Elections and Activist Coalitions in the United States

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Formal models of voting have emphasizd the mean voter theorem, that all parties should rationally adopt identical positions at the electoral mean. The lack of evidence for this assertion is a paradox or contradiction in need of resolution. This article attempts to resolve this paradox by considering an electoral model that includes "valence" or nonpolicy judgements by voters of party leaders. The theorem is used to suggest that Republican success depends on balancing the opposed demands of economic and social conservatives. Democrat success in future elections resides in overcoming the policy demands of economic liberals and gaining support from cosmopolitans—the socially liberal but economically conservative potential supporters of the party.

Models of elections tend to give two quite contradictory predictions about the nature of political competition. In two-party competition, if the "policy space" involves two or more independent issues, then "pure strategy Nash equilibria" generally do not exist, and it is presumed that instability of some kind is possible.[1] That is to say, whatever position is picked by one party there always exists another policy point which will give the second party a majority over the first.

On the other hand, the earlier electoral models based on the work of Hotelling (1929) and Downs (1957) suggest that parties will converge to an electoral center (at the electoral *median*) when the policy space has a single dimension. Although a pure strategy Nash equilibrium generically fails to exist in competition between two agents under majority rule in high enough dimension, there will exist mixed strategy equilibria whose support is located near the electoral center.[2] These various and contrasting theoretical results are in need of resolution: Will democracy tend to generate centrist compromises, or can it lead to disorder?

Partly as a result of these theoretical difficulties with the "deterministic" electoral model, and also because of the need to develop empirical models of voter choice, attention has focused on "stochastic" vote models. A formal basis for such models is provided by the notion of "quantal response equilibria" (McKelvey and Palfrey 1995). In such models, the behavior of each voter is characterized by a vector of choice probabilities determined by the candidate positions.[3] A standard result in this class of models is that all parties converge to the electoral origin when the parties are motivated to maximize vote share or plurality (in the two-party case).[4] The predictions as regards convergence are at odds with the general perception that the principal parties in the United States implement very different policies when in office.

The focus of this article is the apparent paradox that actual political systems display neither *chaos* nor *convergence*. The key idea is that the convergence result need not hold if there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes 1992). The average weight given to the perceived quality of the leader of the

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[1] See Plott (1967), McKelvey (1976, 1979), McKelvey and Schofield (1987), Saari (1997), and Austen-Smith and Banks (1999).

[2] See McKelvey (1986) and Banks, Duggan, and LeBreton (2002).

[3] Hinich (1977), Enelow and Hinich (1984, 1989), and Coughlin (1992), and Lin, Enelow, and Dorussen (1999). For an empirical comparison of deterministic and probabilistic models, see Burden (1997).

[4] See McKelvey and Patty (2006) and Banks and Duggan (2005).

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j^{th} party is called the party's *valence*. In empirical models, a party's valence is usually assumed to be independent of the party's position and adds to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past or is likely to be able to govern well in the future (Penn 2003).

The early empirical work by Poole and Rosenthal (1984) on U.S. presidential elections included these valence terms and noted that there was no evidence of candidate convergence. Formal models of elections incorporating valence have been developed recently, but the theoretical results to date have generally been restricted to the case of a one-dimensional policy space and two parties.[5]

This article offers a general model of elections based on the assumption that there are two kinds of valence. The first kind is a fixed or *exogenous valence*, which for a party j is denoted λ_j. As in empirical work, we assume that λ_j is held constant at the time of an election and so is independent of the party's position. The second kind of valence is known as *activist valence*. When party j adopts a policy position z_j, in the policy space, X, then the activist valence of the party is denoted $\mu_j(z_j)$. Implicitly we adopt a model originally due to Aldrich (1983a, 1983b, 1995) and Aldrich and McGinnis (1989). In this model, activists provide crucial resources of time and money to their chosen party, and these resources are dependent on the party position.[6] The party then uses these resources to enhance its image before the electorate, thus affecting its overall valence. Although activist valence is affected by party position, it does not operate in the usual way by influencing voter choice through the distance between a voter's preferred policy position, say x_i, and the party position. Rather, as party j's activist support, $\mu_j(z_j)$, increases due to increased contributions to the party in contrast to the support $\mu_k(z_k)$ received by party k, then (in the model) all voters become more likely to support party j over party k.[7]

The problem for each party is that activists are likely to be more extreme than the typical voter. By choosing a policy position to maximize activist support, the party will lose centrist voters. The party must therefore determine the "optimal marginal condition" to maximize vote share. The Theorem presented below gives this as a (first order) *balance condition*. Moreover, because activist support is denominated in terms of time and money, it is reasonable to suppose that the activist function will exhibit decreasing returns. For example, in an extreme case, a party that has no activist support at all may benefit considerably by a small policy move to favor a particular interest group. On the other hand, when support is very substantial, then a small increase due to a policy move will little affect the electoral outcome. For this reason we consider that it is reasonable to assume that the functions themselves are concave, so their Hessians are everywhere negative-definite. The Theorem points out that when these functions are sufficiently concave, then the vote-maximizing model will exhibit a Nash equilibrium.

It might be objected that a model based on maximizing probability of winning the election is to be preferred to one based on vote share maximization. It is certainly true that the equilibria of the two models will differ.[8] However, even existence of equilibria in the former class of models is extremely difficult. The vote-maximizing model presented here has the virtue that it leads to definite answers about the likelihood of convergence, in the general multiparty situation, even with three or more parties. A corollary of the theorem shows that when activist valence is identically zero, then there is a necessary condition which must be satisfied for the model to exhibit an equilibrium where all parties locate at the electoral origin. As we discuss in the next section for the two-party case, when this condition is violated, then electoral logic will force one of the parties to move away from the origin so as to increase vote share. Such a move, by creating asymmetry between the parties, is the trigger for differential activist support for the parties.

We see a further virtue of the model to be its emphasis on the importance of electoral valence. It is intrinsic to the model that voters evaluate candidates not only in terms of the voters' preferences over intended policies, but also in terms of judgements over their capacity to carry out these policies. While the model is relatively simple, it turns out that the analysis of the trade-off between preference and judgement is technically quite demanding. Even so, the model suggests fruitful lines of research over the formation of electoral judgements and over the logic of activist support.

