5

An Activist Model of Democracy

Norman Schofield
Washington University

5.1. Introduction: A Stochastic Model of Elections

The focus of this paper is that actual political systems do not appear to satisfy the property of *convergence to an electoral center* that is often predicted by formal vote models. The key theoretical idea is that the convergence result need not hold if there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes 1992). The average weight given to the perceived quality of the leader of a party is called the party's *valence*. In empirical models, a party's valence is usually assumed to be *exogenous*, and independent of the party's position. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn 2003).

The motivation for the development of the *activist stochastic electoral model*, which is presented in this paper, is based on a set of empirical results from multinomial logit electoral estimation for the Netherlands, Britain and the United States.[1] These empirical analyses, coupled with theoretical results, indicate that the stochastic model with exogenous valence cannot fully account for the divergence observed in a number of elections in these polities.

Theorem 5.2, presented below, gives the necessary and sufficient conditions for convergence to the electoral mean in the stochastic model with exogenous valence. The necessary condition is that a convergence coefficient, c, is bounded above by the dimension, w, of the policy space, while a sufficient condition is that the coefficient is bounded above by 1. This coefficient is defined in terms of the difference in exogenous valences, the "spatial coefficient" and the electoral variance. The empirical work on Britain and the Netherlands indicates that the necessary condition was

[1] The details of the estimations are presented in Schofield and Sened (2006).

satisfied. Indeed the *mean voter theorem* should have been valid, even though there was empirical evidence that the parties did not converge to an electoral mean.

Section 5.2 of this paper presents the formal results on the extension of the standard stochastic model based on exogenous valence by adopting the assumption that there are two kinds of valence. The first kind is the usual *exogenous valence*, which for a party j is denoted λ_j. As in empirical work, this formal model assumes that λ_j is held constant at the time of an election, and so is independent of the party's position. The second kind of valence is known as *activist valence*. When party j adopts a policy position z_j, we denote the activist valence of the party by $\mu_j\ (z_j)$. Implicitly the model is an extension of one originally due to Aldrich (1983a,b). In this model, activists provide crucial resources of time and money to their chosen party. The party then uses these resources to enhance its image before the electorate, thus affecting its valence. Although activist valence is affected by party position, it does not operate in the usual way by influencing voter choice through the distance between a voter's preferred policy position, say x_i, and the party position. Rather, as party j's activist support, $\mu_j\ (z_j)$, increases due to increased contributions to the party in contrast to the support $\mu_k\ (z_k)$ received by party k, then (in the model) all voters become more likely to support party j over party k. The problem for each party is that activists are likely to be more extreme than the typical voter. By choosing a policy position to maximize activist support, the party will lose centrist voters. The party must therefore calculate the optimal marginal condition to maximize vote share.The main result, Theorem 5.1, gives this as a (first order) *balance condition*. Moreover, because activist support is denominated in terms of time and money, it is reasonable to suppose that the activist function will exhibit decreasing returns, so that the functions themselves are concave, and their Hessians are everywhere negative-definite. Theorem 5.1 asserts that when these functions are sufficiently concave, then the activist vote maximizing model will exhibit a *Pure Strategy Nash Equilibrium*. Theorem 5.2 presents the results when each party attempts to maximize an expected vote share, where this is defined in terms of a weighted sum of the voter probabilities. In principle, these voter weights can be deduced from the elctoral model utilized in the polity. Theorem 5.2 specializes to the egalitarian case where all voter weights are identical, and obtains the necessary and sufficient conditions for the validity of the mean voter theorem. This model is applicable to electoral systems based on proportional representation.[2]

[2] The proof of Theorem 5.1 is given in the working paper version and is available at http://polisci.wustl.edu/sub_page.php?s=3&m=0&d=24.

Section 5.3 presents the empirical work on elections in the Netherlands in 1977 and in Britain in 1997. A brief illustration is provided of the application of the model to recent elections in the United States. The concluding section 5.4 argues that there is, in general, no centripetal tendency towards an electoral center. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and Riker (1953) and note that under proportional electoral methods, there is very little motivation for interest groups to coalesce. Another way of expressing, in simplified form, the difference between proportional representation and plurality rule is this: under proportional electoral methods, bargaining to create winning coalitions occurs *after* the election. Under plurality rule, if interest groups do not form a coalition *before* the election, then they can be obliterated. This obviously creates a pressure for activist groups to coalesce. Other work (Schofield and Ozdemir 2008) uses this idea to explore the difference between plurality rule and proportional representation that has been pointed out by Duverger (1954).

5.2. A Political Economy Model of Leader Support

The model presented here is an extension of the standard multiparty stochastic model, modified by inducing asymmetries in terms of valence.

The key idea underlying the formal model is that political leaders attempt to estimate the effects of their policy positions on the support they receive. Each leader, whether autocrat or opposition, chooses the policy position as best response to opposing position(s), in order to obtain sufficient support either to retain power or to gain power. The stochastic model essentially assumes that a leader cannot predict support precisely, but can estimate an expected support. In the model with valence, the stochastic aspect of the model is associated with the weight given by each citizen, *i*, to the average perceived quality or valence of the party leader.

