}$ converges to some point in the core, $\mathbb{C}(Q)$.

Now let $\text{CON}(W)^P$ stand for all "smooth" convex preference profiles for the set of political agents $P = \{1, \ldots, p\}$. Thus, $q \in CON(W)^P$ means $q = (q_1, \ldots, q_p)$ where each $q_j : W \to 2(W)$ is a convex preference, whose indifference surfaces are smooth. In particular, this means we can represent the preference profile q by a C^2-utility profile $U \in \mathbb{U}(W, \mathbb{R}^p)$. Let rep:$\text{CON}(W)^P \to \mathbb{U}(W, \mathbb{R}^p)$ be the representation map.

Definition 8 The heart and the uncovered set

(i) *Let* $\mathcal{D}$ *be a fixed set of decisive coalitions and* W *be a Fan space. Let* $q \in \text{CON}(W)^P$ *be a smooth preference profile. Define*

$$\sigma_{\mathcal{D}}(q) = \cup_{M \in \mathcal{D}} \{\cap_{i \in M} q_i\} : W \to 2(W)$$

to be the preference correspondence induced by $\mathcal{D}$ *at the profile* q. *The core of the political game given by* $\mathcal{D}$ *at* q, *written* $\mathbb{C}_{\mathcal{D}}(q)$, *is* $\mathbb{C}(\sigma_{\mathcal{D}}(q))$.

(ii) *The heart of* $\mathcal{D}$ *at* q, *written* $\mathcal{H}_{\mathcal{D}}(q)$, *is defined to be* $\mathcal{H}(\sigma_{\mathcal{D}}(q))$. *The uncovered set of* $\mathcal{D}$ *at* q, *written* $\mathbb{C}^*_{\mathcal{D}}(q)$, *is* $\mathbb{C}^*(\sigma_{\mathcal{D}}(q))$.

(iii) *The Pareto set of the profile* q *is* $\mathbb{C}_P(q) = \mathbb{C}(\sigma_P(q))$ *where*

$$\sigma_P(q) : \{\cap_{i \in P} q_i\} : W \to 2(W)$$

is the Pareto, or strict unanimity, preference correspondence.

(iv) *A correspondence $Q : W \to Z$ is lower hemi continuous (lhc) with respect to topologies on W, Z iff for any open set $Y \subset Z$ the set*

$$\{x \in W : Q(x) \cap Y \neq \phi\}$$

is open in W.

(v) *A continuous selection g for Q is a function $g : W \to Z$, continuous with respect to the topologies on W, Z such that $g(x) \in Q(x), \forall x \in W$, whenever $Q(x) \neq \phi$.*

(vi) *A correspondence $\mathcal{H}$: $\mathrm{CON}(W)^P \to 2(W)$ is called C^2-lower hemi continuous (C^2-lhc) if the map*

$$\mathcal{H}\circ\mathrm{rep}^{-1} : \mathbb{U}(W, \mathbb{R}^p) \to \mathrm{CON}(W)^P \to 2(W)$$

is also lhc with respect to the C^2-topology on $\mathbb{U}(X, \mathbb{R}^p)$.

Schofield (1999a, b, c) has shown that the heart is non-empty, Paretian and C^2-lhc. The heart correspondence can then be shown to admit a continuous selection (Michael 1956).

Theorem 3 summarizes the technical properties of the heart correspondence.

Theorem 3 *Let W be a Fan space, and $\mathcal{D}$ any voting rule. Then $\mathcal{H}_\mathcal{D}$: $\mathrm{CON}(W)^P \to 2(W)$ is C^2-lhc. Moreover, for any $q \in \mathrm{CON}(W)^P$, the set $\mathcal{H}_\mathcal{D}(q)$ is closed, non-empty and is a subset of the Pareto set $\mathbb{C}_P(q)$. Moreover $\mathcal{H}_\mathcal{D}$ admits a continuous selection $g_\mathcal{D}$: $\mathrm{CON}(W)^P \to W$ such that $g_\mathcal{D}(q) \in \mathbb{C}(\sigma_\mathcal{D}(q))$ whenever $\mathbb{C}(\sigma_\mathcal{D}(q))$ is non-empty. Indeed, $g_\mathcal{D}$ can be factored to give a C^2 -differentiable map*

$$g_\mathcal{D}\circ\mathrm{rep}^{-1} : \mathbb{U}(W, \mathbb{R}^p) \to \mathrm{CON}(W)^P \to W.$$

This last property means that if U is a C^2-differentiable profile then the induced profile $U \circ g_\mathcal{D}$ is also C^2-differentiable.

