Models of elections tend to give two quite contradictory predictions about the result of political competition. In two-party competition, if the ‘policy space’ involves two or more independent issues, then ‘pure strategy Nash equilibria’ generally do not exist and instability or chaos may occur (see Plott, 1967; McKelvey, 1976; 1979; Schofield, 1978; 1983; 1985; McKelvey and Schofield, 1986; 1987; Saari, 1997; Austen-Smith and Banks, 1999). That is to say, whatever position is picked by one party, there always exists another policy point which will give the second party a majority over the other. Moreover, vote maximizing strategies could lead political candidates to wander all over the policy space. On the other hand, the earlier electoral models based on the work of Hotelling (1929) and Downs (1957) suggest that parties will converge to an electoral centre (at the electoral median) when the policy space has a single dimension. (An equilibrium can also be guaranteed as long as the decision rule requires a sufficiently large majority – Schofield, 1984; Strnad, 1985; Caplin and Nalebuff, 1988 – or when the electoral distribution has a certain concavity property – Caplin and Nalebuff, 1991.) Although a pure strategy Nash equilibrium generically fails to exist in competition between two agents under majority rule in high enough dimension, there will exist mixed strategy Nash equilibria (Kramer, 1978) whose support lies within a subset of the policy space known as the ‘uncovered set’ (see McKelvey; 1986; Banks, Duggan and Le Breton, 2002). These various and contrasting theoretical results can be seen as a paradox: will democracy tend to generate centrist compromises, or can it lead to chaos? This question is of fundamental importance in a world in which many countries are experimenting with democracy for the first time.

Partly as a result of these theoretical difficulties with the ‘deterministic’ electoral model, and also because of the need to develop empirical models of voter choice (Poole and Rosenthal, 1984), attention has focused on ‘stochastic’ vote models. A formal basis for such models is provided by the notion of ‘quantal response equilibria’ (McKelvey and Palfrey, 1995). In such models, the behaviour of each voter is modelled by a vector of choice probabilities (Lin, Enelow and Dorussen, 1999). A standard result in this class of models is that all parties converge to the electoral origin when the parties are motivated to maximize vote share or plurality (in the two-party case) (see McKelvey and Potty, 2006; Banks and Duggan, 2005). The predictions concerning convergence are at odds with empirical evidence that parties appear to diverge from the electoral centre (Merrill and Grofman, 1999; Adams, 2001; Schofield and Sened, 2006). The paradox that actual political systems display neither chaos nor convergence is the subject of this article. The key idea is that the convergence result need not hold if there is an asymmetry in the electoral perception of the ‘quality’ of party leaders (Stokes, 1992). The average weight given to the perceived quality of the leader of the party is called the party’s ‘valence’. In empirical models this valence is independent of the party’s position, and adds to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn, 2003). The early empirical model of US presidential elections by Poole and Rosenthal (1984) included these valence terms. The authors noted that there was no evidence of candidate convergence.