Definition 5.1. The *Stochastic Model* $E(\mathrm{l}, \mathrm{m}, \beta; \Psi)$ *with Activist Valence.*

The data of the spatial model is a distribution, $\{x_i, \in W: \mathrm{i} \in P\}$, of voter ideal points for the members of the *selectorate*, P, of size p. By the selectorate we mean those citizens who have some potential to influence political choice. We assume that W is an open, convex subset of Euclidean space, R^w, with w finite. Each of the leaders in the set $N = \{1,..., j,..., n\}$ chooses a policy, $z_j \in W$, to declare. Let $\mathbf{z} = (z_1,..., z_n) \in W^n$ be a typical vector of leader positions.

Given $\mathbf{z}$, each citizen, i, is described by a vector

$$\mathbf{u}_i\,(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1),\ldots, u_{ip}\,(x_i, z_n))$$

where

$$u_{i1}(x_i, z_1) = \lambda_j + \mu_j\,(z_j) - \beta\,\|\,x_i - z_j\,\|^2 + \varepsilon_j = u^*_{ij}\,(x_i, z_1) + \varepsilon_j. \tag{5.1}$$

Here $u^*_{ij}\,(x_i, z_1)$ is the observable component of utility. The term, λ_j, is the fixed or *exogenous valence* of leader j, while the function $\mu_j\,(z_j)$ is the component of valence generated by activist contributions to leader j. The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of the Euclidean metric, $\|a - b\|$, on W. The vector $\varepsilon = (\varepsilon_1,\ldots, \varepsilon_j,\ldots,\ \varepsilon_n)$ is the stochastic error, whose multivariate cumulative distribution will be denoted by Ψ.

It is assumed that the exogenous valence vector

$$\mathbf{l} = (\lambda_1, \lambda_2,\ldots, \lambda_n) \text{ satisfies } \lambda_n \geq \lambda_{n-1} \geq \ldots \geq \lambda_2 \geq \lambda_1$$

Citizen behavior is modelled by a probability vector. The probability that a citizen i chooses leader j at the vector $\mathbf{z}$ is

$$\rho_{ij}\,(\mathbf{z}) = \Pr[\,[\,u_{ij}(x_i, z_j) > u_{i1}\,(x_i, z_l)\,],\ \textit{for all } l \neq j] \tag{5.2}$$
$$= \Pr[\varepsilon_l - \varepsilon_j < u^*_{ij}\,(x_i, z_j) - u^*_{il}\,(x_i, z_j),\ \textit{for all } l \neq j] \tag{5.3}$$

Here Pr stands for the probability operator generated by the distribution assumption on ε.

The *expected support* of leader j is.

$$V_j\,(\mathbf{z}) = \frac{\sum_{i \in P} s_{ij}\,\rho_{ij}\,(\mathbf{z})}{\sum_{i \in P} s_{ij}} \tag{5.4}$$

The weights $\{s_{ij}\}$ allow for the possibility that individuals belong to different constituencies and have differing political power. Without loss of generality, we normalize and assume for each j that $\sum_{i \in P} s_{ij} = 1$.

In democratic polities based on proportional representaion we can assume that each $s_{ij} = \frac{1}{p}$ for all i, j. We call this the *egalitarian* case. In non-democratic polities the weights s_{ij} may differ widely. The differentiable function $V : W^n \to \mathrm{R}^n$ is called the *leader profile function.*

In the following it is assumed that the stochastic errors have the Type I extreme value (or Gumbel) distribution, Ψ (Train, 2003). The formal model based on Ψ parallels the empirical models based on multinomial logit (MNL) estimation.

Definition 5.2. *The Extreme Value Distribution,* Ψ.

The cumulative distribution, Ψ, has the closed form

$$\Psi(x) = \exp\left[-\exp[-x]\right]$$

The difference between the Gumbel and normal (or Gaussian) distributions is that the latter is perfectly symmetric about zero.

With this distribution assumption, it follows, for each voter i and leader j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{n} \exp u^*_{ik}(x_i, z_k)} \tag{5.5}$$

In this stochastic electoral model it is assumed that each leader j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, \ldots, z_{j-1}, \ldots, z_{j+1}, \ldots, z_n)$.

Definition 5.3. *Equilibrium Concepts.*

(i) A strategy vector $\mathbf{z}^* = (z^*_1, \ldots, z^*_{j-1}, z^*_j, z^*_{j+1}, \ldots, z^*_n) \in W^n$ *is a local strict Nash equilibrium (LSNE) for the profile function* $V : W^n \to \mathrm{R}^n$ *iff, for each leader* $j \in N$, *there exists a neighborhood* W_j *of* z^*_j *in* W *such that*

$$V_j(z^*_1, \ldots, z^*_{j-1}, z^*_j, z^*_{j+1}, \ldots, z^*_p) > V_j(z^*_1, \ldots z^*_{j-1}, z_j, z^*_{j+1} \ldots, z^*_n)$$

for all $z_j \in W_j - \{z^*_j\}$.

(ii) A strategy vector $\mathbf{z}^* = (z^*_1, \ldots, z^*_{j-1}, z^*_j, z^*_{j+1}, \ldots, z^*_n)$ *is a local weak Nash equilibrium (LNE) iff, for each agent j, there exists a neighborhood* W_j *of* z^*_j *in W such that*

$$V_j(z^*_1, \ldots, z^*_{j-1}, z^*_j, z^*_{j+1}, \ldots, z^*_n) \geq V_j(z^*_1, \ldots z^*_{j-1}, z_j, z^*_{j+1} \ldots, z^*_n)$$

for all $z_j \in W_j$.