In the next section, we use the activist vote-maximizing model in an informal fashion to explain the partisan realignment that has occurred in U.S. politics since 1960. This section has the additional purpose of suggesting that neither chaos nor convergence is a likely phenomenon. The section following presents the formal

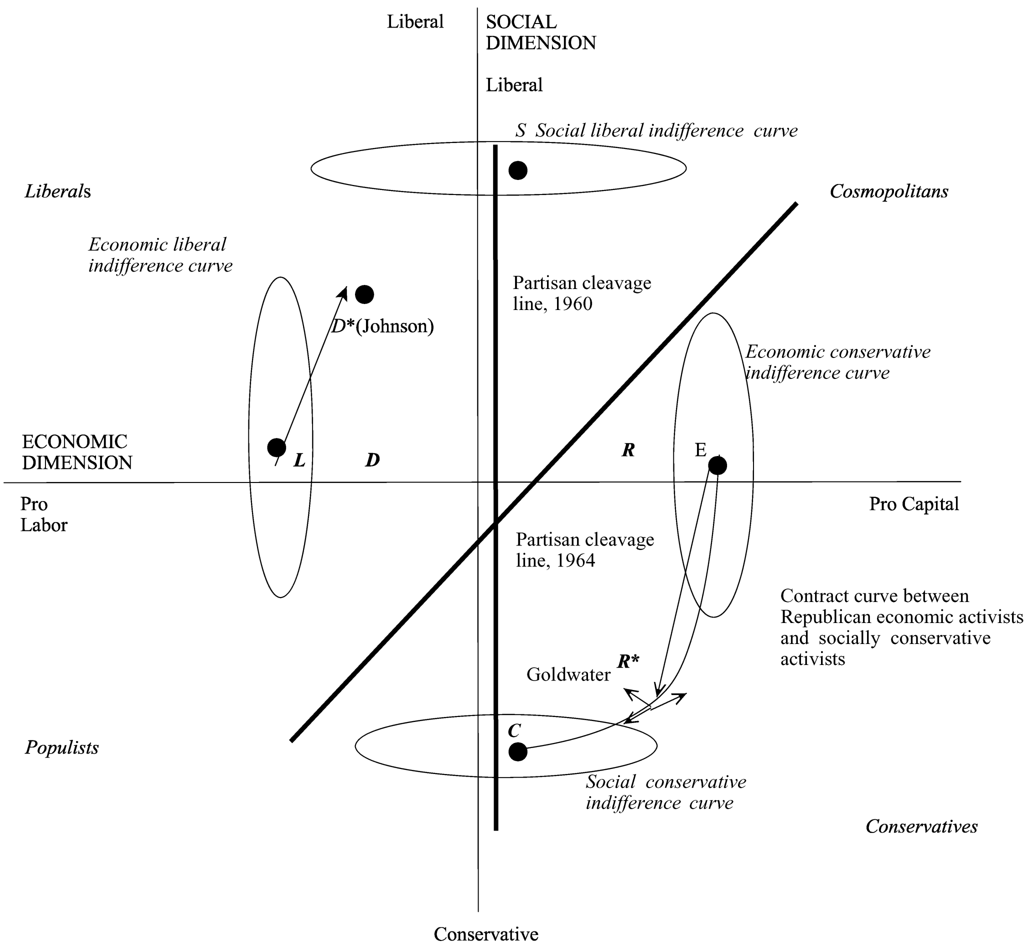
[5]Ansolabehere and Snyder (2000), Aragones and Palfrey (2002), Groseclose (2001), and Londregan and Romer (1993).

[6]For convenience, we assume that $\mu_j(z_j)$ is only dependent on z_j, and not on z_k, $k \neq j$, but this is not a crucial assumption.

[7]In other words, it is not the source of the resources that matters, just the amount.

[8]Duggan (2000), Patty (2002, 2005, 2007).

FIGURE 1 Illustration of Policy Moves by Republican and Democratic Candidates, in a Two-Dimensional Policy Space



model in detail, together with a statement of the Theorem and its Corollary, and examples. A concluding section draws out some inferences for the nature of political competition.

Activist Support for the Parties

To illustrate the stochastic model, consider Figure 1, and suppose first that the economic dimension alone is relevant for political policymaking. We assume that there is an electoral distribution of voter ideal points, whose mean we take as the electoral origin. If we ignore activism for the moment, then the results of the following section show that there are two very different possibilities, depending on the parameters of the model. There is a "convergence coefficient" (labelled c) defined by the valences, λ_{dem} and λ_{rep}, together with the variance of the electoral distribution and the spatial coefficient, β. (This coefficient determines the importance of policy in the utility functions of voters.) If the valences are sufficiently similar (as expressed by an inequality given in terms of c), then both parties will position themselves at the electoral origin, and both will gain about 50% of the vote. In this case, potential activists are unlikely to be motivated to contribute to the parties. As long as $c \leq 1$, then this convergent situation is stable.

On the other hand, if the valences differ, with $\lambda_{dem} > \lambda_{rep}$, say, and if the electoral variance and β are both sufficiently large, so $c > 1$, then the lower valence candidate will vacate the origin in order to increase vote share. For purposes of exposition, we may suppose that conservative economic activists have the preferred position, E. If the Republican candidate moves away from the origin, to a position similar to R, then economic conservative activists would be induced to support this candidate. The asymmetry induced by this support will cause liberal economic activists at L to support the Democratic candidate. Then

R will be pulled further towards E, while D will be pulled towards L. Moreover, if the marginal effect of activists for the Republicans is greater than for the Democrats, then the optimal candidate positions, R and D, will satisfy $|R| > |D|$. This model implies that once the convergent equilibrium is destroyed because of some exogenous change in parameters, and activists become motivated to support the appropriate parties, then convergence can never be recreated.

Note that in terms of the model, there is no reason why R should be to the right, and L to the left. However, once the move is made in one direction or the other, then activist support will tend to reinforce the left-right positioning of the parties.

This simple marginal calculation becomes more interesting when there is a second "social" dimension of policy. Consider the initial positions R and D, on either side of, and approximately equidistant from the origin, as in the figure. Both Social Conservatives, represented by C, and Social Liberals, represented by S, would be indifferent between both parties. A Democratic candidate by moving to position D^* will benefit from activist support of the social liberals, but will lose some support from the liberal economic activists. Note that the figure is based on the supposition that activists are characterized by ellipsoidal indifference contours, reflecting the different saliences they put on the policy axes. The "contract curve" between the two activist groups, centered at L and S, represents the set of conflicting interests or "bargains" that can be made between these two groups over the policy to be followed by the candidate. It can be shown (Miller and Schofield 2003) that this contract curve is a *catenary* whose curvature is determined by the eccentricity of the utility functions of the activist groups. We therefore call this contract curve *the Democratic activist catenary*. If we let u_L, and u_S, denote the utility functions of the two representative pro-Democrat activists, then this catenary is a one-dimensional curve given by equation

$$\left[\alpha_L \frac{du_L}{dz} + \alpha_S \frac{du_S}{dz}\right] = 0, \tag{1}$$

where α_L, α_S are appropriate parameters. A move by the Democratic candidate to a position such as D^* in Figure 1 will maximize the total contributions to the candidate. Of course, the position D^* will depend on the relative willingness of the two activist groups to contribute. In the same way, the *Republican activist catenary* is given by the contract curve between economic conservative activists, positioned at E, and social conservative activists, positioned at C. Denoting the utility functions of these activists by u_E, u_C, respectively, the contract curve between these two activists is given by the equation

$$\left[\alpha_E \frac{du_E}{dz} + \alpha_C \frac{du_C}{dz}\right] = 0. \tag{2}$$

