

Switching Equilibria*

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Abstract

Models of elections typically indicate that all parties, in equilibrium, will adopt positions at the electoral center. This situation, however, is seldom observed. This chapter outlines how an integrated theory of party strategy may be constructed based on a theorem for existence of Local Nash Equilibrium (LNE) in a stochastic electoral model where political agents have differing valence, or electorally perceived quality. In a model based on exogenous valence, it is shown that, under proportional representation, a high valence political leader may be motivated to switch party so as to control a centrist policy position. Under plurality rule, political agents may be forced to switch party as a result of the conflicting demands of activist coalitions.

3.1 A Model of Electoral Competition and Party Switching

A fully developed formal theory of politics would connect the nature of the electoral system, the motivations of parties concerning policy and prerequisites, and the process of government formation, in a way which makes sense of the empirical phenomena. A number of attempts have been made to model the motivations of parties in a game-theoretic framework. For example, one class of models is based on the Downsian framework, where parties compete via policy declarations to the electorate to maximize the number of seats they obtain. Since parties are assumed in these models to be indifferent to policy objectives, viewing policy solely as a means to gain power, symmetry would suggest that one equilibrium would be the situation where all parties declare the same position. In the standard spatial model

(Downs 1957; Hotelling 1929), convergence of this kind is a typical result. There is substantial evidence that convergence by all parties to an electoral center is an extremely unlikely phenomenon (Schofield and Sened 2006).

Partly as a result of this theoretical difficulty with the formal electoral model, and also because of the need to develop empirical models of voter choice, this chapter focuses on a “stochastic” vote model. In such a model, the behavior of each voter is characterized by a vector of choice probabilities determined by the candidate positions (Hinich 1977; Lin, Enelow, and Dorussen 1999). Again, a standard result in the class of stochastic models is that all parties converge to the electoral origin when the parties are motivated to maximize vote share or plurality (in the two party case) (Banks and Duggan 2005; McKelvey and Patty 2006).

Recent formal analysis (Schofield 2006; 2007) shows that the convergence result need not hold if there is an asymmetry in the electoral perception of the “quality” of the political agents (Stokes 1992). The average weight given to the perceived quality of a political agent is called the agent’s *valence*.¹ In empirical models, adding valence to the model increases the statistical significance of the model (Schofield and Sened 2006). In these empirical models, valence is usually assumed to be *exogenous*, in the sense of being independent of the agent’s position. The early empirical work by Poole and Rosenthal (1984) on US presidential elections included these exogenous valence terms and noted that there was no evidence of candidate convergence.

This chapter discusses this general model of elections based on the assumption that there are two kinds of valence. The first kind is an intrinsic or *exogenous valence*, which for a political agent j is denoted λ_j . As in empirical work, we assume that λ_j is held constant at the time of an election and so is independent of the agents’s current position.² The second kind of valence is known as *activist valence*. When agent j adopts a policy position z_j , in the policy space, X , then the activist valence of the agent is denoted $\mu_j(z_j)$. Implicitly, we adopt a model originally due to Aldrich (1983a; 1983b) and Aldrich and McGinnis (1989). In this model, activists provide crucial resources of time and money to their chosen political agent, and these resources are dependent on the agents policy position.³ The agent then uses these resources to enhance its image before the electorate, thus affecting its overall valence. Although activist valence is affected by agent position, it does not operate in the usual way by influencing voter choice through the distance between a voter’s preferred policy position,

say x_i , and the agent position. Rather, as agent j 's activist support, $\mu_j(z_j)$, increases due to increased contributions to the agent in contrast to the support $\mu_k(z_k)$ received by agent k , then (in the model) all voters become more likely to support agent j over agent k .⁴

The problem for each political agent is that activists are likely to be more extreme than the typical voter. By choosing a policy position to maximize activist support, the agent will lose centrist voters. The agent must therefore determine the "optimal marginal condition" to maximize vote share. Theorem 1, presented in the following text, gives this as a (first order) *balance condition*. Moreover, because activist support is denominated in terms of time and money, it is reasonable to suppose that the activist function will exhibit decreasing returns. For example, in an extreme case, a political agent who has no activist support at all may benefit considerably by a small policy move to favor a particular interest group. On the other hand, when support is very substantial, then a small increase due to a policy move will little affect the electoral outcome. For this reason it is reasonable to assume that the functions themselves are concave, so their Hessians are everywhere negative-definite. Theorem 1 points out that when these functions are sufficiently concave, then the vote maximizing model will exhibit a Nash equilibrium.

Because analysis of the stochastic model with activist valence is quite complex, this chapter also presents Theorem 2, for the case when activist valence is nonexistent. This theorem, valid for the case when only exogenous valence is relevant, gives the necessary and sufficient conditions for convergence of political agents to the electoral center.

In applying these result to the question of the switching of the positions and party affiliations of political agents, Theorem 2 is used to examine the Israeli Knesset in the elections between 1996 and 2006. These elections can be considered paradigmatic examples of political positioning under an electoral system that is highly proportional.

To understand these elections, we assume that parties adopt positions that are similar to their vote maximizing positions, as given by the stochastic vote model with exogenous valence. To further examine the nature of coalition bargaining, we use the idea of the *core*, presented earlier in Laver and Schofield (1990), to argue that a dominant party, located at the center of the policy space, can control the formation of government. In the postelection phase, the positions of the parties are assumed to be given, as is the distribution of seats. This distribution defines a set of winning coalitions. The compromise sets

of these various winning coalitions may intersect, to define a single policy point, the core. If the core is stable under small perturbations in the positions of the parties, then it is said to be structurally stable. If a party's position is at the structurally stable core, then we shall call this party the core party. Under these circumstances, it is argued that the core party will dominate the formation of government. If the compromise sets do not intersect, then they will define a domain in the policy space, known as the heart. This model of the heart can then be used to characterize the general pattern of coalition bargaining and government formation.