Formal models of elections incorporating valence have been developed (Ansolabehere and Snyder, 2000; Groseclose, 2001; Aragones and Palfrey, 2002), but the theoretical results to date have been somewhat inconclusive. Extension to the multiparty case is of interest because of recent empirical models of voting in the Netherlands and Germany (Schofield et al., 1998; Quinn, Martin, and Whitford, 1999; Quinn, and Martin, 2002), Britain (Schofield, 2005a; 2005b), Israel (Schofield, Sened and Nixon, 1998; Schofield and Sened, 2002; 2005; 2006) and Italy (Giannetti and Sened, 2004). All these empirical models have suggested that dergence is generic. Most of these empirical models have been based on the ‘multinomial logit’ assumption that the stochastic errors had a ‘Type I extreme value distribution’ (Dow and Endersby, 2004). Schofield (2007) provides a ‘classification theorem’ for the formal vote model based on the same stochastic distribution assumption. The ‘policy space’ is assumed to be of dimension w, and there is an arbitrary number, p, of parties. The party leaders exhibit differing valence. A ‘convergence coefficient’ incorporating all the parameters of the model can be defined. Instead of using the notion of a Nash equilibrium, the result is given in terms of the existence of a ‘local Nash equilibrium’. It is shown that there are necessary and sufficient conditions for the existence of a ‘pure strategy vote maximizing local Nash equilibrium’ (LNE) at the mean of the voter distribution. When the necessary condition fails, then parties in equilibrium will adopt diverse positions. In general, parties whose leaders have the lowest valence will take up positions furthest from the electoral mean. Moreover, because a pure strategy Nash equilibrium (PNE) must be a local equilibrium, the failure of existence of the LNE at the electoral mean implies non-existence of such a centrist PNE. The failure of the necessary condition for convergence has a simple explanation: if the variance of the electoral distribution is sufficiently large in contrast to the expected vote share of the lowest-valence party at the electoral mean, then this party has an incentive to move away from the origin towards the electoral periphery. Other low-valence parties will follow suit, and the local equilibrium will result with parties distributed along a ‘principal electoral axis’.
An empirical study of voter behaviour for Israel for the election of 1996 (based on Schofield and Sened, 2005) is used to show that the necessary condition for party convergence failed for this election. The equilibrium positions obtained from the formal result, under vote maximization, are in general comparable with, but not identical to, the estimated positions. The two highest-valence parties (Labour and Likud) were symmetrically located on either side of the electoral origin, while the lowest-valence parties were located far from the origin. In such a polity, based on a proportional electoral system, it is generally necessary to form coalition governments. The existence of small, low-valence, radical parties on the electoral periphery may create serious difficulties in the formation of majority government. It is possibly for this reason that Ariel Sharon, formerly leader of Likud, and Shimon Peres, formerly leader of Labour, in 2005 formed Kadima, a new centrist party. This article also presents results from analysis of the 1997 election in Britain (Schofield, 2005a; 2005b). In this case the empirical estimates of the parameters of the model, taken together with the formal analysis, suggest that convergence should have occurred. Instead the Conservative Party was estimated to be at a position far from the electoral centre. It is suggested that the discrepancy between the formal and the empirical models can be accommodated by considering the effect of activists on the optimal party position. Since concerned activists will raise funds for the party, but only if the party adopts a policy position that accords with activists’ concerns, there is a tension between activist demands and the electoral concerns of the party leadership. The model based on activist support estimates the marginal trade-off generated by opposed activist groups within a party. It is suggested that the low valence of recent Conservative leaders obliged them to seek support from activists supporting British sovereignty against the European Union, and thus to take up radical positions on the second, ‘European’ axis.

In contrast, the apparent move by the Labour Party towards the electoral centre between 1992 and 1997 was a consequence of the increase of the electoral valence of Tony Blair, the leader of the party, rather than a cause of this increase. Recent work by Miller and Schofield (2003) using this model suggests that, in the United States, the movement of presidential candidates in a two-dimensional policy space generated by economic and social dimensions is the result of contending and opposed activist groups. The underlying premise of the notion of the local Nash equilibrium, used in these models, is that party leaders will not consider ‘global’ changes in party policies, but will instead propose small changes in the party position in response to changes in beliefs about electoral response. These models regard elections as the aggregation of both electoral evaluation or ‘valence’ and electoral preferences. Valence can be regarded as that element of a voter's choice which is determined by judgement rather than preference. This accords well with the arguments of James Madison in Federalist 10 of 1787 (Rakove, 1999) and of Condorcet (1785) in his treatise on social choice theory. Schofield (2005c; 2006) provides a discussion of the relevance of these valence models for the constitutional basis of the US polity.

**Empirical analysis for Israel**

Figure 1 shows the estimated positions of the parties at the time of the 1996 Israeli election. Figure 1 also gives the estimated distribution of voter ideal points for the 1996 election, based on a factor analysis of the survey responses derived from the survey of Arian and Shamir (1999). The two dimensions of policy deal with attitudes to the Palestine Liberation Organization (PLO) (the horizontal axis) and religion (the vertical). The party positions were obtained from analysis of party manifestos (Schofield, Sened and Nixon, 1998; Schofield and Sened 2005; 2006). With the use of information on the individual voter intentions, it is possible to construct a multinomial logit model (based on the Type I extreme value distribution). Figure 1

Voter distribution and estimated party positions in the Knesset at the 1996 election
The model assumes that the voter utility vector has the form \( u_i(x_i, z_j) \), where

\[ u_i(x_i, z_j) = u_i^w(x_i, z_j) + \varepsilon_j \]

and

\[ u_i^w(x_i, z_j) = \lambda_j - \|x_i - z_j\|^2. \]

Here the position of voter \( i \) is \( x_i \) while the position of party \( j \) is \( z_j \). The term \( \|x_i - z_j\| \) is the distance between these two points. The valences of the \( p \) parties are given by the vector \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p) \) and are ranked

\[ \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1. \]

The error terms \( \{\varepsilon_j\} \) have the Type I extreme value distribution, \( \varepsilon \).