We now make the assumption that the marginal contributions of the activists of both parties affect the two parties' activist valences, μ_{dem} and μ_{rep} by the marginal equations

$$\frac{d\mu_{dem}}{dz}\bigg|_z = \left[\alpha_L(z)\frac{du_L}{dz}\bigg|_z + \alpha_S(z)\frac{du_S}{dz}\bigg|_z\right], \tag{3}$$

$$\frac{d\mu_{rep}}{dz}\bigg|_z = \left[\alpha_E(z)\frac{du_E}{dz}\bigg|_z + \alpha_C(z)\frac{du_C}{dz}\bigg|_z\right]. \tag{4}$$

The four coefficients in these equations parametrize the willingness of activists to contribute, as well as the effects these contributions have on party valences. These assumptions allow us to determine the first-order conditions for maximizing vote share. The main theorem of this article is that the first-order condition for the candidate positions (z^*_{dem}, z^*_{rep}) to be a Nash equilibrium in the vote-share maximizing game is that it satisfies a *balance equation.* This means that, for each party $j = dem$ or rep, there is a weighted electoral mean for party j, given by the expression

$$M_j = \sum_i \alpha_{ij} x_i, \tag{5}$$

and which is determined by the set of voter preferred points $\{x_i\}$. Notice that the coefficients $\{\alpha_{ij}\}$ for candidate j will depend on the position of the other candidate, k. The *balance equation* for each j is given by

$$\left[M_j - z^*_j\right] + \frac{1}{2\beta}\left[\frac{d\mu_j}{dz_j}\bigg|_z\right] = 0. \tag{6}$$

The locus of points satisfying this equation is called the *balance locus* for the party. It is also a catenary obtained by shifting the appropriate activist catenary towards the weighted electoral mean of the party. The gradient vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for party j* (at the position z^*_j) and represents the marginal effect of the activist groups on the party's valence. The gradient term $[M_j - z^*_j]$ is the *marginal electoral pull of party j* (at z^*_j). Obviously, this pull is zero at $z^*_j = M_j$. Otherwise, it is a vector pointing towards M_j.

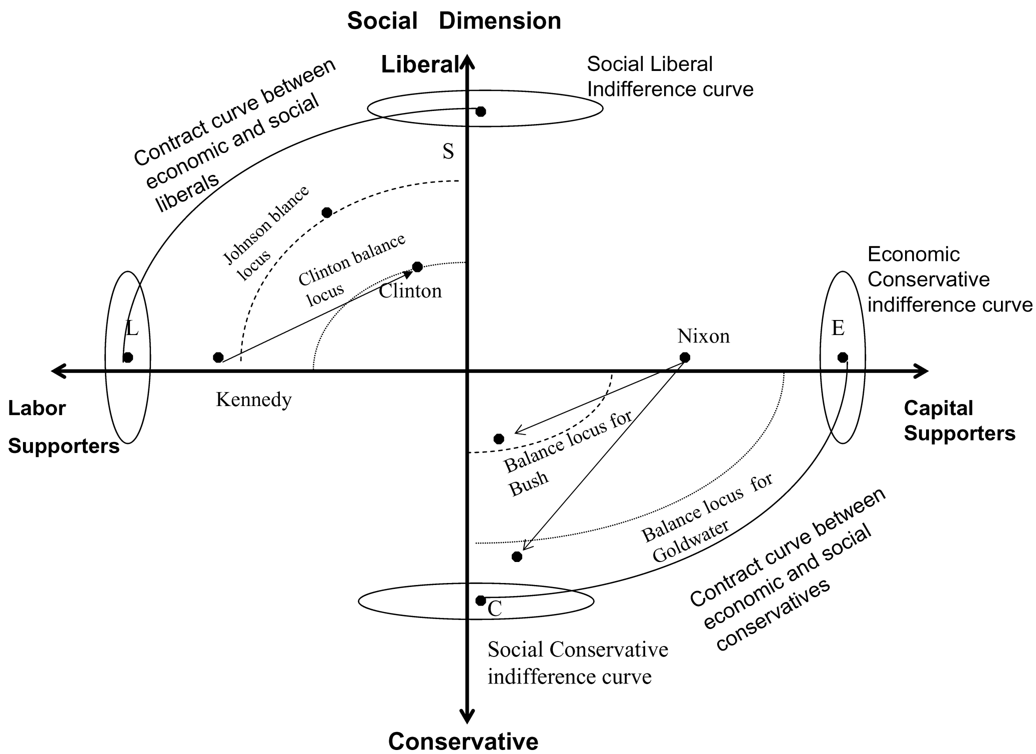
To illustrate, the pair of positions (D^*, R^*) in Figure 1 are party positions that maximize activist contributions. At R^* the figure shows three gradient vectors (defined by the two marginal activist pulls toward E and C, and the marginal electoral pull, pointing towards the electoral center. These three vectors must sum to zero for the balance condition to be satisfied. The locations of the balance loci for the two parties depend on the difference between the exogenous valences, λ_{dem} and λ_{rep}. In particular if λ_{dem} -λ_{rep} is increased for some exogenous reason, then the relative marginal activist effect for the Republicans becomes more important, while for the Democrats it becomes less important.

The positioning of R^* in the electoral quadrant labelled "Conservatives" in Figure 1 and of D^* in the liberal quadrant is meant to indicate the realignment that has occurred since the election victory of Kennedy over Nixon in 1960. By 1964 Lyndon Johnson had moved away from a typical New Deal Democratic position, L, to a position comparable to D^*. By doing so, he brought about a transformation that eventually lost the south to the Republican party. According to the model just presented, such a move by Johnson was a rational response to civil rights demands, by increasing activist support. Although the support by the social liberals would be small initially at position L, the rate of increase of support (associated with a move along the Democratic catenary) would be large in magnitude. Conversely, the initial rate of the loss of support from Labor would be relatively small. A move by Johnson away from L along the catenary would thus lead to a substantial increase in overall activist support. Moreover, the empirical analysis in Schofield, Miller, and Martin (2003) suggests that λ_{dem} for Johnson was large in contrast to λ_{rep} for Goldwater. We can therefore infer that Goldwater's dependence on activist support was greater than Johnson's. This is reflected in Figure 2, where the balance locus for Goldwater is shown to be further from the electoral origin than the balance locus for Johnson. Thus the magnitude of $\frac{du_C}{dz}$ provides an explanation of why socially conservative activists responded so vigorously to the new Republican position adopted by Goldwater, and came to dominate the Republican primaries in support of his proposed policies. These characteristics of the balance solution appear to provide an explanation for Johnson's electoral landslide in 1964. This is indicated by the partisan cleavage line in Figure 1 for 1964, which separates those voters who were estimated to vote for Johnson, against those estimated to vote for Goldwater (Schofield, Miller, and Martin 2003). The change in the Democrat/Republican cleavage lines in this period gives a geometric representation of the political realignment that occurred and is still occurring.