Because the coalition situation is very different depending on whether the core is empty or nonempty, it is argued that a party leader will have a very strong incentive to seek the central core, but only when the party leader has a very high valence. It is consistent with this argument that the policy move by Ariel Sharon, the leader of Likud in the Israel Knesset, in the months before the March 2006 election, was possible precisely because Sharon had such a high exogenous valence. To make such a policy move, Sharon had to switch out of the party, Likud, and create a new party called *Kadima* ("Forward"). It is implicit in this argument that political moves by other low valence parties would have had an insignificant effect on the political configuration.

In contrast, I suggest that the role of activists in a plurality electoral system provides extensive scope, in principle, for party switching. To apply Theorem 1, first consider presidential elections in the United States. To illustrate the idea, consider figure 3.1, and suppose that the economic dimension alone is relevant for political policymaking. We assume that there is an electoral distribution of voter ideal points, whose mean we take as the electoral origin. If we ignore activism for the moment, then the results of the following section show that there are two very different possibilities, depending on the parameters of the model. If the exogenous valences, λ_{dem} and λ_{rep} , of the Democrat and Republican candidates are sufficiently similar, then both candidates will position themselves at the electoral origin, and both will gain approximately 50 percent of the vote. In this case, potential activists are unlikely to be motivated to contribute to the parties.

On the other hand, if the valences differ sufficiently, with $\lambda_{dem} > \lambda_{rep}$, say, then the lower valence candidate will vacate the origin to increase vote share. For purposes of exposition, we may suppose that conservative economic activists have the preferred position, E. If the Republican candidate moves away from the origin, to a position

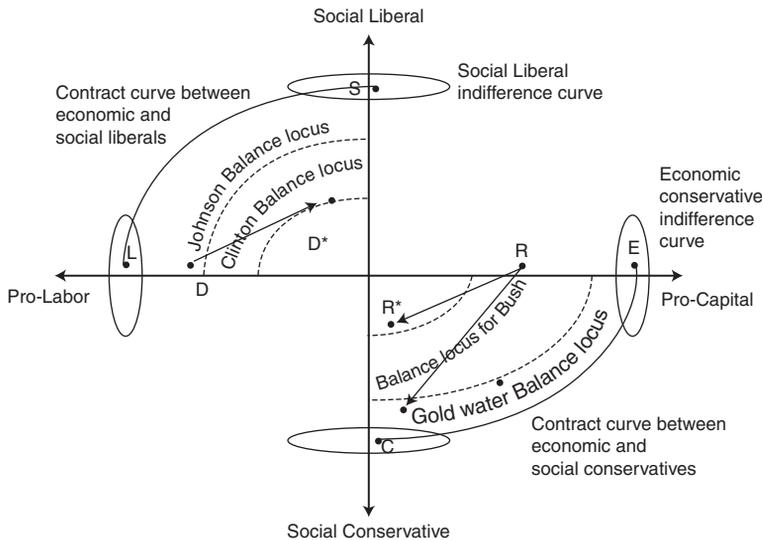


Figure 3.1 Activist coalitions in the United States.

similar to R, then conservative economic activists would be induced to support this candidate. The asymmetry induced by this support will cause liberal economic activists at L to support the Democratic candidate. Then R will be pulled further toward E, while D will be pulled toward L. Moreover, if the marginal effect of activists for the Republicans is greater than for the Democrats, then the optimal candidate positions, R and D, will satisfy $|R| > |D|$. This model implies that once the convergent equilibrium is destroyed because of some exogenous change in parameters, and activists become motivated to support the appropriate parties, then convergence can never be recreated.

Note that, in terms of the model, there is no reason why R should be to the right, and L to the left. However, once the move is made in one direction or the other, then activist support will tend to reinforce the left-right positioning of the parties.

This simple marginal calculation becomes more interesting when there is a second “social” dimension of policy. Consider the initial positions R and D, on either side of, and approximately equidistant from the origin, as in the figure. Both Social Conservatives, represented by C, and Social Liberals represented by S, would be indifferent between both parties. A Democratic candidate by moving to position D* will

benefit from activist support by the social liberals, but will lose some support from the liberal economic activists. Note that the figure is based on the supposition that activists are characterized by ellipsoidal indifference contours, reflecting the different saliences (or emphases) they put on the policy axes. The contract curve between the two activist groups, centered at L and S, represents the set of conflicting interests or “bargains” that can be made between these two groups over the policy to be followed by the candidate. Miller and Schofield (2003) showed that this contract curve between the economic and social liberals is a *catenary* whose curvature is determined by the eccentricity of the utility functions of the activist groups (See also Schofield and Miller 2007; Miller and Schofield 2008). We therefore call this contract curve *the Democratic activist catenary*. In the same way, the *Republican activist catenary* is given by the contract curve between economic conservative activists, positioned at E, and social conservative activists, positioned at C.

Theorem 1 gives the balance condition that a pair of positions, such as (D^*, R^*) in figure 3.1, constitute a Nash equilibrium for the vote maximizing game between the agents. These equilibrium positions will lie on what are called the *balance locii* of the agents. These balance locii will depend on the exogenous valences of the two candidates, as well as on the relative willingness of the various activist groups to provide support. For example, because the exogenous valence of Goldwater was much lower than Johnson in the 1964 election (Schofield, Miller, and Martin 2003), we can infer that the balance locus of Goldwater was further from the origin than that of Johnson (as illustrated in figure 3.1). The figure also presents plausible balance locii for Bush and Clinton in the elections of 2000 and 1992, respectively.

This model of activist support for presidential candidates (Miller and Schofield 2003; 2007; Schofield and Miller 2007) provides a more complex model of candidate positioning than the more simple model with exogenous valence. It suggests that candidates must juggle the opposed interests of competing activist groups. In recent US elections, the issues that concern voters and activist groups have involved the so-called war against terror, immigration, stem cell research, and so forth. These crosscutting issues create the possibility that a candidate for congressional office will jump or switch party, when the candidate believes that the party is overly dominated by certain activist groups whose policy positions are in opposition to the candidate. For example, a traditional Republican, such as John Danforth of Missouri,

with a policy position at E, may feel that his party is dominated by activists at C, possibly causing him to switch to a Democrat party that has adopted policies close to S. This model is applied in section 3.4 to discuss such recent examples of party switching by candidates in congressional and gubernatorial elections.