For convenience, I say $g_\mathcal{D}$ is a smooth *Paretian selection which converges to the core.*

To use Theorem 3 to model coalition bargaining, I assume as before that the preferred position of the *leader* (or *agent*) for party j determines the declaration z_j of the party. I shall assume that the outcome of bargaining is an element of $W = (X \times \Delta_P)$, namely a policy choice x and a distribution $(\delta_1, \ldots \delta_p)$ of the total perquisites. Thus, the leader of party j is described by the utility function

$$U_j^{\alpha_j}(z_j, \delta_j) = - \| z_j - x_j \|^2 + \alpha_j \delta_j.$$

This implies that the leader can be described by a smooth, strictly convex preference correspondence $q_j^{\alpha_j}(z_i) : X \times \Delta_P \to X \times \Delta_P$. Let $\alpha = (\alpha_1, .., \alpha_p)$, $\mathbf{z} = (z_1, \ldots, z_p)$ and $q^\alpha(\mathbf{z})$ denote the profile of leader preferences. The Pareto set $\mathbb{C}_P(q^\alpha(\mathbf{z}))$ in $X \times \Delta_P$ is the unanimity choice of this preference profile. As in the previous section, I now consider a family $\mathbb{D} = \{\mathcal{D}_1, \ldots, \mathcal{D}_t, \ldots, \mathcal{D}_T\}$ of decisive coalitions. I call each set, $\mathcal{D}_t$ the *voting rule induced by the election.* For each $\mathcal{D}_t$, I define the heart of the voting rule on the space $W = X \times \Delta_P$ as $\mathcal{H}_{\mathcal{D}_t}(q^\alpha(\mathbf{z})$. This set I write as $\mathcal{H}_t^\alpha(\mathbf{z})$. I write the core $\mathbb{C}(\sigma_{\mathcal{D}_t}(q^\alpha(\mathbf{z})))$ as $\mathbb{C}_t^\alpha(\mathbf{z})$. Theorem 3 can then be applied to show that each correspondence $\mathcal{H}_t^\alpha$ is C^2- lhc and admits a C^2-selection which converges to the core, $\mathbb{C}_t^\alpha(\mathbf{z})$. The family of correspondences $\{\mathcal{H}_t^\alpha\}$ I write as $\mathcal{H}_{\mathbb{D}}^\alpha$.

To extend these concepts to the situation where the electoral outcome is a lottery, I again use the definition of $\tilde{W} = \mathrm{Bor}(X \times \Delta_P)$, the set of all lotteries over $X \times \Delta_P$, endowed with the weak topology. Now let $\tilde{\mathcal{H}}_t^\alpha : X^p \to 2(\tilde{W})$ be the extension of the heart correspondence to this space, so $\tilde{\mathcal{H}}_t^\alpha(\mathbf{z})$ is the set of lotteries over the set $\mathcal{H}_t^\alpha(\mathbf{z})$ with the induced topology. Then lhc of $\mathcal{H}_t^\alpha$ implies lhc of $\tilde{\mathcal{H}}_t^\alpha$ (Schofield 1999a, b).

Theorem 4 *For a fixed voting rule, $\mathcal{D}_t$, there exists a smooth selection $\tilde{g}_t^\alpha : X^p \to \tilde{W}$ of the correspondence $\tilde{\mathcal{H}}_t^\alpha : X^p \to 2(\tilde{W})$,which converges to the core.*

As in the previous section, $\tilde{g}_t^\alpha$ is meant to capture the notion of *coalition risk* at the vector $\mathbf{z}$ of party positions and at the decisive structure $\mathcal{D}_t$. Convergence to the core is intended to capture the following logic. If the core $\mathbb{C}_t^\alpha(\mathbf{z})$ is non-empty, then the selection $\tilde{g}_t^\alpha(\mathbf{z})$ must put all probability weight on this set, guaranteeing that this is the outcome. In such a situation there is no coalition risk.