The response by Republican candidates after this election, while taking advantage of the political realignment, has brought about something of a dilemma for both parties. This can be seen by considering in detail the balance condition at the position R^*, in Figure 1. At that point there are two gradients, $\frac{du_E}{dz}|_z, \frac{du_C}{dz}$, pointing away from the electoral origin. Because the distance from E is significant, the marginal contribution from economic conservative activists will be negative as the Republican position moves down the catenary. Further movement down the Republican catenary, in response to social conservative activism, will induce some activists located near E to recalculate the logic of their support. Indeed, members of the business community who can be designated "cosmopolitans," who are economically conservative but relatively liberal in their social values, must be concerned about the current policy choices of the Republican president. It is worth giving a long quote from John Danforth in which he expresses these concerns.

> When government becomes the means of carrying out a religious program, it raises obvious questions under the First Amendment. But even in the absence of constitutional issues, a political party should resist identification with a religious movement. While religions are free to advocate for their own sectarian causes, the work of government and those who engage in it is to hold together as one people a very diverse country. At its best, religion can be a uniting influence, but in practice, nothing is more divisive. For politicians to advance the cause of one religious group is often to oppose the cause of another.
>
> Take stem cell research. Criminalizing the work of scientists doing such research would give strong support to one religious doctrine, and it would punish people who believe it is their religious duty to use science to heal the sick.
>
> During the 18 years I served in the Senate, Republicans often disagreed with each other. But there was much that held us together. We believed in limited government, in keeping light the burden of taxation and regulation. We encouraged the private sector, so that a free economy might thrive. We believed that judges should interpret the law, not legislate. We were internationalists who supported an engaged foreign policy, a strong national defense and free trade. These were principles shared by virtually all Republicans.
>
> But in recent times, we Republicans have allowed this shared agenda to become secondary to the agenda of Christian conservatives. As a senator, I worried every day about the size of the federal deficit. I did not spend a single minute worrying about the effect of gays on the institution of marriage. Today it seems to be the other way around.
>
> The historic principles of the Republican Party offer America its best hope for a prosperous

FIGURE 2 Realignment 1960–2006



and secure future. Our current fixation on a religious agenda has turned us in the wrong direction. It is time for Republicans to rediscover our root. (2005)

As this quote suggests, there are many potential economic advantages to be gained from medical advances, particularly those resulting from stem cell research. Acquiescence to the policy demands of social conservatives means these gains will be forgone.

In parallel, a Democratic position further along the Democrat catenary, particularly one associated with a Democratic candidate who has exogenous valence higher than the Republican opponent, would bring into being a new gradient vector associated with activist support for the Democratic candidate, from cosmopolitan economic activists. Small moves by such a candidate would induce a significant increase in contributions.

The dynamic logic of this electoral model is that both parties will tend to move in a clockwise direction as they attempt to maximize electoral response by obtaining support from their respective activist groups. The model suggests that eventually the Democratic candidate will be located close to *S* while the Republican candidate will be close to *C*. From then on, populists will dominate the Republican Party and cosmopolitans will dominate the Democratic, Party.

The model that we propose therefore suggests that realignment of party positions from (D, R) about 1960 to positions close to (S, C) in 2006 takes about two generations. Indeed, Schofield, Miller, and Martin (2003) suggest that a slow realignment has been going on since at least the election of 1896. The position of the Democratic candidate, William Jennings Bryan, in 1896 may resemble that of future Republican presidential candidates, while the position of the Republican candidate, William McKinley, in 1896 may turn out to be similar to the positions of future Democratic candidates.

The reason such a realignment is very slow to come into being is due to the power of party activists who support the existing realignment at a given time. Consider the New Deal party alignment, based on economic conservatism/liberalism. The Republican activists were small businesses, professionals, and middle-class people who felt they had a lot to lose by more government regulation/redistribution. Most of the New Deal supporters were northern labor and southern agrarian interests who had something to gain by challenging the McKinley northeastern/business coalition. Many of these Democratic activists resisted the realignment that eventually occurred in the 1960s, when the Democratic Party sponsored civil rights and other socially liberal legislation. This caused the Republicans to react with their southern strategy, attracting socially conservative voters who were alienated by this version of liberalism.

The question remains, however: Why didn't this realignment occur in the fifties or the late forties? The slow pace was not because there were no advocates. There were always voices calling for the Democratic Party to be more liberal on the social dimension: for example, civil rights supporters like Hubert Humphrey in 1948 who wanted the Democrats to incorporate a more activist stance. The slow pace was also not due to lack of incentive for Republicans: Many Republican strategists realized they had a lot to gain if the Democratic Party split on the issue of civil rights.

However, the New Deal economic activists (both Democratic and Republican) all had something to lose by allowing a realignment based on the social dimension. The typical northern Democratic activist—perhaps a member of a labor union—was in a strong position of power in the party. The Democratic leadership in Congress knew that they had to consult the unions on major legislation and defer to them in large part. Similarly, southern farmers knew that they were getting a flow of benefits from their pivotal position in the Democratic party. They also knew that they would have to forego that position of party dominance if the Democrats ever became serious about civil rights. Thus, New Deal activists had every reason to try to maintain control of the party machinery (caucuses, the nomination process, etc.) in order to prevent what many nevertheless regarded as inevitable: the eventual emergence of a strong civil rights bill. An example of how New Deal activists kept control of the Democratic Party machinery is the nomination of Adlai Stevenson in 1952 and 1956. He was forced to promise not to talk about civil rights before southern Democrats would even consider his nomination. Northern activists were happy to agree to this in order to keep the Democratic coalition together.

The same thing could be said of the traditional GOP activists of the New Deal era. They did not have the same control of the legislative process since they were out of power in Congress most of the time, but they still had a lot to lose by a party realignment that emphasized social rather than economic dimensions of policy. Of course, there were many northeastern Republicans who were liberal on civil rights, who supported civil rights legislation, and who hated the Goldwater revolution. The Goldwater revolution was a revolution in the Republican Party precisely because it took power away from the traditional GOP activists and handed it over to western (and eventually southern) business and ideological interests. The traditional GOP activists hated the outcome. The business of the Republican Party became something quite different from the comfortable agenda of the Eisenhower period: the fight for fiscal conservatism and low taxes.

In short, party realignment takes time because the position of a party is not simply controlled by vote-maximizing politicians. Policy choice is constrained by party elites who control party machinery and would rather lose a few elections than change the orientation of the party.