The general conclusions that can be drawn from the formal models are

(i) Under proportional rule, and with exogeneous valence alone, there will be a *centripetal* tendency toward the electoral center that will make itself felt only for political agents with high valence. In particular, if a party leader has a very high valence, like as Sharon in Israel, then this leader will have an incentive to leave his old party so as to be able to take up a dominant position at the core of the political configuration. Low valence party leaders, in contrast, will be subject to a centrifugal tendency toward the electoral periphery. In a situation where the core is empty, then policy repositioning by low valence party leaders will have a negligible effect on the political configuration.

(ii) In an electoral system based on plurality rule, such as the United States, activist groups have a significant effect on candidate positioning. Policy positions adopted by candidates will involve compromises between the more extreme positions preferred by activists, and the centrist positions that win votes. In a two-dimensional context, candidates for office may be obliged to switch parties (and choose new policy positions) to gain activist support from groups that previously opposed the candidate. In particular, when policy concerns are polarized, and activist groups play a major role, then party switching may induce a dramatic realignment of the two-party system.

3.2 Local Nash Equilibrium in the Stochastic Electoral Model

The purpose of this section is to present a model of positioning of parties in electoral competition so as to account for the generally observed phenomenon of nonconvergence. The model presented here is an extension of the multiparty stochastic model of Lin, Enelow, and Dorussen (1999) and McKelvey and Patty (2006), constructed by inducing asymmetries in terms of *valence*. The basis for this extension is the varied empirical evidence that valence is a significant component of the judgments made by voters of political agents. The stochastic model essentially assumes that political agents cannot predict

vote response precisely, but can estimate an expected vote share. In the model with valence, the stochastic element is associated with the weight given by each voter, i , to the average perceived quality or “valence” of the political agent. At a particular election, valence will be due to the “integral” of the agent’s competence in the past and expectations about future competence. Valence may also be effected by the resources available to the agent, and in particular the degree of positive media exposure that the agent can exploit.

There are a number of possible choices for the appropriate game form for multiparty competition. The simplest one, which is used here, is that the utility function for agent j is proportional to the vote share, V_j , of the agent. With this assumption, we can examine the conditions on the parameters of the stochastic model that are necessary and sufficient for the existence of a Pure strategy Nash Equilibrium (PNE) for this particular game form. This chapter uses calculus techniques to estimate optimal positions. As usual with this form of analysis, we can obtain the necessary conditions for the existence of local optima, or local pure strategy Nash equilibria (LNE). Clearly, any PNE will be a LNE, but not conversely. However, in the models considered here, it is evident that the necessary condition fails, leading to the inference that PNE are typically nonexistent. Although the true utility functions of party leaders are unknown, the comparison of LNE, obtained by simulation of empirical stochastic models, with the estimated positions of parties in the various polities that have been studied suggest that the valence model provides a close first order approximation to the political game.

Definition 1. *The Stochastic Vote Model $M(\lambda, \mu, \beta; \Psi)$ with Activist Valence for a policy space X of dimension w .*

(i) Each of the parties in the set $P = \{1, \dots, j, \dots, p\}$ chooses a policy, $z_j \in X$ to declare.

Let $\mathbf{z} = (z_1, \dots, z_p) \in X^p$ be a typical vector of party policy positions. Given \mathbf{z} , each voter, i , is described by a vector

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \dots, u_{ip}(x_i, z_p)),$$

$$\text{where } u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta \|x_i - z_j\|^2 + \epsilon_{ij}. \quad (1)$$

$$= \mathbf{u}_{ij}^*(x_i, z_j) + \epsilon_{ij}. \quad (2)$$

Here $\mathbf{u}_{ij}^*(x_i, z_j)$ is the observable component of utility. The term, λ_j , is the intrinsic or *exogenous valence* of party j , while the function

$\mu_j(z_j)$ is the component of valence generated by activist contributions to agent j . It is assumed that the resources contributed by activists are used by the political agent to enhance its image through advertising and the like.

The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of a metric induced from the Euclidean norm, $\|\cdot\|$, on X . The vector $\epsilon_i = (\epsilon_{i1}, \dots, \epsilon_{ij}, \dots, \epsilon_{ip})$ is the stochastic error. It is assumed the multivariate cumulative distribution of each ϵ_i is identical. This distribution is denoted by Ψ .

(ii) The exogenous valence vector

$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_p) \quad \text{satisfies} \quad \lambda_p \geq \lambda_{p-1} \geq \dots \geq \lambda_2 \geq \lambda_1.$$

(iii) Voter behavior is modeled by a probability vector. The probability that a voter i chooses party j at the vector z is

$$\rho_{ij}(z) = \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j] \quad (3)$$

$$= \Pr [\epsilon_{il} - \epsilon_{ij} < u_{ij}^*(x_i, z_j) - u_{il}^*(x_i, z_l), \text{ for all } l \neq j]. \quad (4)$$

Here \Pr stands for the probability operator generated by the distribution assumption on ϵ .

(iv) The *expected vote share* of agent j is

$$V_j(z) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(z). \quad (5)$$

The differentiable function $V : X^p \rightarrow \mathbb{R}^p$ is called the *party profile function*.

Definition 2. *Equilibrium Concepts.*

(i) A strategy vector $z^* = (z_1^*, \dots, z_p^*) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile function $V : X^p \rightarrow \mathbb{R}^p$ iff, for each agent $j \in P$, the policy z_j^* strictly locally maximizes $V_j(\dots z_{j-1}^*, -, z_{j+1}^* \dots)$.

(ii) A strategy vector z^* is a *weak Nash equilibrium* (PNE) iff, for each agent j , z_j^* weakly but globally maximizes $V_j(\dots z_{j-1}^*, -, z_{j+1}^* \dots)$.

The most common assumption in empirical analyses is that Ψ is the *Type I extreme value distribution* (sometimes called Gumbel). The theorems presented in this chapter are based on this assumption. This distribution assumption is the basis for much empirical work based on multinomial logit (MNL) estimation. Dow and Endersby (2004: 111) also suggest that “researchers are justified in using MNL specifications.”