(iii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$ *is a srict or weak, pure strategy Nash equilibrium (PSNE or PNE) iff* W_j *can be replaced by W in (i),(ii) respectively.*

(iv) The strategy z_j^* *is termed a "local strict best response", a "local weak best response", a "global weak best response", a "global strict best response", respectively to* $\mathbf{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_n^*)$. W

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}^*$ to be a LNE and thus a PNE, without having to invoke concavity. Of particular interest is the vector

$$x_j^* = \frac{\sum_{i \in P} s_{ij} x_i}{\sum_{i \in P} s_{ij}} = \sum_{i \in P} s_{ij} x_i \tag{5.6}$$

In the *egalitarian* case, all $s_{ij} = 1/p$, and we can transform coordinates so that in the new coordinate system, $x^* = \sum_{i \in P} x_i = 0$. We shall refer to $\mathbf{z}_0 = (0, \ldots, 0)$ as the joint *electorate origin*.

Theorem 5.1 shows, even in the egalitarian case, that $\mathbf{z}_0 = (0, \ldots, 0)$ will generally not satisfy the first order condition for a LSNE, namely that the differential of V_j, with respect to z_j be zero. However, if the activist valence function is identically zero, so that only exogenous valence is relevant, then the first order condition at $\mathbf{z}_0$ will be satisfied.

It follows the definition of the Gumbel distribution, that for voter i, with ideal point, x_i, the probability, $\rho_{ij}(\mathbf{z})$, that i picks j at $\mathbf{z}$ is given by

$$\rho_{ij}(\mathbf{z}) = \left[1 + \Sigma_{k \neq j}[\exp(f_{jk})]\right]^{-1} \tag{5.7}$$

$$\text{where } f_{jk} = \lambda_k + \mu_k(z_k) - \lambda_j - \mu_j(z_j) + \beta \, || x_i - z_j ||^2 - \beta \, || x_i - z_k ||^2.$$

Schofield (2006a) shows that the first order condition for $\mathbf{z}^*$ to be a LSNE is that it be a *balance solution*.

Definition 5.4. *The balance solution for the model* $E(\lambda, \mathrm{m}, \beta; \Psi)$.
Let $[\rho_{ij}(\mathbf{z})] = [\rho_{ij}]$ *be the matrix of voter probabilities at the vector* $\mathbf{z}$*, and let*

$$\alpha_{ij} = \frac{s_{ij}\,[\rho_{ij} - \rho_{ij}^2]}{\sum_{k \in P} s_{kj}\,[\rho_{kj} - \rho_{ij}^2]} \tag{5.8}$$

be the matrix of coefficients. The balance equation for z_j^* is given by expression

$$z_j^* = \frac{1}{2\beta}\frac{d\mu_j}{dz_j} + \sum_{i=1}^{p} \alpha_{ij}\, x_i \tag{5.9}$$

The vector ${}_i\alpha_{ij}\, x_i$ is called the weighted electoral mean for leader j, and can be written

$$\sum_{i=1}^{p} \alpha_{ij}\, x_i = \frac{dE_j^*}{dz_j} \tag{5.10}$$

Notice first that the weight α_{ij} shows how the citizen *i* influences leader *j* in his choice of policy position. Moreover, the weights for leader *j* depend on the vector of positions $\{\mathbf{z}_{-j}\}$ of leaders other than *j*. The balance equation can be rewritten as

$$\left[\frac{dE_j^*}{dz_j} - z_j^*\right] + \frac{1}{2\beta}\frac{d\mu_j}{dz_j} = 0 \tag{5.11}$$

The bracketed term on the left of this expression is termed the *marginal electoral pull of leader j* and is a gradient vector pointing towards this leader's weighted electoral mean. This position is that point where the electoral pull is zero. The vector $\frac{d\mu_j}{dz_j}$ is called *the marginal activist pull for leader j.*

If $\mathbf{z}^* = (z_1^*, \ldots z_j^*, \ldots z_n^*)$ is such that each z_j^* satisfies the balance equation then call $\mathbf{z}^*$ the *balance solution.*

Theorem 5.1.[3] *Consider the electoral model E(l, m, β; Ψ) based on the Type I extreme value distribution, and including both exogenous and activist valences. The first order condition for $\mathbf{z}^*$ to be an LSNE is that it is a balance solution. If all activist valence*

[3] The proof of Theorem 5.1 can be found in Schofield (2006a).

functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then the balance solution will be a PNE.

We emphasize that the *marginal electoral pull of leader j* is a gradient vector pointing towards the weighted electoral mean of the leader, and represents the *centripetal* pull to the center. *The marginal activist pull for leader j* represents the *centrifugal force* generated by the resources made available by activists.

In principle, this model can be used to examine the equilibrium position of a political leader, responding to activist demands, and balancing the pull of the selectorate, in order to gain resources that can be used to compete with political opponents. Even without activists, convergence to a centrist position, as in the Downsian model, is impossible if the population is sufficiently heterogenous in its beliefs or preferences.