Federalism also has an effect. The Goldwater revolution in the GOP started in the sixties and was resisted by socially liberal northeastern Republicans. By 1980, they had definitively lost control of the national Republican Party. However, they had not lost control of the local Republican parties in states such as Pennsylvania and Maine. As a result, social liberals from the northeast (like Jeffords and Chaffee, for example) remained a diminishing minority voice in the national Republican Party even as the Republican Party came to stand for huge deficits, opposition to abortion, and constitutional amendments to ban gay marriage. The 2006 election saw the defeat of a few more of these socially liberal Republicans. This has made the Republican Party somewhat more socially conservative than it had been before. In 2006, all positions of congressional leadership in the GOP are held by southerners and westerners. In contrast, the Democratic Party is now solidly in control of what used to be the heart of the Republican Party: the Northeast.

Federalism in U.S. politics therefore acts to slow down the pace of realignment.

The Formal Stochastic Model

The electoral model presented here is an extension of the multiparty stochastic model of Lin, Enelow, and Dorussen (1999) and McKelvey and Patty (2006), modified by inducing asymmetries in terms of valence. The justification for developing the model in this way is the empirical evidence that valence is a natural way to model the judgements made by voters of party leaders and candidates. There are a number of possible choices for the appropriate model for multiparty competition. The simplest one, which is used here, is that the utility function for the candidate of party j is proportional to the vote share, V_j, of the party in the presidential election.[9] With this assumption,

[9]For refining the model, and for empirical analysis, it would be more appropriate to use the share of the electoral college votes, or a combination of this and the party vote shares in the elections to

we can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE). Because the vote share functions are differentiable, we use calculus techniques to obtain conditions for positions to be locally optimal. Thus we examine what we call *local pure strategy Nash equilibria* (LNE). From the definitions of these equilibria it follows that a PNE must be a LNE, but not conversely.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of policy choices, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that candidates cannot predict vote response precisely, but that they can estimate the effect of policy proposals on the expected vote share. In the model with valence, the stochastic element is associated with the weight given by each voter, i, to the average perceived quality or valence of the candidate.[10]

Definition 1. *The Stochastic Vote Model* $V(\lambda, \mu, \beta; \Psi)$ *with Activist Valence.*

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, N, of size n. We assume that X is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Without loss of generality, we adopt coordinate axes so that $\frac{1}{n}\Sigma x_i = 0$. By assumption $0 \in X$, and this point is termed the *electoral mean*, or alternatively, the *electoral origin*. Each of the parties in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare prior to the specific election to be modeled.

Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of candidate policy positions.

Given $\mathbf{z}$, each voter, i, is described by a vector $\boldsymbol{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$ where

$$\begin{aligned} u_{ij}(x_i, z_j) &= \lambda_j + \mu_j(z_j) - \beta||x_i - z_j||^2 + \epsilon_j \\ &= u^*_{ij}(x_i, z_j) + \epsilon_j. \end{aligned} \tag{7}$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. Again, the constant term, λ_j, is the fixed or *exogenous valence* of party j. The function $\mu_j(z_j)$ is the component of valence generated by activist contributions to agent j. The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of a metric induced from the Euclidean norm, $\| \cdot \|$, on X. The vector $\epsilon = (\epsilon_1, \ldots, \epsilon_j, .., \epsilon_p)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by Ψ.

It is assumed that the exogenous valence vector

$$\lambda = (\lambda_1, \lambda_2, \ .., \ \lambda_p) \text{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1.$$

Voter behavior is modeled by a probability vector. The probability that a voter i chooses party j at the vector $\mathbf{z}$ is

$$\rho_{ij}(\mathbf{z}) = \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j]. \tag{8}$$

$$= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j]. \tag{9}$$

Here Pr stands for the probability operator generated by the distribution assumption on ϵ. The *expected vote share* of agent j is

$$V_j(\mathbf{z}) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(\mathbf{z}). \tag{10}$$

The differentiable function $V : X^p \to \mathbb{R}^p$ is called the *party profile function.* □

The most common assumption in empirical analyses is that Ψ is the *Type I extreme value distribution* (sometimes called Gumbel (maximum)). The theorem in this article is based on this assumption. This distribution assumption is the basis for much empirical work based on multinomial logit estimation (Dow and Endersby 2004).

Definition 2. *The Type I Extreme Value Distribution*, Ψ.

(i) The cumulative distribution, Ψ, has the closed form

$$\Psi(h) = \exp\left[-\exp\left[-h\right]\right],$$

with probability density function

$$\psi(h) = \exp[-h] \exp\left[-\exp\left[-h\right]\right]$$

and variance $\frac{1}{6}\pi^2$.

(ii) For each voter i, and party j, the probability that a voter i chooses party j at the vector $\mathbf{z}$ is

$$\rho_{ij}(\mathbf{z}) = \frac{\exp\left[u^*_{ij}(x_i, z_j)\right]}{\sum_{k=1}^{p} \exp u^*_{ik}(x_i, z_k)}. \tag{11}$$

See Train (2003, 79). □

In this stochastic electoral model it is assumed that each party j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p)$.

Congress. We adopt this simplifying assumption in order to present the essential structure of the formal model.

[10] Earlier one-dimensional models by Londregan and Romer (1993) and Groseclose (2001) included valence and supposed that candidates were uncertain about the median voter location. Here, the voter is certain about the candidate location, but uncertain about how to vote, and this uncertainty implies that the agents are uncertain about voter choices.

Definition 3. *Equilibrium Concepts.*

(i) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, \ldots, z_p^*) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile function $V : X^p \to \mathbb{R}^p$ iff, for each party $j \in P$, there exists a neighborhood X_j of z_j^* in X such that

$$V_j\left(z_1^*, \ldots z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*\right) > V_j\left(z_1^*, \ldots, z_j, \ldots, z_p^*\right) \text{ for all } z_j \in X_j - \left\{z_j^*\right\}.$$

(ii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, \ldots, z_p^*)$ is a *local weak Nash equilibrium* (LNE) iff, for each agent j, there exists a neighborhood X_j of z_j^* in X such that

$$V_j\left(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*\right) \geq V_j\left(z_1^*, \ldots, z_j, \ldots, z_p^*\right) \text{ for all } z_j \in X_j.$$

(iii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, \ldots, z_p^*)$ is a *pure strategy Nash equilibrium* (PNE) iff X_j can be replaced by X in (ii).

(iv) The strategy z_j^* is termed a *local strict best response*, a *local weak best response*, or a *global best response*, respectively to $\mathbf{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_{j+1}^*, \ldots, z_p^*)$ depending on which of the above conditions is satisfied. □

From the definitions, it follows that if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}^*$ to be a PNE.

Notice that in this model, each agent is uncertain about the precise electoral outcome, because of the stochastic component of voter choice. Nonetheless, we presume that each agent uses opinion poll data, etc., to estimate expected vote share, and then responds to this information by searching for a "local equilibrium" policy position in order to gain as many votes as possible.