Definition 3. The Type I Extreme Value Distribution, Ψ .

The cumulative distribution, Ψ , has the closed form

$$\Psi(b) = \exp [-\exp [-b]].$$

It follows from (2) that for each voter i , and agent j , the probability that a voter i chooses party j at the vector \mathbf{z} is

$$\rho_{ij}(\mathbf{z}) = [1 + \sum_{k \neq j} [\exp(f_k)]]^{-1} \quad (6)$$

$$\text{where } f_k = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta \|x_i - z_j\|^2 - \beta \|x_i - z_k\|^2.$$

It can be shown that the first order condition for \mathbf{z}^* to be a LSNE is that it be a *balance solution*.

Definition 4. The balance solution $\mathbf{z}^* = (z_1^*, \dots, z_j^*, \dots, z_p^*)$ for the model $M(\lambda, \mu, \beta; \Psi)$ is given by the set of equations, $j = 1, \dots, p$:

$$\left[\sum_{i=1}^n \alpha_{ij} x_i - z_j^* \right] + \frac{1}{2\beta} \frac{d\mu_j}{dz_j} = 0 \quad (7)$$

$$\text{where } [\alpha_{ij}] = \left[\frac{\rho_{ij} - \rho_{ij}^2}{\sum_i (\rho_{ij} - \rho_{ij}^2)} \right] \quad (8)$$

and $[\rho_{ij}] = [\rho_{ij}(\mathbf{z}^*)]$ is the matrix of choice probabilities at \mathbf{z}^* .

The vector $\sum_{i=1}^n \alpha_{ij} x_i$ is the *weighted electoral mean* of agent j . The bracketed term on the left of expression (7) is termed the *marginal electoral pull of agent j* and is a gradient vector pointing toward the weighted electoral mean of the agent. Thus, the weighted electoral mean is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called the *marginal activist pull for agent j* .

The following theorems are proved in Schofield (2006, 2007). See also Schofield (2008).

Theorem 1. Consider the electoral model $M(\lambda, \mu, \beta; \Psi)$ based on the Type I extreme value distribution, and including both exogenous and activist valences.

(i) The first order condition for \mathbf{z}^* to be an LSNE is that it is a balance solution.

(ii) If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great modulus, then the balance solution will be a PNE. \square

The following theorem classifies the model $M(\lambda, \beta; \Psi)$, when all activist functions are identically zero.

To state the theorem, the coordinates are first transformed, so that in the new coordinates, $x^s = \frac{1}{n} \sum x_i = 0$. The vector $\mathbf{z}_0 = (0, \dots, 0)$ is termed the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends on the distribution of voter ideal points. These are encoded in the electoral covariance matrix.

Definition 5: *The electoral covariance matrix*, ∇ is the w by w matrix $\frac{1}{n} [(\xi_r, \xi_s)]_{r,s=1,\dots,w}^{r=1,\dots,w}$ where $\xi_r = (x_{1r}, x_{2r}, \dots, x_{nr})$ is the n vector of the r^{th} coordinates of voter ideal points, and (ξ_r, ξ_s) refers to scalar product.

Write $v_s^2 = \frac{1}{n} (\xi_s, \xi_s)$ for the electoral variance on the s^{th} axis and

$$v^2 = \sum_{s=1}^w v_s^2 = \frac{1}{n} \sum_{s=1}^w (\xi_s, \xi_s) = \text{trace}(\nabla)$$

for the total electoral variance.

The formal model just presented, and based on Ψ is denoted $M(\lambda, \beta; \Psi)$.

Theorem 2.

(i) At the vector $\mathbf{z}_0 = (0, \dots, 0)$ the probability $\rho_{ij}(\mathbf{z}_0)$ that i votes for agent j is independent of i and is given by

$$\rho_j = \left[1 + \sum_{k \neq j} \exp[\lambda_k - \lambda_j] \right]^{-1} \quad (9)$$

(ii) The Hessian of the vote share function of the low valence party, 1, at the vector \mathbf{z}_0 is

$$C_1 = [\beta(1 - 2\rho_1)\nabla - I], \quad (10)$$

where I is the ω by ω identity matrix. (11)

(iii) The condition for the joint origin to be a LSNE in the model $M(\lambda, \beta; \Psi)$ is that C_1 has negative eigenvalues. \square

Corollary 1 Assume X is two dimensional. Then, for the model $M = M(\lambda, \beta; \Psi)$, the sufficient condition for the joint origin to be a LSNE is that the coefficient $c(\lambda, \beta; \Psi) = 2\beta[1 - 2\rho_1]\text{trace}(\nabla)$ is strictly less than 1.

Corollary 2 In the case that X is w -dimensional, the necessary condition for the joint origin to be a LSNE is that $c(\lambda, \beta; \Psi) < w$.

3.3 Empirical Analyses for Israel

3.3.1 The Israeli Knesset in 1996 and 2006

To illustrate Theorem 2, consider the case of Israel in 1996. Figure 3.2 shows the estimated positions of the parties at the time of the 1996 election in a two dimensional policy space (see Schofield and Sened 2006 for details).

Table 3.1 presents summary statistics of the 1996 election, together with valence estimates based on a stochastic model derived from a voter survey (Arian and Shamir 1999). It is readily shown that one of the eigenvalues of the NRP is positive. Indeed it is obvious that there is a principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. The formal analysis indicates that low valence parties should position themselves on this eigenspace as illustrated in the simulation given in figure 3.3.