(The cumulative distribution, \( \Phi \), takes the closed form \( \Phi(x) = \exp(-\exp(-x)) \).)

The probability that a voter \( i \) chooses party \( j \) is

\[ \rho_i(x_i, z_j) = \Pr \left( u_i^w(x_i, z_j) > u_i^w(x_i, z_j') \right), \text{ for all } j'. \]

Here \( \Pr \) stands for the probability operator associated with \( \varepsilon \). The expected vote share of agent \( j \) is
This model is denoted \( M(\lambda, \beta, \gamma) \). A local pure strategy Nash equilibrium (LNE) is simply a vector \( z = (z_1, \ldots, z_p) \) of party positions with the property that each \( z_j \) locally maximizes \( V_j(z) \), taking the other party positions \( A \). A necessary condition for \( z = (0, \ldots, 0) \) to be a pure strategy Nash equilibrium (PNE) is that it be a LNE and thus that all Hessians have eigenvalues at \( z^* \) that are non-positive. This can be expressed as a single necessary condition on a ‘convergence coefficient’ defined terms of the Hessian of the vote share function of the party with the lowest valence (Schofield, 2006b). Since the lowest-valence party is the National Religious Party (NRP) (for the 1996 model for Israel), a necessary condition for the NRP vote share to be maximized at the origin is that both eigenvalues of this Hessian be non-positive. However, the calculation given below shows that that one of the eigenvalues was positive. It follows that the NRP position that maximizes its vote share is not at the origin. Thus the convergent position \((0, \ldots, 0)\) cannot be a Nash equilibrium to the vote maximizing game.

Indeed it is obvious that there is a principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. It follows that low-valence parties should then position themselves on this eigenspace, as illustrated in the simulation given in Figure 2.

To present the calculation, we use the fact that the valence of the NRP was −4.52. The spatial coefficient is \( \delta = 1.12 \). Because the valences of the major parties are 4.15 and 3.14, the formal analysis implies that, when all parties are at the origin, the vote share, \( g_{\text{NRP}} \), can be computed to be

\[
\frac{g_{\text{NRP}}}{1 + e^{4.15 + 4.52 - 3.14 - 4.52}} = 0
\]

Moreover, the Hessian of the NRP at the origin depends on the electoral variance and this is

\[
C_{\text{NRP}} = 2(1.12) \begin{vmatrix}
1 & 0.501 \\
0.501 & 0.732 \\
\end{vmatrix} - \begin{vmatrix}
1.24 & 1.32 \\
1.34 & 0.64 \\
\end{vmatrix}
\]

The eigenvalues of the NRP Hessian at the origin are 2.28 and −0.40, giving a saddle point. Thus, the origin cannot be a Nash equilibrium. The ‘convergence coefficient’ can be calculated to be 3.88, larger than the necessary upper bound of 2.0. The major eigenvector for the NRP is \((1.0, 0.8)\), and along this axis the NRP vote share function increases as the party moves away from the origin. The minor, perpendicular axis is given by the vector \((-1.25)\) and on this axis the NRP vote share decreases. Figure 2 gives one of the local equilibria in 1996, obtained by simulation of the model. The figure makes it clear that the vote maximizing positions lie on the principal axis through the origin and the point \((1.0, 0.8)\). In all, five different LNE were located. However, in all the equilibria the two high-valence parties, Labour and Likud, were located at precisely the same positions, as shown in Figure 2. The only difference between the various equilibria was that the positions of the low-valence parties were permuted among them.

Figure 2 suggests that the simulation was compatible with the predictions of the formal model based on the extreme value distribution. All parties were able to increase vote shares by moving away from the origin, along the principal axis, as determined by the large, positive principal eigenvalue. In particular, the simulation confirms the logic of the above analysis. Low-valence parties, such as NRP and Shas, in order to maximize vote shares must move far from the electoral centre. Their optimal positions will lie in either the north-east quadrant or the south-west quadrant. The vote maximizing model, without any additional information, cannot determine which way the low-valence parties should move. As noted above, the simulations of the empirical models found multiple LNE essentially differing only in permutations of the low-valence party positions.

In contrast, since the valence difference between Labour and Likud was relatively low, their optimal positions would be relatively close to, but not identical to, the electoral mean. The simulation for the elections of 1988 and 1992 are also compatible with this theoretical inference. Figure 2 also suggests that every party, in local equilibrium, should adopt a position that maintains a minimum distance from every other party. The formal analysis, as well as the simulation exercise, suggests that this minimum distance depends on the valences of the neighbouring parties. Intuitively it is clear that, once the low-valence parties vacate the origin, then high-valence parties like Likud and Labour will position themselves almost symmetrically about the origin, and along the major axis.