It follows from (11) that for voter i, with ideal point, x_i, the probability, $\rho_{ij}(\mathbf{z})$, that i picks j at $\mathbf{z}$ is given by

$$\rho_{ij}(\mathbf{z}) = \left[1 + \Sigma_{k \neq j}\left[\exp(f_k)\right]\right]^{-1} \tag{12}$$

where $f_k = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta||x_i - z_j||^2 - \beta||x_i - z_k||^2.$

We use (12) to show that the first-order condition for $\mathbf{z}^*$ to be an LSNE is that it be a *balance solution.*

Definition 4: *The Balance Solution for the Model* $V(\lambda, \mu, \beta; \Psi)$.

Let $[\rho_{ij}(\mathbf{z})] = [\rho_{ij}]$ be the n by p matrix of voter probabilities at the vector $\mathbf{z}$, and let

$$[\alpha_{ij}] = \left[\frac{\rho_{ij} - \rho_{ij}^2}{\Sigma_{k=1}^n\left(\rho_{kj} - \rho_{kj}^2\right)}\right] \tag{13}$$

be the n by p matrix of weighting coefficients.

The *balance equation* for z_j^* is given by expression

$$z_j^* = \frac{1}{2\beta}\frac{d\mu_j}{dz_j} + \sum_{i=1}^n \alpha_{ij} x_i. \tag{14}$$

The vector $\sum_i \alpha_{ij} x_i$ is a convex combination of the set of voter ideal points. This vector is called the *weighted electoral mean* for party j. Define

$$M_j(\mathbf{z}) = \sum_i \alpha_{ij} x_i. \tag{15}$$

The balance equations for $j = 1, \ldots, p$ can then be written as

$$\left[M_j(\mathbf{z}) - z_j^*\right] + \frac{1}{2\beta}\frac{d\mu_j}{dz_j} = 0. \tag{16}$$

The bracketed term on the left of this expression is termed the *marginal electoral pull of party j* and is a gradient vector pointing towards the weighted electoral mean, $M_j(\mathbf{z})$, of the party. This weighted electoral mean is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for party j.*

If $\mathbf{z}^*$satisfies the system of balance equations, for all j, then call $\mathbf{z}^*$ a *balance solution.* □

For the following discussion we note again that by suitable choice of coordinates, the equiweighted electoral mean $\frac{1}{n}\Sigma x_i = 0$, and is termed the electoral origin.

The following theorem is proved in Schofield (2006).

Theorem. Consider the electoral model $V(\lambda, \mu, \beta; \Psi)$ based on the Type I extreme value distribution and including both exogenous and activist valences.

(i) The first-order condition for z* to be an LNE is that it is a balance solution.

(ii) If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then a balance solution will be an LNE. □

Notice that if X is open, then this first-order condition at $\mathbf{z}^*$ is necessary for $\mathbf{z}^*$ to be a PNE. We implicitly assume that any relevant $\mathbf{z}^*$ will lie in the interior of X.

In the case that the activist valence functions are identically zero, we write the model as $V(\boldsymbol{\lambda}, \beta; \Psi)$.The key consideration for this model is whether the electoral origin is an LNE. The Corollary shows that when the activist valence functions are identically zero, then for each fixed j, all α_{ij} in (15) are identical. Thus, when there is only exogenous valence, the balance solution satisfies $z_j^* = \frac{1}{n}\Sigma x_i$, for all j. In this case, the marginal electoral pull is zero at the origin and the joint origin $\mathbf{z}_0 = (0, \ldots, 0)$ satisfies the first-order condition.

However, when the valence functions $\{\mu_j\}$ are not identically zero, then it is the case that generically $\mathbf{z}_0$ cannot satisfy the first-order condition. Instead the vector $\frac{d\mu_j}{dz_j}$ "points towards" the position at which the activist valence is maximized. When this marginal or gradient vector, $\frac{d\mu_j}{dz_j}$, is increased (as activists become more willing to contribute to the party), then the equilibrium position is pulled away from the weighted electoral mean of party j, and we can say the "activist effect" for the party is increased. In the two-party case, if the activist valence functions are fixed, but the exogenous valence, λ_j, is increased, or λ_k, (for $k \neq j$) is decreased, then the weighted electoral mean, $M_j(\mathbf{z})$, approaches the electoral origin. Thus the local equilibrium of party j is pulled towards the electoral origin. We can say the "electoral effect" is increased.

The second-order condition for an LSNE at $\mathbf{z}^*$ depends on the negative definiteness of the Hessian of the activist valence function. If the eigenvalues of these Hessians are negative at a balance solution, and of sufficient magnitude, then this will guarantee that a vector $\mathbf{z}^*$ which satisfies the balance condition will be an LSNE. Indeed, this condition can ensure concavity of the vote share functions, and thus of existence of a PNE.

We now apply the Theorem to the model $V(\boldsymbol{\lambda}, \beta; \Psi)$.

Definition 5. *The Convergence Coefficient of the Model* $V(\boldsymbol{\lambda}, \beta; \Psi)$.

Assume the space X has dimension w.

(i) Define

$$\rho_1 = \left[1 + \sum_{k=2}^{p} \exp[\lambda_k - \lambda_1]\right]^{-1}. \tag{17}$$

(ii) Let X be endowed with an orthogonal system of coordinate axes $(1, \ldots, s, \ldots, t, \ldots, w)$. For each coordinate axis let $\xi_t = (x_{1t}, x_{2t}, \ldots, x_{nt}) \in \mathbb{R}^n$ be the vector of the t^{th} coordinates of the set of n voter ideal points. Let $(\xi_s, \xi_t) \in \mathbb{R}$ denote scalar product. The covariance between the s^{th} and t^{th} axes is denoted $(v_s, v_t) = \frac{1}{n}(\xi_s, \xi_t)$ and $v_s^2 = \frac{1}{n}(\xi_s, \xi_s)$ is the electoral variance on the s^{th} axis.

(iii) The symmetric $w \times w$ *electoral covariation matrix* ∇ is defined to be $[(\xi_s, \xi_t)]_{t=1,..w}^{s=1,..w}$., while the *electoral covariance matrix* is defined to be $\nabla^* = \frac{1}{n}\nabla$.

(iv) The *total electoral variance* is

$$v^2 = \sum_{s=1}^{w} v_s^2 = \frac{1}{n}\sum_{s=1}^{w}(\xi_s, \xi_s) = \text{trace}\,(\nabla^*).$$

□

(v) The *convergence coefficient of the model* $V(\boldsymbol{\lambda}, \beta\,; \Psi)$ is

$$c(\lambda, \beta; \Psi) = 2\beta[1 - 2\rho_1]v^2. \tag{18}$$

□

It is readily shown that at $\mathbf{z}_0 = (0, \ldots, 0)$, the probability, $\rho_{ij}(\mathbf{z}_0)$, that i votes for party j is independent of i. It follows that, for the lowest valence party, the probability $\rho_{i1}(\mathbf{z}_0) = \rho_1$, where ρ_1 is given by (17). Obviously if all valences are identical then $\rho_1 = \frac{1}{p}$, as expected. The effect of increasing λ_j, for $j \neq 1$, is clearly to decrease ρ_1, and therefore to increase $c(\boldsymbol{\lambda}, \beta; \Psi)$. The following result is proved in Schofield (2007).