In the case $\mu_j = 0$ for all j, the balance condition becomes

$$z_j = \sum_{i \in P} s_{ij} x_i \tag{5.12}$$

In the egalitarian case with all weights $\{s_{ij}\}$ identical, then first order balance condition becomes

$$z_j^* = \frac{1}{p} \sum_{i=1}^{p} x_i \tag{5.13}$$

By a change of coordinates we choose $\frac{1}{p}\Sigma x_i = 0$. In this case, the marginal electoral pull is zero at the origin and the joint origin $\mathbf{z}_0 = (0,..., 0)$ satisfies the first order condition. However, since $\mathbf{m} = \mathbf{0}$, we cannot use the concavity of $\mathbf{m}$ to assert the existence of equilibrium. Schofield (2007) shows that if $\mathbf{m} = \mathbf{0}$, then there is a coefficient, c, defined in terms of all model parameters and the electoral covariance matrix of the voter preferred points such that $c < w$ is a necessary condition for $\mathbf{z}_0$ to be a LSNE in the egalitarian stochastic vote model.

Definition 5.5. *The Electoral Covariance Matrix,* ∇_0.

Let $W = R^w$ *be endowed with a system of coordinate axes* $r = 1,..., w$. *For each coordinate axis let* $\xi_r = (x_{1r}, x_{2r},..., x_{pr})$ *be the vector of the* r^{th} *coordinates of the set of* p *voter bliss points. The scalar product of* ξ_r *and* ξ_s *is denoted* $(\xi_r \cdot \xi_s)$.

The symmetric $w \times w$ *electoral covariance matrix about the origin is denoted* ∇_0 *and is defined by*

$$\nabla_0 = \frac{1}{p}\left[(\xi_r \text{-} \xi_s)\right]^{r=1,\ldots,w}_{s=1,\ldots,w}$$

Let $(\sigma_r, \sigma_s) = \frac{1}{p}(\xi_r, \xi_s)$ *be the electoral covariance between the* r^{th} *and* s^{th} *axes, and* $\sigma_s^2 = \frac{1}{p}(\xi_s, \xi_s)$ *be the electoral variance on the* s^{th} *axis, with*

$$\sigma^2 = \sum_{s=1}^{w} \sigma_s^2 = \frac{1}{p}\sum_{s=1}^{w} (\xi_s \text{-} \xi_s) = trace\,(\nabla_0)$$

the total electoral variance.

Theorem 5.2. *(i) The Hessian of the egalitarian vote share function of party j at* $\mathbf{z}_0$ *is a positive multiple of the w by w characteristic matrix.*[4]

$$C_j = 2\beta\ (1 - 2\rho_j)\ \nabla_0 - I \tag{5.14}$$

where I is the w by w identity matrix.

(ii) The necessary and sufficient condition for $\mathbf{z}_0$ *to be an LSNE is that all* C_j *have negative eigenvalues. Since* C_1 *must also have negative eigenvalues, it follows that a necessary condition for* $\mathbf{z}_0$ *to be an LNE is that a convergence coefficient, c, defined by*

$$c = 2\beta\ (1 - 2\rho_1)\ \sigma^2$$

is bounded above by the dimension, w.

(iii) In two dimensions, a sufficient condition is that c is bounded above by 1. *In higher dimensions a sufficient condition can be expressed by appropriate bounds on the cofactors of* C_1.

While maximization of vote share is an appropriate maximand under proportional egalitarian rule, a more appropriate maximand under plurality rule would be a seat share function

$$S_j(\mathbf{z}) = S_j(V_1(\mathbf{z}),\ldots, V_j(\mathbf{z}),\ldots V_n(\mathbf{z}))$$

which might very well be a logistic function of $V_j(\mathbf{z})$. The techniques of the proof of Theorem 5.1 and Theorem 5.2 can be extended to this more general case.

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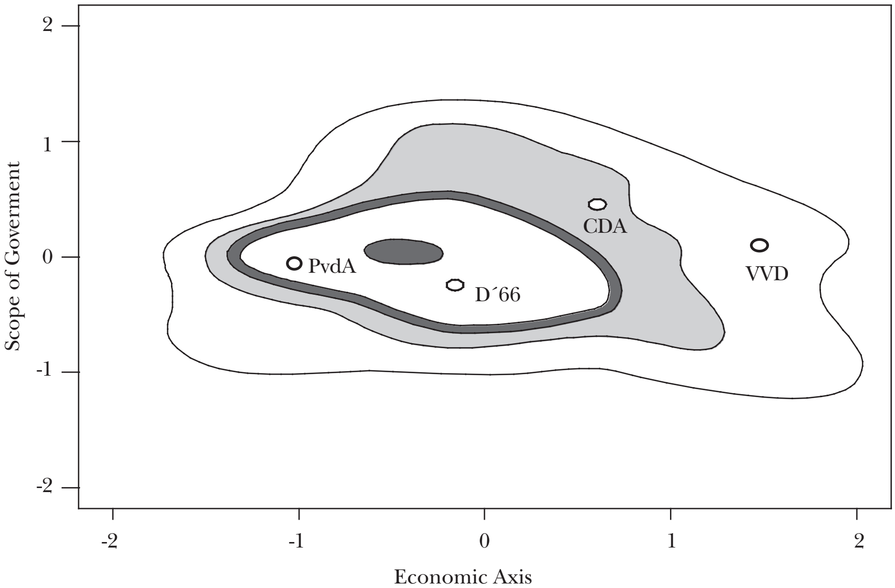
[4] The proof of Theorem 5.2 can be found in Schofield (2007).