As table 3.1 indicates, the lowest valence party was the NRP with valence -4.52 . The spatial coefficient is $\beta = 1.12$, and the electoral variances on the two axes are 1.0 and 0.732 respectively. Computations for the model $M(\Psi)$ give the following:

$$\rho_{NRP} \simeq \frac{1}{1 + e^{4.15+4.52} + e^{3.14+4.52}} \simeq 0.$$

$$\nabla = \begin{bmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{bmatrix}$$

$$C_{NRP} = 2(1.12)\nabla - I = \begin{bmatrix} 1.24 & 1.32 \\ 1.32 & 0.64 \end{bmatrix}$$

$$c(\lambda, \beta; \Psi) = 3.88.$$

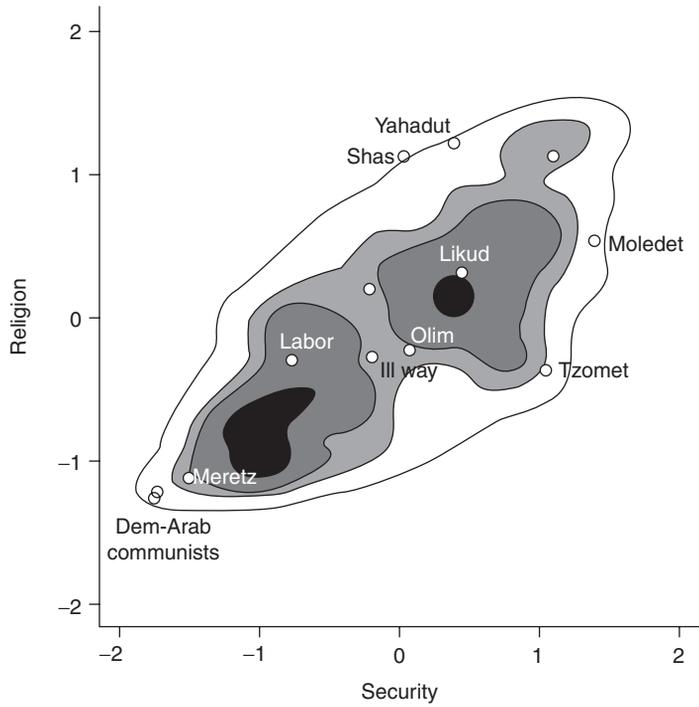


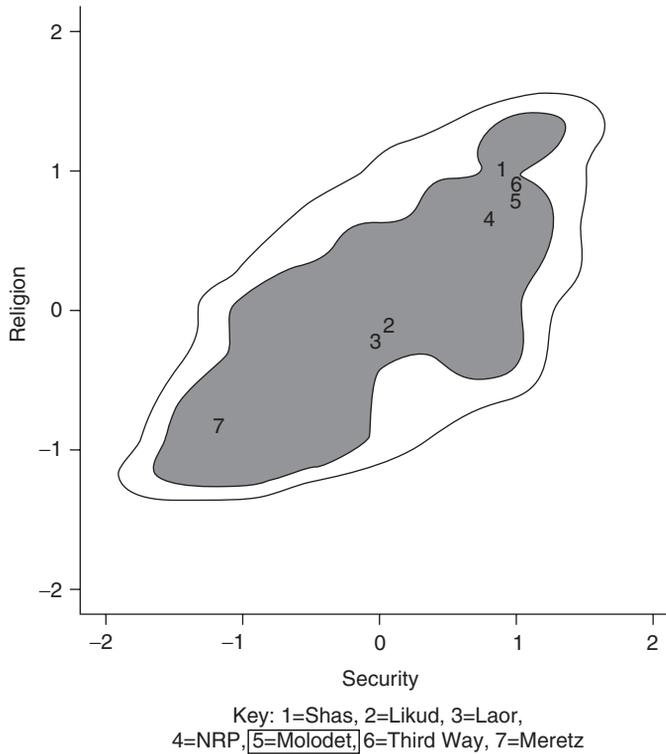
Figure 3.2 Party positions in the Israeli Knesset 1996.

Table 3.1 Seats, votes, and valences in the Knesset

Party	1996	1996	1996	1996	1999	2003
	National Vote %	Sample Vote %	Seats	Valence ¹	Seats	Seats
Others Left	7.3	0	9	-	10	9
Meretz	7.6	6.0	9	0	10	6
Labor	27.5	44.0	34	4.15	28	21
Center	3.2	1.8	11	-2.34	18	15
Likud	25.8	43.0	32	3.14	23	47 ^a
Shas	8.7	2.0	10	-2.96	17	11 ^b
NRP	8.0	5.1	9	-4.52	5	6
Moledet	2.4	1.8	2	-0.89	4	-
Yahadut	3.7	1.8	4	-	5	5 ^b

Note:

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Figure 3.3 A local Nash equilibrium in the Knesset in 1996.

The eigenvalues of C_{NRP} are 2.28 and -0.40 , giving a saddlepoint, and a value for the coefficient, $c(\lambda, \beta; \Psi)$, of 3.88. This exceeds the necessary upper bound of 2.0, required for an equilibrium at the joint origin. We immediately infer that all local equilibria are nonconvergent. The major eigenvector for the NRP is $(1.0, 0.8)$, and along this axis the NRP vote share function increases as the party moves away from the origin. The minor, perpendicular axis is given by the vector $(1, -1.25)$ and on this axis the NRP vote share decreases as the party moves away from the origin. Figure 3.3, gives one of the local equilibria in 1996, obtained by simulation of the model. The figure makes it clear that the vote maximizing positions lie on the principal axis through the origin and the point $(1.0, 0.8)$. Five different LSNE were located: in all cases, the two high valence parties, Labor and Likud, were located at almost precisely the same positions. The only

difference between any two of these various equilibria were that the positions of the low valence parties were somewhat perturbed.

The simulations of vote maximizing positions were compatible with the predictions of the formal model based on the extreme value distribution. All parties were able to increase vote shares by moving away from the origin, along the principal axis, as determined by the large, positive principal eigenvalue. In particular, the simulation confirms the logic of the above analysis. Low valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center. Their optimal positions will lie either in the northeast quadrant or the southwest quadrant.

In contrast, since the valence difference between Labor and Likud was relatively low in the elections of 1988, 1992, and 1996, the relevant eigenvalues on the major axis, for their Hessians at the origin, were also low (but still positive) and their optimal positions would be relatively close to, but not identical to, the electoral mean. Simulation for the elections of 1988, 1992, and 1996 were compatible with this theoretical inference. It should be noted that the positions of Labor and Likud are particularly closely matched by their positions in the simulated vote maximizing equilibria.

Clearly, the configuration of equilibrium party positions will fluctuate as the valences of the large parties change in response to exogenous shocks. The logic of the model remains valid however, since the low valence parties will be obliged to adopt relatively radical positions to maximize their vote shares.