Corollary.

The joint origin $\mathbf{z}_0$ satisfies the first-order condition to be an LNE for the model $V(\boldsymbol{\lambda}, \beta; \Psi)$. A necessary condition for $\mathbf{z}_0$ to be an LNE for the model $V(\boldsymbol{\lambda}, \beta; \Psi)$ is that $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$. In the case $w = 1$ or 2, a sufficient condition for $\mathbf{z}_0$ to be an LSNE is that $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$.

Proof.

Since $\{\mu_j\}$ are all identically zero, then by (12), we see that $\rho_{ij} = \rho_j$ is independent of i at $\mathbf{z}_0 = (0, \ldots, 0)$. By (13) this implies $\alpha_{ij} = \frac{1}{n}$, for all j, and so

$$z_j = \frac{1}{n}\sum_i x_i = 0$$

satisfies the first-order condition. This proves $\mathbf{z}_0$ satisfies the first-order condition.

It can then be shown that the Hessian of ρ_{i1} is

$$\frac{d^2\rho_{i1}}{dz_1^2} = \left[\rho_1 - \rho_1^2\right]\left\{4\beta^2[1 - 2\rho_1][\nabla_{i1}(z_1)] - 2\beta I\right\}.$$

Here $[\nabla_{i1}(z_1)] = [(x_i - z_1)^T\,(x_i - z_1)]$ is the w by w matrix of cross-product terms about the point z_1. When $z_1 = 0$ then

$$\frac{1}{n}\sum_i \nabla_{i1}(0) = \nabla^*$$

is the electoral covariance matrix about the origin. The Hessian of V_1 is now given by

$$\frac{1}{n}\sum_i \frac{d^2\rho_{i1}}{dz_1^2} = \left[\rho_1 - \rho_1^2\right]\left\{[1 - 2\rho_1][4\beta^2\nabla^*] - 2\beta I\right\}.$$

By assumption $1 > \rho_1 > 0$ so $[\rho_1 - \rho_1^2] > 0$. Moreover, $\beta > 0$ so the eigenvalues of the Hessian of V_1 can be identified with the eigenvalues of

$$C_1 = [2\beta[1-2\rho_1](\nabla^*) - I].$$

Suppose now that

$$\text{trace}[2\beta[1-2\rho_1](\nabla^*) - I] > 0].$$

Since $v^2 = \text{trace}(\nabla^*)$, this can be written as $c(\boldsymbol{\lambda}, \beta; \Psi) > w$. A standard result (Schofield 2003) implies that at least one eigenvalue of C_1 will then be positive. This implies that V_1 cannot be an LNE. Thus $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$ is a *necessary* condition for $\mathbf{z}_0$ to be an LNE.

Now consider the case $w = 1$ or 2. A similar argument shows that the condition

$$\text{trace}[2\beta[1-2\rho_1](\nabla^*)] < 1$$

is a *sufficient* condition for C_1 to have negative eigenvalues. Moreover, $\lambda_j \geq \lambda_1$ implies that $\rho_j \geq \rho_1$, for all j, so $trace(C_1) \geq trace(C_j)$ and $\det(C_1) \geq \det(C_j)$, for $j = 2, \ldots, p$,
where $C_2, \ldots, C_p$, are the Hessians of the other parties at $\mathbf{z}_0$.

Thus if C_1 has negative eigenvalues then so do $C_2, \ldots, C_p$. This implies that $z_1 = z_2 = \cdots = z_p = 0$ will all be mutual local strict best responses. Thus the sufficient condition for $\mathbf{z}_0$ to be an LSNE is that $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$. □

In higher dimensions, a sufficient condition for an LSNE can be obtained in terms of the cofactors of C_1 .

Example 1 in One Dimension.

We can illustrate the Corollary, in the case the necessary condition fails, by assuming that X is a compact interval, $[-a, +a] \subset \mathbb{R}$. Suppose further that there are three voters at $x_1 = -1$, $x_2 = 0$, and $x_3 = +1$. Then $v^2 = \frac{2}{3}$, and the necessary condition for an LNE at $\mathbf{z}_0 = (0, 0)$ is

$$c(\lambda, \beta; \Psi) = 2\beta[1-2\rho_1]\frac{2}{3} \leq 1 \tag{19}$$

where $\rho_1 = [1 + \exp[\lambda_2 - \lambda_1]]^{-1}$. This condition fails if $\beta > \beta_0$ where

$$\beta_0 = \frac{3[\exp(\lambda_2 - \lambda_1) + 1]}{4[\exp(\lambda_2 - \lambda_1) - 1]}. \tag{20}$$

Obviously if $\lambda_2 = \lambda_1$, then $\beta_0 = \infty$. Suppose that λ_2 is increased but λ_1 is kept constant, so that β_0 approaches $\frac{3}{4}$. When $\beta > \beta_0$ then $\mathbf{z}_0$ fails to be an equilibrium, and z_1^* must move away from the origin, either towards x_1 or x_3.

To see this, suppose $\lambda_2 = 1$ and $\lambda_1 = 0$. Then $\beta_0 = 1.62$. To illustrate the Corollary, we let $\beta = 2.0$. Then at $\mathbf{z}_0 = (0, 0)$ we find

$$\rho_{i1}(\mathbf{z}_0) = [1 + [\exp(1)]]^{-1} = 0.268$$

so $V_1(\mathbf{z}_0) = 0.268$.

Now consider $\mathbf{z} = (z_1, z_2) = (+0.1, 0)$. Because candidate 1 is further than candidate 2 is from both x_1 and x_2 we find

$$\rho_{11} = [1 + [\exp(1.41)]]^{-1} = 0.196$$

while

$$\rho_{21} = [1 + [\exp(1.02)]]^{-1} = 0.265.$$

However, candidate 1 is closer than 2 to x_3 and so ρ_{31} increases to

$$\rho_{31} = [1 + [\exp(1 - 0.38)]]^{-1} = 0.351.$$

Thus

$$V_1(\mathbf{z}) = \frac{1}{3}[0.196 + 0.265 + 0.351] = 0.271.$$

Hence candidate 1 can slightly increase vote share by moving way from the origin. Obviously the joint origin cannot be an equilibrium.