5.3. Empirical Models

5.3.1. Netherlands 1977 and 1981

Next we consider a multinomial logit (MNL) model for the elections of 1977 and 1981 in the Netherlands (Schofield, Martin, Quinn and Whitford 1998; Quinn, Martin and Whitford 1999) using data from the middle level Elites Study (ISEIUM 1983).There are four main parties: Labor (PvdA), Christian Democratic Appeal (CDA), Liberals (VVD) and Democrats (D'66), with approximately 38 percent, 36 percent, 20 percent and 6 percent of the popular vote in 1977. Figure 5.1 gives the estimate of the density contours of the electoral distribution of voter bliss points based on the Rabier Inglehart (1981) Euro-barometer survey.

FIGURE 5.1: Party positions in the Netherlands in 1977



The estimated exogenous valences were normalized, by choosing the D'66 to have exogenous valence $\lambda_{D66} = 0$. The other valences are $\lambda_{VVD} = 1.015$, $\lambda_{CDA} = 1.403$

and $\lambda_{PvdA} = 1.596$. To compute the D'66 Hessian, we note that the electoral variance on the first axis is $\sigma_1^2 = 0.658$, while on the second it is $\sigma_2^2 = 0.289$. The covariance (σ_1, σ_2) is negligible.

The spatial coefficient $\beta = 0.737$ for the model with exogenous valence. Thus the probability of voting for each of the parties, as well as the Hessians when all parties are at the origin, can be calculated as follows:

$$\rho_{D66} = \frac{1}{1 + e^{1.015} + e^{1.043} + e^{1.596}} = 0.078.$$

$$2\beta\,(1 - 2\rho_{D66}) = 2 \times 0.737 \times 0.844 = 1.244$$

$$\text{Hence } C_{D66} = (1.244)\ C_{D66} = (1.244)\begin{bmatrix} 0.658 & 0 \\ 0 & 0.289 \end{bmatrix} - I$$

$$= \begin{pmatrix} -0.18 & 0 \\ 0 & -0.64 \end{pmatrix},$$

$$\text{so} c = 2 \times 0.622 \times 0.947) = 1.178$$

Although the convergence coefficient exceeds 1.0, so the sufficient condition, given by Theorem 5.2 is not satisfied, the necessary condition of the Theorem is satisfied, and the eigenvalues for the characteristic matrix for D'66 can be seen to be negative. Thus the joint origin is an LSNE for the stochastic model with exogenous valence.

In a similar way, we can compute the other probabilities, giving

$$(\rho_{D66}, \rho_{VVD}, \rho_{CDA}, \rho_{PvdA}) = (0.078, 0.217, 0.319, 0.386)$$

This vector can be identified as the expected vote shares of the parties when all occupy the electoral origin. Note also that these expected vote shares are very similar to the sample vote shares

$$(S^*_{D66}, S^*_{VVD}, S^*_{CDA}, S^*_{PvdA}) = (0.104, 0.189, 0.338, 0.369),$$

as well as the average of the national vote shares in the two elections.

$$(E^*_{D66}, E^*_{VVD}, E^*_{CDA}, E^*_{PvdA}) = (0.094, 0.199, 0.356, 0.352)$$

These national vote shares can be regarded as approximations of the expected vote shares. Quinn and Martin (2002) performed a simulation of the empirical model and showed that the joint origin was indeed a PSNE for the vote-maximizing model with the exogenous valence values estimated by the MNL model. Moreover, the positions given in figure 5.1 could not be an LSNE of the stochastic model with exogenous valence alone. This conflict between the predicted equilibrium positions of the model and the estimated positions suggest that the activists for the parties played an important role in determining the party positions. Although we do not have data available on the activist valences for the parties, these empirical results indicate that Theorem 5.1 is compatible with the following two hypotheses:

(i) the party positions given in figure 5.1 are a close approximation to the actual positions of the parties;
(ii) each party was at a Nash equilibrium position in an electoral contest involving a balance for each party between the centripetal electoral pull for the party and the centrifugal activist pull on the party.

5.3.2. The Election in the United Kingdom in 1997

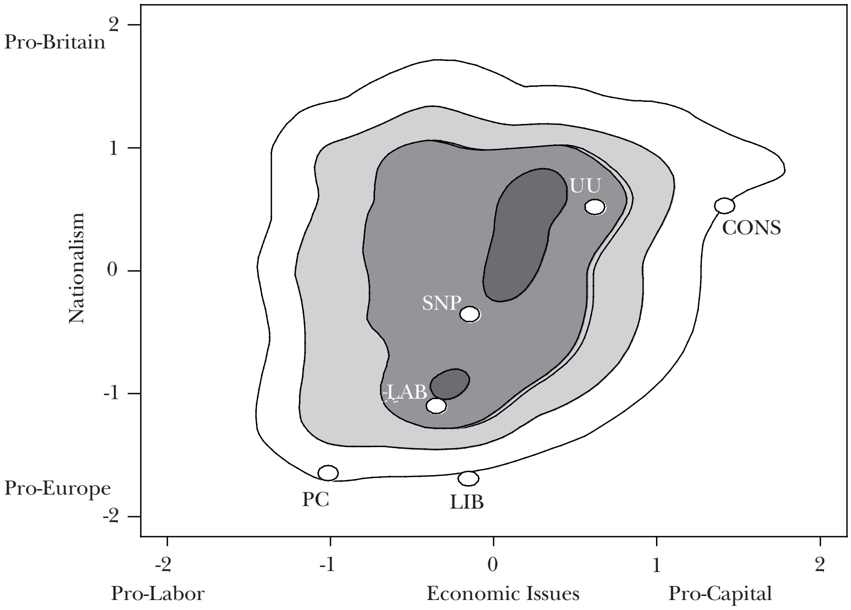
Figure 5.2 shows the estimated positions of the parties, based on a survey of Party MPs in 1997 (Schofield 2005a,b). In addition to the Conservative Party (CONS), Labor[5] Party (LAB) and Liberal Democrat Party (LIB) responses were obtained from Ulster Unionists (UU), Scottish Nationalists (SNP) and Plaid Cymru (PC). The first axis is economic, the second axis concerned attitudes to the European Union (pro-Europe to the "south" of the vertical axis, and pro-Britain to the "North"). The electoral model with exogenous valence was estimated for the election in 1997.