Comparison between Figure 1, of the estimated party positions, and Figure 2, of simulated equilibrium positions, reveals a notable disparity particularly in the position of Shas. In 1996 Shas was pivotal between Labour and Likud, in the sense that, to form a winning coalition government, either of the two larger parties required the support of Shas. It is obvious that the location of Shas in Figure 1 suggests that it was able to bargain effectively over policy and, presumably, prequisites. Indeed, it is plausible that the leader of Shas was aware of this situation, and incorporated this awareness in the utility function of the party.

The close correspondence between the simulated LNE based on the empirical analysis and the estimated actual political consensus suggests that the true utility function for each party \( j \) has the form \( g_j(x) = V_j(x) + \delta_j(x) \), where \( \delta_j(x) \) may depend on the beliefs of party leaders about the post-election coalition possibilities, as well as the effect of activist support for the party.

This hypothesis leads to the further hypothesis that, for the set of feasible strategy profiles in the Israel polity, \( \delta_j(x) \) is ‘small’ relative to \( V_j(x) \). A formal model to this effect could indicate that the LNE for \( \{U_j\} \) would be close to the LNE for \( \{V_j\} \). Note, however, that this perturbation of the party utility function causes parties to leave the main electoral axis. It is possibly for this reason that coalition politics in Israel has been very complex.

The Likud Party, under Ariel Sharon, was constrained by the religious parties in its governing coalition. This apparently caused Sharon to leave Likud to set up a new centrist party, Kadima (‘Forward’) with Shimon Peres, previously leader of Labour. The reason for this reconfiguration was the victory on 10 November 2005 of Amir Peretz over Peres for leadership of the Labour Party, and Peretz’s move to the left along the principal electoral axis.

Consistent with the model presented here, Sharon’s intention was to position Kadima very near the electoral centre on both dimensions, to take advantage of his high valence among the electorate. Sharon’s subsequent hospitalization had an adverse effect on the valence of Kadima, under its new leader, Ehud Olmert. Even so, in the election of 28 March 2006 Kadima took 29 seats, against 19 seats for Labour, and only 12 for Likud. One surprise was a new centrist pensioners’ party with 7 seats. Because Kadima with Labour and the other parties of the left had 70 seats, Olmert was able to put together a majority coalition on 28 April, including the Orthodox party Shas. As Figure 1 illustrates, Shas is centrist on the security dimension, indicating that this was the key issue of the election.

**Empirical analysis for Britain**

This section analyses the general election in Britain in 1997 in order to suggest how activists for the parties may influence party positioning. The analysis shows that the valence model as presented above cannot always explain divergence of party positions. For example, Figure 3 shows the estimated positions of the party leaders, based on a survey of party MPs in 1997 (Schofield, 2005a; 2005b). In addition to the Conservative Party, Labour Party, and Liberal Democrats, responses were obtained from Ulster Unionists, Scottish Nationalists and Plaid Cymru (Welsh Nationalists).

The axis is economic, the second pro or anti the European Union. The electoral model was estimated for the election in 1997, using only the economic dimension.

**Figure 3**

Estimated party positions in the British Parliament for a two-dimensional model. Notes: Highest-density contours of the voter sample distribution at the 95%, 75%, 50% and 10% levels. CONS: Conservative Party; LAB: Labour Party; LIB: Liberal Democrats; PC: Plaid Cymru (Welsh Nationalists); SNP: Scottish National Party; UU: Ulster Unionist Party. Source: MP survey data and a National Election Survey.
For this election, we have $\rho_{\text{lib}} = \frac{e^{0.24}}{1 + e^{0.24} + e^{0.97}} \approx 0.14$ so the probability $\rho_{\text{lib}}$ that a voter chooses the Liberal Democrats is

The Hessian for this party at the origin is $\mathbf{C}_{\text{lib}} = -0.28$, which is compatible with a Nash equilibrium at the origin. Extending the model to two dimensions gives a Hessian $\mathbf{C}_{\text{lib}} = \begin{pmatrix} 0.72 & 1 \\ 0 & 1.5 \end{pmatrix}$ at the origin. The Hessian for the Liberal Democrats is $\mathbf{C}_{\text{lib}} = \begin{pmatrix} 0 & -0.28 \\ 0 & 1 \end{pmatrix}$

According to the formal model, all parties should have converged to the origin on the first axis. Because the eigenvalue for the Liberal Democrats is positive on the second axis, we have an explanation for its position away from the origin on the Europe axis. However, there is no explanation for the location of the Conservative Party so far from the origin on both axes. Schofield (2005a; 2005b) adapts the activist model of Aldrich (1983a; 1983b) wherein the falling exogeneous valence of the Conservative Party leader increases the marginal importance of two opposed activist groups in the party: one group 'pro-capital' and one group 'anti-Europe', as in Figure 4.