If $\lambda_2 - \lambda_1$ is further increased, then the equilibrium position, z_1^*, must move further out towards x_3. Notice that the vote share function $V_1(z_1, z_2)$ is clearly neither concave nor quasi concave in z_1, so the usual methods of proving existence of equilibria cannot be used.

Example 2 in Two Dimensions.

Suppose we now consider the two-dimensional case with

$$x_1 = (-1, 0), \quad x_2 = (0, 0) \quad x_3 = (+1, 0),$$

$$x_4 = (0, 1), \quad x_5 = (0, -1).$$

Using Definition 5, it follows that

$$\xi_1 = (-1, 0, +1, 0, 0), \text{ and } \xi_2 = (0, 0, 0, +1, -1).$$

The electoral covariance matrix is then

$$\nabla^* = \frac{1}{5}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{21}$$

so $v^2 = \frac{4}{5}$. The crucial condition for a local equilibrium at the origin is that the Hessian of the vote share function of player 1 has negative eigenvalues. The Hessian is given by the matrix

$$\frac{2\beta[1-2\rho_1]}{5}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{22}$$

The necessary condition is that the trace of this matrix is negative. In fact, because of the symmetry of the example, the necessary condition on each eigenvalue becomes

$$2\beta[1-2\rho_1]\frac{2}{5} \leq 1.$$

This condition fails if $2\beta[1-2\rho_1]\frac{2}{5} > 1$, in which case both eigenvalues will be positive. Thus, if $\beta > \beta_1$, where

$$\beta_1 = \frac{5}{4(1-2\rho_1)} = \frac{5[\exp(\lambda_2 - \lambda_1) + 1]}{4[\exp(\lambda_2 - \lambda_1) - 1]}$$

then the electoral origin is a minimum of the vote share function of player 1. Thus player 1 can move away from the origin, *in any direction,* to increase vote share. Schofield (2005) and Schofield and Sened (2005, 2006) show that typically there will exist a "principal" high variance electoral axis. Simulation of empirical models with exogenous valence and p parties shows that the lowest valence player will move away from the origin on this axis when the convergence condition $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$ is violated. In this case, an LSNE will exist, but not at the electoral origin, and will satisfy the condition $\|z_1^*\| > \|z_p^*\|$. In other words, in equilibrium, the highest-valence party will adopt a position closer to the electoral origin, while low-valence parties will move to the electoral periphery.

These two simple examples provide the justification of the assertion made in the second section of this article that when λ_2 and λ_1 are substantially different, in terms of v^2 and β, then the joint origin becomes unstable. Note, however, that the joint origin will be an equilibrium as long as λ_2 and λ_1 are similar, or v^2 and β are "small enough."

Concluding Remarks

The main purpose of this article has been to show that convergence need not be expected in party competition. Since activist support is of key importance in elections in the United States, we expect party candidates to move away from the electoral origin towards the activist catenaries. There is substantial evidence that it is necessary to employ a two-dimensional policy model with both economic and social axes in order to understand American politics. This implies that party success requires the formation of coalitions among actors who have conflicting policy preferences over at least one dimension of policy. A successful party coalition is a "coalition of enemies." This can be seen in the remarks by Republican John Danforth, quoted previously, clearly revealing the increasing hostility between evangelical Christians and the business community within the GOP.

Since at least the Reagan era, the winning Republican coalitions have consisted of social conservatives (located at C in Figure 1) and economic conservatives (located at E). However, many of the most salient issues of the twenty-first century threaten to split this Republican coalition. Stem cell research is important to the probusiness conservatives, but anathema to social conservatives. Mexican immigration helps business by keeping labor prices low, but is regarded with hostility by social conservatives (many of them wage earners) who see it as a threat to their own livelihood as well as to traditional American values.

The model developed in this article incorporates just this kind of intraparty tension to explain the paradox of nonconvergence in American party politics. Social conservative activists use new issues like stem cell research and immigration to pull the Republican party along the catenary toward their ideal point at C. Republicans like Danforth see this move as a forfeiture of traditional probusiness ideals: "limited government," "keeping light the burden of taxation and regulation." They exert a contrary "tug" toward E. The combined effects of these two activist vectors must be balanced by an electoral pull toward the center to create a party equilibrium that is characterized by divergent party locations.

The equilibria of the Democratic and Republican parties are of course interdependent. To the extent that social conservatives are successful in moving the Republican party location toward their ideal point, the model clearly indicates the Democratic best response. The cosmopolitans—like Danforth—have the most to lose by the increasing commitment of the GOP to socially conservative causes. Many of these are professionals or business executives, with sympathy for women's rights, environmentalism, and civil liberties, who have always supported Republicans after consulting their pocketbook. But if the Republican Party is seen as the party of strict immigration control (and thus higher wages) and restrictions on stem cell research (and thus foregone opportunities for growth in new biotechnology industries), then they may change their support.

The model indicates that, as the Republican party moves toward *C*, then disaffected cosmopolitans may be increasingly tempted to contribute resources to pull the Democrats toward *S*. A Democratic Party candidate who maintains traditional social liberal positions (in support of African-American voters and women's rights) while moving toward a moderate position in economic policy could obtain significant resources from activists located in that quadrant. Just as the Reagan Republicans constructed a winning majority by picking up social conservatives, so may a Democratic candidate find it possible to construct a new majority by attracting economic conservatives who are also social liberals. Taking an emphatic stand in favor of stem cell research and in favor of immigrants' rights could be the kind of signal needed to accomplish this goal. It would at least put pressure on the fragmenting Republican coalition.

As the formal model suggests, the other determinant of party location is party valence. The relative party valence determines whether the trade-offs between party activists occur close to the origin, or at some distance from it. The party with the lower exogenous valence is forced to move out in order to attract what noncentrist support it can from committed policy activists. Current poll results indicate a shrinking valence for the Republican Party. In

May 2006, only 37% of Americans thought the Republican Party came closer to sharing their moral values (compared to 50% for Democrats), and only 22% of Americans thought the Republican Party was more likely to protect their civil liberties (compared to 62% for Democrats).[11] These figures indicate that $\lambda_{dem} - \lambda_{rep}$ is increasing, forcing the Republican Party to move out to appeal to its activist base. From the standpoint of this model, the ideal Democratic candidate would be a well-liked economic moderate who is clearly differentiated from the Republicans on the basis of social (not economic) liberal policy.

Current events illustrate this article's focus on the use of nonpolicy electoral "valence" and the policy demands of party activists to resolve the paradoxical nonconvergence of the two-party system in the United States. The formal model demonstrates how a divergent equilibrium emerges as each party attempts to balance the centripetal pull of electoral politics against the centrifugal pulls from distinct coalitions of party activists. Exogenous shocks, and the emergence of newly salient issues, such as stem cell research, can change the parameters that determine the exact location of a party equilibrium. The ability of each party to maintain a fragile coalition of party activists with quite different agendas may vary, creating opportunities for the other party to attract increasingly disaffected activists.

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