For 1997 $(\lambda_{con}, \lambda_{lab}, \lambda_{lib}, \beta)_{1997} = (+1.24, 0.97, 0.0, 0.5)$ so

$$\rho_{lib} = \frac{e^0}{e^0 + e^{1.24} + e^{0.97}} = \frac{1}{7.08} = 0.14$$

[5] We use the U.S. spelling for this party.

FIGURE 5.2: Party positions in the United Kingdom



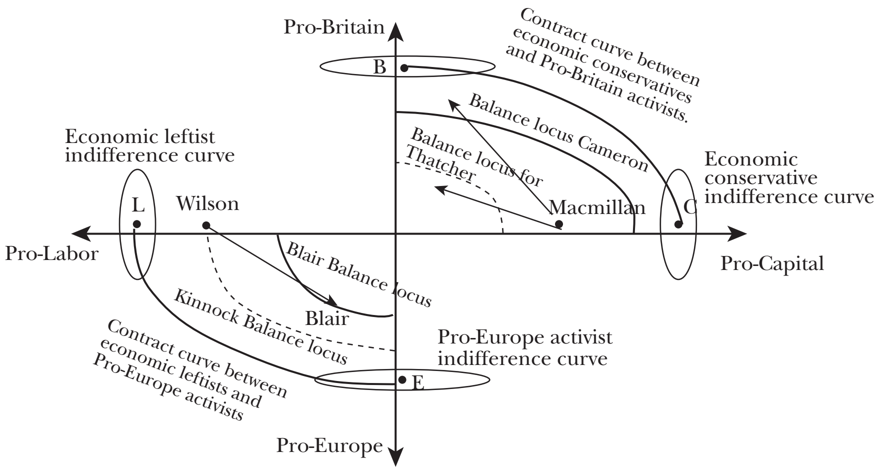
Since the electoral variance is 1.0 on the first economic axis and 1.5 on the European axis, we obtain

$$A_{lib} = \beta\ (1 - 2\rho_{\mathrm{lib}}) = 0.36 \text{ and}$$

$$C_{lib} = \left[(0.72) \begin{pmatrix} 1.0 & 0 \\ 0 & 1.5 \end{pmatrix} \right] - I = \begin{bmatrix} -0.28 & 0 \\ 0 & +0.08 \end{bmatrix}$$

The convergence coefficient can be calculated to be 1.8, so the sufficient condition fails. Although the necessary condition is satisfied, the origin is clearly a saddlepoint for the Liberal Democrat Party. Note that the second "European" axis is a "principal electoral axis" exhibiting greater electoral variance. This axis is the eigenvector associated with the positive eigenvalue. Because the covariance between the two electoral axes is negligible, we can infer that, for each party, the eigenvalue of the Hessian at the origin is negative on the first or minor "economic" axis. According to the formal model with exogenous valence, all parties should have converged to the origin on this minor axis. Because the eigenvalue for the Liberal Democrat Party is positive on the second axis, we have an explanation for its position away from the origin on the Europe axis in figure 5.2. However there is no explanation for the location of the Conservative Party so far from the origin on both axes. Figure 5.3 gives an illustration taken from Schofield (2005) based on the empirical model for Britain for recent elections. The Labor Party benefits from resources from two potential activist groups, with preferred policy positions at L and E. The contract curve is the curve connecting these preferred positions of an activist group (L) on the economic left and an activist group (E), supporting a membership of a strong European Union. At the same time, the falling exogenous valence of the Conservative Party leader increased the marginal importance of two opposed activist groups in the party: one group "pro-capital" (at C) and one group "pro-Britain" (at B). Figure 5.3 suggests that the Labor Party position has moved from a location denoted "Wilson" along the balance locus to "Blair" , while the Conservative party has shifted postion from "Macmillan" along different balance loci to "Thatcher" in the 1980's and more recently along the "Cameron" balance locus.