I now repeat the analysis of the previous section for the case of a game form $\tilde{g}^\alpha = \{\tilde{g}_t^\alpha, \pi_t\}$ obtained as a selection from the heart correspondence. First let K be some compact convex subset of $\mathbb{R}^p$ for the parameters α, and let $\tilde{g}$ be a general game form that specifies the game form $\tilde{g}^\alpha = \{\tilde{g}_t^\alpha, \pi_t\}$ for each $\alpha \in K$.

Definition 9 *The game form $\tilde{g}$, which specifies $\{\tilde{g}_t^\alpha, \pi_t\}$ at $\alpha \in K$ is heart compatible over K iff each component $\tilde{g}_t^\alpha : X^p \to \tilde{W}$ is a smooth selection of the heart correspondence $\tilde{\mathcal{H}}_t^\alpha$ $X^p \to 2(\tilde{W})$.*

Theorem 5 *There exists a game form $\tilde{g}$ which is heart compatible and has the following property: If the induced utility profiles are given by $\{U^g : X^p \to R^p\}$ then there is an open dense set in*

$$\{U^g : X^p \to \mathbb{R}^p\} \cap \mathbb{U}_b(X^p, \mathbb{R}^p)$$

such that each profile in this set exhibits a locally isolated LSNE.

In applying this theorem, it will prove useful to consider the notion of a *structurally stable core* for the particular case when non policy perquisites are zero.

Definition 10 *Consider the case $\alpha = (\alpha_1, .., \alpha_p) = (0, \ldots, 0)$. If the core $\mathbb{C}_t^0(\mathbf{z})$ at $\mathbf{z}$ and D_t is non-empty then it is said to be structurally stable if, for any $x \in \mathbb{C}_t^0(\mathbf{z})$, there exists a neighborhood Z^* of $\mathbf{z}$ in X^p and a neighborhood X^* of x in X such that $X^* \cap C_t^0(\mathbf{z}^*) \neq \phi$ for all $\mathbf{z}^* \in Z^*$.*

When the core at $\mathbf{z}$ and $\mathcal{D}_t$ is structurally stable then it is denoted as $\mathbb{SC}_t^0(\mathbf{z})$.

In other words, the *core* $\mathbb{C}_t^0(\mathbf{z})$ is structurally stable if a small arbitrary perturbation of the profile $\mathbf{z}$ simply perturbs the location of the core. The symmetry conditions developed by McKelvey and Schofield (1986, 1987) allow us to determine when a core is structurally stable. In general, these symmetry conditions are easiest to use when the policy core coincides with the position of a party.

The preliminary discussion in Sect. 3 suggests that an empirically relevant theory of party behavior can be based on Theorem 5.

6 Concluding remarks

It is now possible to draw some general conclusions about party government formation from the formal models presented in the previous sections of this paper.

1. The stochastic model suggests that low valence parties will tend to be subject to a centrifugal tendency. When the political system is based on proportional representation, then coalition formation involving agreement between high valence, centrist parties and low valence parties may lead to a degree of instability.
2. This tendency can be overcome if the valence of leader of one of the centrist parties is sufficiently high that it can adopt a core position in the policy space. As the example of Sharon indicates, this may necessitate a reconfiguration of the party structure.
3. Because low valence parties may resent the dominance of a core party, they may adjust position in an attempt to destroy the core configuration.

This paper has used the notion of *local Nash equilibrium.* Equilibrium positions will be sensitive to the beliefs that sustain them. It is therefore possible that small changes in beliefs about the electoral response function and the coalition expectations, represented by $\tilde{g}^{\alpha}$, may lead to political moves that destroy the local equilibrium. The recent changes in the political configuration in Israel suggest that small electoral shifts in beliefs can have dramatic consequences.

This suggests that we extend the notion of a structurally stable core, given in Definition 10, and define a *structurally stable LNE* (or *ss*LNE) to be an LNE that is impervious to small perturbations in the set of beliefs. The discussion of the election of March 2006 in Sect. 3 indicates that the creation of the new dominant party, Kadima, may have brought into existence a structurally stable local equilibrium.

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