It is important to note, however, that the positions of the low valence parties in figure 3.2 are similar, but not identical, to their estimated vote maximizing equilibrium position as given in figure 3.3. This strongly suggests that an appropriate model of party positioning assumes that parties are concerned with policy, and adopt positions with a view toward the coalitions that may form after the election. One way to express this inference is as follows: The close correspondence between the simulated LSNE based on the empirical analysis and the estimated actual political configuration suggests that the true utility function for each party j has the form $U_j(\mathbf{z}) = V_j(\mathbf{z}) + \delta_j(\mathbf{z})$, where $\delta_j(\mathbf{z})$ may depend on the beliefs of party members about the postelection coalition possibilities and on the support of activist coalitions. This suggests that in any political equilibrium of the electoral game, a party can position itself at the electoral origin only when its leader has a very high valence in contrast to all other competing political leaders.

3.3.2 Recent Switches in the Israeli Knesset

This section discusses recent changes in the political configuration in Israel. Figure 3.4 provides a schematic representation of the Knesset based on the party positions after the election of 2003. The figure shows Labor with 21 seats, after Am Ehad, with 2 seats, joined Labor in 2003, while Likud has 40 seats after being joined by Olim, with 2 seats. Although Barak, of Labor, became prime minister in 1999, he was defeated by Ariel Sharon, of Likud, in the election for prime minister in 2000. Schofield (1999) and Schofield and Sened (2006) propose a set known as the heart to describe the set of policies that may occur as a result of coalition bargaining. The figure indicates that there are three plausible governments: Likud, Shinui, and a religious party; Likud and Shas/Shinui; and a coalition of the left, led by Labor, with support from Shas/Shinui. These coalition possibilities are encapsulated in the idea of the heart (the triangular domain in figure 3.4). Notice that this domain is insensitive to small changes in location or strength of these parties. Given the vector of leader exogenous valences at the time of the 2003 election,

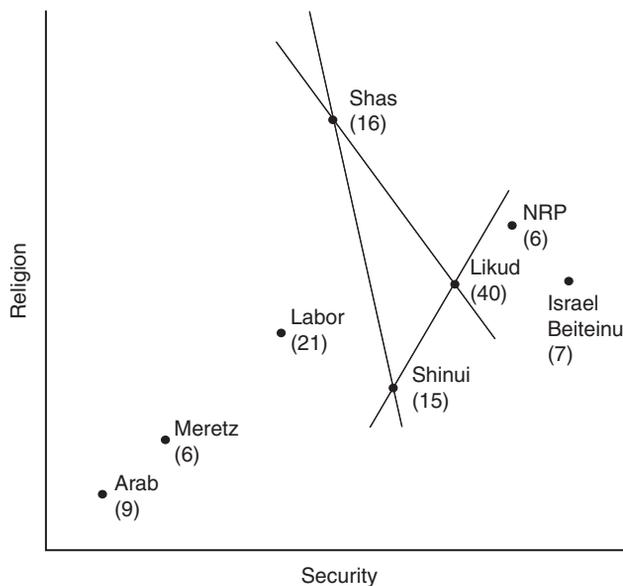


Figure 3.4 Party positions in the Knesset in 2003.

it is reasonable to suppose that the election result would give rise to an empty core. Notice that the party positions given in figure 3.4 are consistent with the empirical and simulation analysis presented earlier. In particular, it is obvious that neither Likud nor Labor are located at the electoral origin.⁵

Now consider figure 3.5, which presents estimates of the party positions at the election of March, 2006. In a highly publicized move before the election, Sharon left the Likud Party and signaled a strong move to the left by allying with Shimon Peres, the former leader of Labor and the author of the Oslo accords, together with a number of other senior Labor Party members, to form the new party, Kadima. This move positioned Kadima at the origin of the electoral space at (0, 0), as shown in figure 3.5. Following Sharon's stroke in January 2006, Ehud Olmert took over as leader of Kadima and was able to win 29 seats. Likud took only 12 seats, while the 4 parties on the upper right of the figure won 38 seats. One surprise of the election was the appearance of a Pensioners' party with 7 seats. The consequence of this sequence of events is that Kadima secured itself a weakly dominant position in the Knesset. The Kadima position in figure 3.5 can

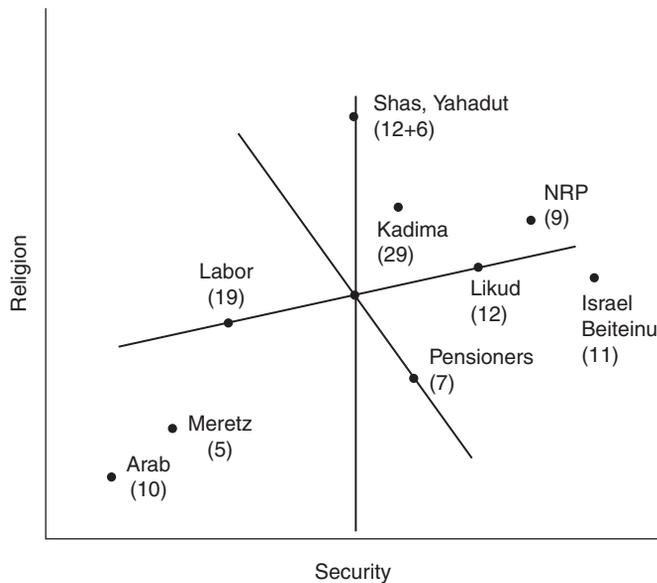


Figure 3.5 Party positions in the Knesset in 2006.

be readily be seen to be a policy core (Laver and Schofield 1990). To see this, note that the convex hull of the policy positions of parties in any winning coalition (with more than 61 seats) must include the position of Kadima. Notice, however, that the core position of Kadima is structurally unstable, in the sense that a small move by Labor on the vertical axis creates the possibility of a counter-Kadima coalition, involving Labor and Likud. As long as Labor is situated at the position given in figure 3.5, then Kadima will be able to veto any attempt to construct a winning coalition government excluding Kadima.