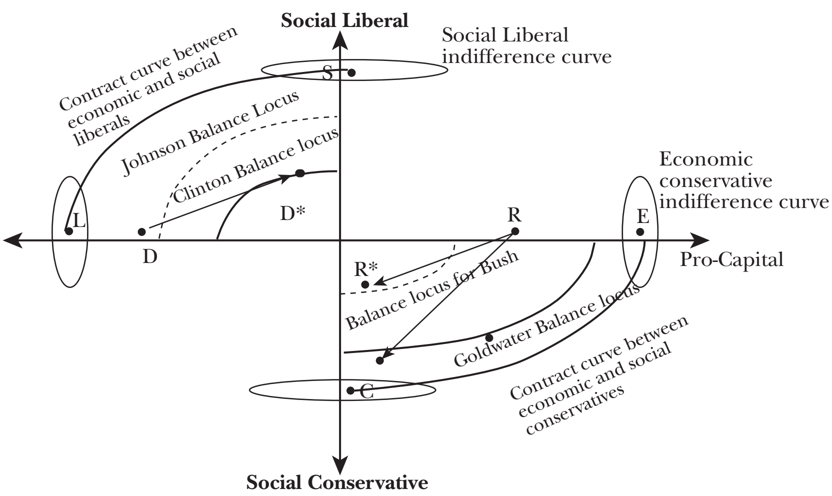
FIGURE 5.3: Balance loci in the United Kingdom



5.3.3. Elections in the United States

Miller and Schofield (2003, 2008) and Schofield and Miller (2007) have used this model (based on an economic axis and a social axis). For example, suppose that the critical condition of Theorem 5.2 fails. As suggested by the notion of a balance locus, candidates for office in a two party system must balance the centripetal electoral gradient against a centrifugal activist gradient. Figure 5.4 illustrates these formal results, by showing the contract curve between R and C for a Republican candidate, and the contract curve between L and S for a Democrat candidate. The equilibrium position for a Republican candidate will depend on the Republican exogenous valence and the position adopted by the opposition candidate. When there is a single economic dimension, then the valence difference between the contenders will separate them on left and right. Potential activist concerns can then bring the second, social dimension into existence. Optimal, or vote maximizing, candidate positions will lie on the two balance loci. In general the optimal position for a low valence candidate like Goldwater will lie on a balance locus farther from the electoral center than that of a candidate like Bush whose valence is relatively higher. As figure 5.4 suggests, the changing configuration of centripetal and centrifugal forces appears to lead to a slow rotation in the configuration of the parties. Schofield, Miller and Martin (2003) argue that a political realignment (Sundquist 1973) occurs when the two party configuration is changed suddenly (as the result of a constitutional quandary). The historical analysis suggests that this has tended to occur in a clockwise direction since the election of McKinley in 1896 (See Schofield 2006b).

FIGURE 5.4: Balance loci in the U.S.



To provide a quick test of whether the convergence condition holds in the United States, consider table 5.1, which presents a one dimensional multinomial logit (MNL) model of the 1992 presidential contest between Clinton, Perot and G. H. W. Bush.[6]

TABLE 5.1: MNL model of the 1992 presidential election in the US
(normalized w.r.t Perot)

		Coef.	Std. dev.	z	prob	95%	Conf. Interval
	β coeff.	0.12	0.02	−5.34	0.00	0.08	0.16
Bush	λ_{BUSH}	−1.16	1.02	−1.13	0.26	−3.16	0.85
Clinton	$\lambda_{CLINTON}$	−0.48	0.96	−0.51	0.61	−2.36	1.39
Bush	worsefinan	−0.481	0.259	−1.86	0.063	−0.987	0.026
Clinton		0.122	0.23	0.53	0.596	−0.329	0.573
Bush	worseecon	−0.381	0.244	−1.56	0.118	−0.86	0.097
Clinton		0.669	0.27	2.48	0.013	0.14	1.198
Bush	govjobs	0.117	0.086	1.37	0.172	−0.051	0.285
Clinton		0.067	0.075	0.89	0.372	−0.08	0.215
Bush	govhealth	0.22	0.066	3.34	0.001	0.091	0.35
Clinton		0.069	0.067	1.02	0.306	−0.063	0.2
Bush	black	−0.002	0.084	−0.03	0.979	−0.166	0.162
Clinton		−0,21	0.074	−2.85	0.004	−0.354	−0.065
Bush	abortion	−0.451	0.113	−4.01	0	−0.672	−0.231
Clinton		−0.021	0.117	−0.18	0.857	−0.25	0.208
Bush	term	0.272	0.321	0.85	0.397	−0.357	0.901
Clinton		0.177	0.27	0.65	0.513	−0.352	0.705
Bush	deficit	−1.003	0.268	−3.74	0	−1.528	−0.478
Clinton		−0.418	0.275	−1.52	0.129	−0.958	0.121
Bush	east	−0.277	0.32	−0.86	0.388	−0.905	0.352
Clinton		0.407	0.293	1.39	0.165	−0.168	0.981
Bush	south	0.406	0.302	1.34	0.179	−0.186	0.999
Clinton		0.65	0.3	2.17	0.03	0.062	1.239

[6] The survey was the National Election Survey for 1992. The socio-demographic terms in table 5.1 are self explanatory. The table is based on research by Guido Cataife.