Figure 4
Illustration of vote maximizing positions of Conservative and Labour Party leaders in a two-dimensional policy space
The optimal Conservative position will be determined by balancing the electoral effects of these two groups. The optimal position for this party will be one which is 'closer' to the locus of points that generates the greatest activist support. This locus is where the joint marginal activist pull is zero. This locus of points can be called the 'activist contract curve' for the Conservative Party. Note that in Figure 4 the indifference curves of representative activists for the parties are described by ellipses. This is meant to indicate that preferences of different activists on the two dimensions may accord different saliences to the policy axes. The 'activist contract curve' given in the figure, for Labour, say, is the locus of points satisfying the first order. This curve represents the balance of power between Labour supporters more interested in economic issues (centred at L in the figure) and those more interested in Europe (centred at E). The optimal positions for the two parties will be at appropriate positions that satisfy the optimality condition. According to this model, a party's optimal position will tend to be nearer to the electoral origin when the valence of the party leader is higher. In contrast, a party whose leader has low valence will be more influenced by activist groups, and will tend to adopt a position further from the electoral centre and nearer to the position preferred by the dominant activist group. This model has been applied to the US polity by Miller and Schofield (2003) and Schofield, Miller and Martin (2003).

Proportional representation and plurality rule

Most of the early work in formal political theory focused on two-party competition, and generally concluded that there would be strong centripetal electoral forces causing parties to converge to the electoral centre. The extension of this theory to the multiparty context, common in European
polities, has proved very difficult because of the necessity of dealing with coalition governments (Riker, 1962). However, the symmetry conditions developed by McKelvey and Schofield (1987) showed that a large, centrally located party could dominate policy if it occupied what is known as a ‘core position’. Thus, in situations where there is a stable policy core there would be certainty over the post-election policy outcome of coalition negotiation (Laver and Schofield, 1998). Absent a policy core, the post-election outcome will be a lottery across various possible coalitions, all of which are associated with differing policy outcomes and cabinet allocations. Modelling this post-election ‘committee game’ can be done with cooperative game theoretical concepts (Banks and Duggan, 2000).

Although the non-cooperative stochastic electoral model presented here can give insight into the relationship between electoral preferences and beliefs (regarding the valences of party leaders), it is still incomplete. The evidence suggests that party leaders pay attention not only to electoral responses but also to the post-election coalition consequences of their choices of policy positions. Nonetheless, the combination of the electoral model and post-election bargaining theory (Schofield and Sened, 2002) suggests the following.

In a polity based on a proportional electoral rule, the high-valence parties will be attracted towards the electoral centre. However, if there are two such competing parties of similar valence neither will locate quite at the centre. There may be many low-valence parties, whose equilibrium, vote maximizing positions will be far from the electoral centre. In order to construct winning coalitions, one or other of the high-valence parties must bargain with more ‘radical’ low-valence parties, and this could induce a degree of coalitional instability. However, it is possible that a charismatic leader, such as Sharon in Israel, can adopt a centrist position and dominate politics by controlling the policy core.

In a polity based on a plurality electoral rule, the disproportionality between votes and seats may increase the importance of activist groups. A party with a relatively low-valence leader will be forced to depend on activist support. Consequently, the party will be obliged to move to a more radical position so to attract activist support.

This may provide a reason why Britain’s Labour Party appeared to acquiesce to the demands of its left-wing supporters during the leadership of Michael Foot in 1980–3 and of Neil Kinnock in 1983–92. This led to Labour defeats in the elections between 1983 and 1992. Tony Blair became Labour leader following the death of John Smith in 1994 and his high valence allowed him to overcome union opposition and to craft the centrist ‘New Labour’ policies that led to Labour victories in the elections of 1997, 2001 and 2005.

Concluding remarks

To sum up, these models suggest how the democrat paradox can be resolved: convergence to an electoral centre is not a generic phenomenon, but can occur when a party leader is generally regarded by the electorate to be of superior quality or valence. Chaos does not occur in these models, though a degree of coalitional instability is possible under proportional electoral rule when there is no highly regarded political leader at the policy core.

See Also

- political competition
- rational behaviour
- rational choice and political science

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Bibliography


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