The simulations of the 1996 election in Israel demonstrated that Likud was previously unable to capture the core. By moving Labor to the left, Peretz created the opportunity for Sharon to out maneuver him, allowing Sharon to strategically move to a position that would increase the probability that he would control the core. Because Sharon's own party members would not support him in this move, he had to leave Likud and form Kadima. The formal analysis presented earlier suggests that the location of Kadima at the electoral origin was a vote maximizing position for the party because of the high valence difference between Sharon and Peretz.⁶

It thus appears that Sharon's shift in party and his change of policy has led to a fundamental transformation in the political configuration, from the coalition structure without a core (that had persisted since 1996), to a new configuration, associated with the center, core party, Kadima. Even though Kadima is estimated to be a core party after the 2006 election, Olmert needed the support of Labor to be able to deal with the complex issue of fixing a permanent border for Israel. The debacle in Lebanon severely weakened Olmert's popularity, and in October 2006, the 61 members of the Kadima-Labor governing coalition voted to bring Israel Beiteinu into the coalition. The April 2007 report on the failure of the government during the war with Lebanon in summer 2006 has threatened the stability of the government. Barak won the election for the Labor party leadership on June 12, 2007, and became minister of defense in the Kadima-led government on June 18, 2007, while Shimon Peres became president. On February 3, 2008, Avigdor Lieberman, chairman of Israeli Beiteinu, announced that the party would quit the government because of disagreement over issues such as Jerusalem, the refugees, and the nature of a future Palestinian state. Barak agreed to remain in the coalition, thus maintaining the majority of the coalition. However, as of July 2008, Olmert faces charges of corruption, and he only avoided a vote of no-confidence by agreeing to an election for leader of Kadima in the following September.

Although the Kadima government is structurally unstable, in the sense used here, it has opened negotiations with both Syria, over the question of stability in Lebanon, and with Hamas, using Turkey and Egypt as intermediaries.

3.4 Party Switches in the United States

As figure 3.1 suggests, social conservative activists have come to prominence in US presidential elections. As social conservatives gain control of the Republican political machinery in a given state or electoral district, socially moderate Republican candidates may be forced into the Democratic Party simply because they can no longer hope to win a Republican primary. The normal ambition of politicians transforms socially liberal Republicans into moderate Democrats. The result is increased party polarization on the social dimension and decreased party differences on the economic dimension.

A case in point is John Moore, a longtime executive with Cessna Aircraft in Wichita; a pro-business conservative, he was nevertheless unlikely to win a Republican primary for any statewide position due to his “softness” on social issues. He consequently converted in 2002 and was elected as the Democratic lieutenant governor. Moore retired in 2006, and the open position brought about an even more dramatic development. Mark Parkinson officially switched parties in time to run for the lieutenant governor’s position. Parkinson, a former Republican Party Chairman for the state of Kansas, was elected lieutenant governor on the Democratic ticket. Others in Kansas are going the same route. In 2004, Republican Cindy Neighbor switched parties to run for the state legislature, opposed to a social conservative who had defeated her in the primary in 2004. She was elected in 2006 (Milburn 2006).

These ballot box conversions are not limited to ambitious Kansas moderates. Perhaps the most striking and visible such conversion was that of Jim Webb of Virginia. Webb is a much-decorated Vietnam war veteran who had been Reagan’s Secretary of the Navy. As recently as 2000, he supported Republican Allen to be the US Senator from Virginia. In 2006, however, he was a Democrat running against Allen. Traditional New Deal Democrats were aghast; but Webb defeated Allen, and his presence in the party moves the Democratic center of gravity to the right on economic policy.

Each such switch makes further switches more likely. Although Kansas has been seen as a state in which the Democratic Party is all

but defunct, the conversion of a small number of socially moderate Republicans to the Democratic Party could easily restore a healthy two party competition in Kansas. But in the process, each individual conversion *changes what it means to be a Democrat* (cf. Heller and Mershon this volume). Increasingly, a Democrat is an economic moderate or conservative who is strongly liberal on social issues—not (as in the New Deal), a strong economic liberal whose Democratic affiliation is a response to class conflict.

These observations are not meant to advocate any particular strategy for either party. Rather, they suggest that partisan change continues to have a certain inevitability about it, despite the fond wishes of entrenched party activists. Each partisan realignment has occurred *despite the opposition of existing party activists*.

Populist Democrats in the 1930s, who earlier supported William Jennings Bryan, were suspicious of the ethnic industrial laborers that the New Deal brought into the party. In the same way, traditional Republican activists were aghast when their candidate Rockefeller was booed for criticizing Goldwater-style radicalism at the 1964 convention (Branch 1998, 402).

Partisan realignment is a dynamic process because of the destabilizing influence of vote-maximizing candidates who see opportunities to win elections even at the cost of generating some hostility within the ranks of the preexisting activist cadres. As a result, partisan identities are always changing, even though there is a tendency to see them as fixed and immutable. The Republican Party in 1868 was the post-civil war party of racial equality through strong national government. The Republican Party in 1948 was the party of the balanced budget and civil libertarianism. Neither of these identities proved to be immutable, and the current identities of both parties are again in flux.

The departure of even a small number of pro-business social liberals from the Republican Party—like Jeffords of Vermont or Parkinson in Kansas—has inevitable effects on both parties. Each such departure increases the proportion of social conservatives in the Republican Party, making it easier for social conservatives to dominate both the party primaries and the activists who give the party its image to the nation. This in turn makes it even more difficult for social liberals to hope for a successful career within the GOP. Voters, as well as activists and candidates, adjust. If they are concerned about women's rights or the separation of church and state, they are less likely to vote as Republican and more likely to shift to independent or Democratic status.

At the same time, symmetrical adjustments are made in the Democratic Party. Just as Strom Thurmond's conversion to the Republican Party helped trigger a long list of similar conversions by socially conservative Democrats, so each socially liberal Republican who converts to the Democratic party makes the social issue that triggered the conversion a more salient aspect of the Democratic identity. The Democratic Party and Republican Party become internally more homogeneous as regards economic policy, and more polarized with respect to social policy.