TABLE 5.1 *(cont.)*: **MNL model of the 1992 presidential election in the US**

(normalized w.r.t Perot)

		Coef.	Std. dev.	z	prob	95%	Conf. Interval
Bush	west	–0.307	0.304	–1.01	0.313	–0.904	0.289
Clinton		0.239	0.301	0.79	0.427	–0.35	0.828
Bush	newvoter	0.497	0.325	1.53	0.127	–0.141	1.134
Clinton		–0.283	0.294	–0.96	0.335	–0.858	0.292
Bush	dem	–0.527	0.448	–1.18	0.24	–1.404	0.351
Clinton		1.651	0.319	5.17	0	1.025	2.277
Bush	rep	1.366	0.387	3.53	0	0.608	2.124
Clinton		–0.83	0.365	–2.28	0.023	–1.545	–0.115
Bush	female	0.563	0.231	2.43	0.015	0.11	1.017
Clinton		0.191	0.22	0.87	0.387	–0.241	0.622
Bush	educyrs	0.101	0.055	1.81	0.07	–0.008	0.209
Clinton		0.032	0.052	0.62	0.534	–0.069	0.134
Bush	age 18-29	–1.18	0.39	–3.03	0.002	–1.944	–0.417
Clinton		–0.83	0.377	–2.2	0.028	–1.568	–0.092
Bush	age 30-44	–0.731	0.32	–2.28	0.022	–1.358	–0.103
Clinton		–0.729	0.323	–2.26	0.024	–1.362	–0.095
Bush	age 45-59	–0.453	0.352	–1.28	0.199	–1.143	0.238
Clinton		–0.14	0.346	–0.41	0.685	–0.818	0.538
Log likelihood = –565							
$p = 905$							

TABLE 5.2: **Explanation of Variables**

Variable	Explanation
worsefinan	Whether the voter thinks the national economy got worse.
worseecon	Whether the voter thinks his personal finances got worse.
govjobs	1: The government should see people have jobs. 7: The government should let each person get his own job without intervention.
govhealth	1: The government should provide health plan. 7: Private plans.
govblack	1: The government should help blacks. 7: Blacks should help themselves.

TABLE 5.2 *(cont.)*: **Explanation of Variables**

Variable	Explanation
abortion	1: Always be permited; 4: Never be permitted (2 & 3 intermediate cases).
term	0: Does not favor term limits. 1: Favors.
deficit	The respondent thinks the size of the budget deficit is one of the most important problems.
east	Whether the respondent is from East.
south	Whether the respondent is from South.
west	Whether the respondent is from West.
newvoter	Whether the respondent is a new voter.
dem	Whether the respondent is a democrat.
rep	Whether the respondent is a republican.
female	Whether the respondent is female.
educyrs	Years of education.

Instead of (5.1) we use the expression

$$u_{ij}(x_i, z_j) = \lambda_j - \beta \| x_i - z_j \|^2 + \theta_j^T \eta_i + \varepsilon_j. \tag{5.15}$$

where the k-vector θ_j represents the effect of the k different sociodemographic parameters (class, domicile, education, income, etc.) on voting for the party j while η_i is a k-vector denoting the i^{th} individual's relevant "sociodemographic" characteristics. We use θ_j^T to denote the transpose of θ_j so $\theta_j^T \eta_i$ is a scalar. The terms $\{\lambda_j\}$ are the intrinsic valences, and assumed constant at each election, as in Section 5.2. Using (5.5) we find that the low valence candidate, Bush, has $\lambda_{BUSH} = -1.158$, while $\lambda_{CLINTON} = -0.482$ and $\lambda_{PEROT} = 0$. Thus

$$\rho_{BUSH} = \frac{e^0}{e^0 + e^{1.158-0.482} + e^{1.158}}$$

$$= \frac{1}{1 + e^{.678} + e^{1.158}} = 0.16$$

In the same way, $\rho_{CLINTON} = 0.32$.

The spatial coefficient is $\beta = 0.120$, and the electoral variance is $\sigma^2 = 6.22$. Thus

$$c = 2\beta\ (1 - 2\rho_{BUSH})\ \sigma^2$$

$$= 2\ (0.120)\ (0.68)\ (6.22) = 1.015.$$

Since $w = 1$, the necessary condition fails. The eigenvalue for Bush is + 0.015, which, though small, is positive, signifying that the electoral origin is a minimum for the vote share function of Bush. In contrast,

$$C_{CLINTON} = 2\beta\ (1 - 2\rho_{CLINTON})\ \sigma^2 - 1$$

$$= 2\ (0.120)\ (0.36)\ (6.22) - 1 = -0.91.$$

Thus the electoral origin is a maximum for Clinton's vote share function. In fact, using the [−2.0, +2.0] scale as in figure 5.1 and figure 5.2 gives the electorally perceived positions of the candidates as

$$(z_{CLINTON}, z_{PEROT}, z_{BUSH}) = (-0.31\ {+0.57}, 1.07).$$

On this economic scale, Clinton is just left of center, Perot moderately right of center, and G. H. W. Bush fairly far right of center. These positions are compatible with local Nash equilibrium positions for the vote maximizing model, since Clinton's best response to a Bush position on the right must be to the left. This provides some justification for the validity of the model.

5.4. Concluding remarks

This paper has discussed elections in the Netherlands, Britain and the United States. It is evident that they all display complex and distinct characteristic features. The main empirical point that emerges is that any centripetal tendency towards an electoral center is very weak. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. Indeed, we can follow Duverger (1954) and note that under proportional electoral methods, there is very little motivation for interest groups to coalesce. Another way of expressing, in simplified form, the difference between proportional representation and plurality rule is this: under proportional electoral methods, bargaining to create winning coalitions occurs *after* the election. Under plurality rule, if interest groups do not form a coalition *before* the election, then they can be obliterated. This obviously creates a pressure for activist groups to coalesce.

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