Thus, as social polarization increases between the parties, the economic differences will slowly disappear. As pro-business social liberals join the Democratic Party, it will become increasingly difficult to imagine that party going back to a New Deal identity. Just as the New Deal Democratic Party consisted of segregationists and labor unions united on an antibusiness platform, the emerging Democratic Party will find itself united at a social liberal position, with a centrist position on economic policy. The proportion of Democrats who adopt a traditional antibusiness stance will be reduced. A simple electoral calculus by candidates will tend to move them to a Clinton-style moderate position on economic policy—advocating (among other things), a more inclusive policy toward immigrants, and a more enthusiastic commitment to stem cell and related medical research.

3.5 Concluding Remarks

The theorems presented here, together with analysis of spatial policy maps (Schofield 2008), suggest the following set of tentative hypotheses about the nature of political competition. The examples from Israel and the United States allow for further hypotheses about the role of party switching in affecting coalition formation, under electoral systems based on proportional representation, and in influencing political realignment in plurality electoral systems.

1. The pure spatial model of direct democracy indicates that the occurrence of a core, or unbeaten alternative, is very unlikely in a direct democracy using majority rule, when the dimension of the policy is at least two (Schofield 2008). However, a social choice concept known as the heart, a generalization of the core, will exist, and converges to the core when the core is nonempty (Schofield 1999).
2. A legislative body, made up of democratically elected representatives, can be modeled in social choice terms. Because party strengths will be

disparate, a large, centrally located party may be located at a core position. Such a party, even in a situation with no majority party, will be able to dominate the formation of coalition government, as suggested by the example from Israel.

3. A more typical situation is one with no core party. In such a case, the legislative heart can give an indication of the nature of bargaining between parties as they attempt to form a winning coalition government.
4. This theory of legislative behavior takes as given the position and strengths of the parties. Because a centrally located party may dominate coalitional bargaining, and because such a party should be able to garner a large share of the vote, there would appear to be a strong *centripetal* tendency in all electoral systems based on proportional representation. However, this tendency will be most relevant for a party whose leader has a high exogenous valence.
5. Estimates of party positions indicate that parties adopt quite heterogeneous positions.⁷ This suggests that there is a countervailing or *centrifugal* force that affects all parties other than those with very high valence.
6. Although core parties can be observed in a number of polities with electoral systems based on proportional rule, the dominance of such center parties can be destroyed, particularly if there is a tendency to political fragmentation and social conflict. The stochastic model suggests that only a political leader with a high exogenous valence is able to move to a central position to control the core.
7. Although the model with exogenous valence is able, in some circumstances, to predict the existence of a core party, it is suggested that the model with activist valence gives a better understanding of the extensive dispersion of positions of the parties.⁸
8. This suggests that party location can be modeled as a balancing act between the *centripetal electoral* pull, and the *activist centrifugal* pull. This implies that on occasion a party leader is subject to a conflict between the more extreme policy preferences of activists for his party, and the centripetal tendency implied by his high exogenous valence. A switch of policy position by a leader, such as Sharon in Israel in 2006, can then be rationalized in terms of the centripetal tendency implied by high valence.
9. Under *proportional electoral methods*, there need be no strong tendency forcing activist groups to coalesce in order to concentrate their influence. Thus proportionality in the electoral system, as in Israel and many European polities (see Benoit and Laver 2006; Schofield 2008), leads to activist fragmentation and thus party fragmentation. Belgium is a good example, as discussed in Schofield (2008). More generally, it can be expected that low valence political leaders may switch parties particularly in situations where they perceive that they can gain increased resources from activist groups as a result of switching.

10. Under *plurality rule*, small parties face the possibility of extinction. Unlike the situation in a polity based on proportional rule, an activist group linked to a small party in a plurality polity has little expectation of influencing government policy. Thus activist groups face increasing returns to scale, in terms of the relationship between the size of the political coalition that they influence on the one hand and political power and representation on the other. The activist model of elections presented in this chapter suggests that when there are two dimensions of policy, then there can be a number of principal, opposed activist groups. The nature of the electoral contest generally forces these activist groups to concentrate support for at most two parties, as in the United States.
11. Thus, in the United States, plurality rule induces the two party system, through this effect on activist groups. Although the two party configuration may be in equilibrium at any time, the tension within the activist coalitions induces a slow rotation, and thus political realignment. Candidates for political office, at the federal and local level, will be forced to balance the centripetal electoral effect against the centrifugal effect induced by activist groups.
12. On occasion in the United States, polarization within the two dimensions of policy may induce conflict within and between activist groups. These effects may become so pronounced that the two party system may become less stable.⁹ The illustrations presented in this chapter suggest that switching between parties can become extreme, leading eventually to a political realignment.
13. The well-known relationship between proportional representation and the degree of political fragmentation may be accounted for indirectly as a consequence of the logic forced on activist groups and the effect induced on the political configuration.

3.6 Notes

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1. Stokes (1963: 373) used the term *valence issues* to refer to those that “involve the linking of the parties with some condition that is positively or negatively valued by the electorate.” As he observes, “in American presidential elections... it is remarkable how many valence issues have held the center of the stage.” Stokes observation is validated by recent empirical work on many polities, as well as a study on the psychology of voting (Westen 2007).

2. However, current exogenous valence will depend on the agents previous policy choices, and the outcome of these choices. As an illustration, George Bush's low overall valence, as of July 2008, is a consequence of policy choices in the past that turned out to be imprudent.
3. For convenience, we assume that $\mu_j(z_j)$ is only dependent on z_j , and not on z_k , $k \neq j$, but this is not a crucial assumption.
4. In other words, it is not the source of the resources that matters, just the amount.
5. Jeong et al. (2008) estimate the 2003 election. Their results indicate that Sharon had a very high valence in 2003.
6. We may hypothesize that Sharon was able to position Kadima at the center precisely because of his high valence.
7. This is clearly indicated by the estimates of party position presented in Benoit and Laver (2006).
8. The empirical analyses in Schofield and Sened (2006) for Italy, Netherlands, and the United Kingdom support this inference.
9. Of course, there have been presidential elections, such as 1860, where activist polarization has been so strong that there were four competing candidates.

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