MULTIPARTY DEMOCRACY

Elections and Legislative Politics

NORMAN SCHOFIELD
ITAI SENED

CAMBRIDGE

MULTIPARTY DEMOCRACY

This book adapts a formal model of elections and legislative politics to study party politics in Israel, Italy, the Netherlands, Britain, and the United States. The approach uses the idea of valence—that is, the party leader's nonpolicy electoral popularity—and employs survey data to model these elections. The analysis explains why small parties in Israel and Italy keep to the electoral periphery. In the Netherlands, Britain, and the United States, the electoral model is extended to include the behavior of activists. In the case of Britain, it is shown that there will be contests between activists for the two main parties over who controls policy. Regarding the recent 2005 election, it is argued that the losses of the Labour Party were due to Blair's falling valence. For the United States, the model gives an account of the rotation of the locations of the two major parties over the last century.

Norman Schofield is the William Taussig Professor in Political Economy at Washington University in St. Louis. He served as Fulbright Distinguished Professor of American Studies at Humboldt University Berlin in 2003–4 and held a Fellowship at the Center for Advanced Study in the Behavioral Sciences at Stanford in 1988–9. Professor Schofield is the author of *Architects of Political Change* (Cambridge University Press, 2006), *Mathematical Methods in Economics and Social Choice* (2003), *Multiparty Government* (coauthored with Michael Laver, 1990), and *Social Choice and Democracy* (1985). He received the William Riker Prize in 2002 for contributions to political theory and was co-recipient with Gary Miller of the Jack L. Walker Prize for the best article on political organizations and parties in the *American Political Science Review* for 2002–4.

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For Elizabeth and Sarit

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Preface

This book closes a phase of a research program that has kept us busy for more than ten years. It sets out a theory of multiparty electoral politics and evaluates this theory with data from Israel, Italy, the Netherlands, Britain, and the United States.

Four decades ago, our teacher and mentor, William H. Riker, started this effort with *The Theory of Political Coalitions* (1962). What is perhaps not remembered now is that Riker's motivation in writing this book came from a question that he had raised in his much earlier book, *Democracy in the United States* (1953): Why did political competition in the United States seem to result in roughly equally sized political coalitions of disparate interests? His answer was that minimal-winning coalitions were efficient means of dividing the political spoil. This answer was, of course, not complete, because it left out elections—the method by which parties gain political power in a democracy. His later book, *Positive Political Theory* (1973), with Peter Ordeshook, summed up the theory available at that time, on two-party elections. The main conclusion was that parties would tend to converge to an electoral center—either the median or mean of the electoral distribution. Within a few years, this convenient theoretical conclusion was shown to be dependent on assumptions about the low dimension of the policy space. The chaos results that came in the 1970s were, however, only applicable to two-party elections where there was no voter uncertainty. With voter uncertainty, it was still presumed that the mean voter theorem would be valid. The chaos theorem did indicate that in parliaments where the dimension was low and where parties varied in strength, stability would occur, particularly if there were a large centrally located or dominant party. Indirectly, this led to a reawakening of interest in completing Riker's coalition program. Now, the task was to examine the post-election situation in Parliament, taking

party positions and strengths as given, and to use variants of rational choice theory to determine what government would form. While a number of useful attempts were made in this endeavor, they still provided only a partial solution, since elections themselves lay outside the theory. One impediment to combining a theory of election with a theory of coalition was that the dominant model of election predicted that parties would be indistinguishable—all located at the electoral mean, and all of equal size.

A key theoretical argument of this book is that this mean voter theorem is invalid when voters judge parties on the basis of evaluation of competence rather than just proposed policy. Developing this new theorem came about because of an apparent paradox resulting from work with our colleagues Daniela Giannetti, Andrew Martin, Gary Miller, David Nixon, Robert Parks, Kevin Quinn, and Andrew Whitford. On the basis of logit and probit models of the Netherlands, it was found by simulation that parties could have increased their vote by moving to the center. However, when the same simulation was performed using an empirical model for Israel in 1988, no such convergence was observed. Some later work on the United States then brought home the significance of Madison's remark in *Federalist 10* about "the probability of a fit choice." The party constants in the estimations could be viewed as valences, modelling the judgments made of the parties by the electorate. These judgments varied widely in the case of Israel, somewhat less so in Italy and Britain, and even less so in the Netherlands. The electoral theorem presented in Chapter 3 shows that if electoral uncertainty is not too high and electoral judgments are sufficiently varied, parties will, in equilibrium, locate themselves in different political "niches," some of which will be far from the electoral center. Immediately we have an explanation both for the occurrence of radical parties and for Duverger's (1954) hypothesis about the empty electoral center.

This book attempts to combine the resulting theory of elections with a theory of government formation, applicable both to polities with electoral systems based on proportional representation (or PR), such as Israel, Italy, and the Netherlands, but also to polities such as Britain and the United States with electoral systems based on plurality or "first past the post." Essentially we propose that, under PR, pure vote maximization is tempered by the beliefs of party leaders about the logic of coalition formation. Under the plurality electoral mechanism, party coalitions must typically occur before the election, and this induces competition between the activists within each party. Naturally, this model raises many new topics of theoretical concern, particularly since we combine notions of both

non-cooperative game theory and social choice theory. Because the theoretical model presented here is quite abstract and technically demanding, we suggest that only the first section of Chapter 3 is covered on first reading. The formal sections of this chapter on electoral uncertainty and on the heart can be left for reading after the more substantive chapters have been examined.

Over the years, we have been fortunate to receive a number of National Science Foundation awards, most recently grant SES 0241732. Schofield wishes to express his appreciation for this support and for further support from the Fulbright Foundation, from Humboldt University, and from Washington University during his sabbatical leave from 2002 to 2003. We are also very grateful to the Weidenbaum Center at Washington University for research support. We thank Martin Battle and Dganit Ofek for research collaboration, and Alexandra Shankster, Cherie Moore, and Ben Klemens for help in preparing the manuscript. John Duggan made a number of perceptive remarks on the proof of the electoral theorem. Jeff Banks was always ready with insights about our earlier efforts to develop the formal model. Jim Adams and Michael Laver shared our enthusiasm for modelling the political world.

Our one regret is that Jeffrey Banks, Richard McKelvey, and William Riker are not here to see the results of our efforts. They would all have enjoyed the theory, and Bill, especially, would have appreciated our desire to use theory in an attempt to understand the real world. This book is dedicated to the memory of our friends.

Norman Schofield and Itai Sened.
St. Louis, Missouri, April 14, 2006

1

Multiparty Democracy

1.1 INTRODUCTION

When Parliament first appeared as an innovative political institution, it was to solve a simple bargaining problem: Rich constituents would bargain with the King to determine how much they wished to pay for services granted them by the King, such as fighting wars and providing some assurances for the safety of their travel and property rights.

In the modern polity, governments have greatly expanded their size and the range and sphere of their services, while constituents have come to pay more taxes to cover the ever-growing price tag of these services. Consequently, parliamentary systems and parliamentary political processes have become more complex, involving more constituents and making policy recommendations and decisions that reach far beyond decisions of war and peace and basic property rights. But the center of the entire bargaining process in democratic parliamentary systems is still Parliament.

Globalization trends in politics and economics do not bypass, but pass *through* local governments. They do not diminish but increase pressure and demands put on national governments. These governments that used to be sovereign in their territories and decision spheres are now constantly feeling globalization pressures in every aspect of their decision-making processes. Some of these governments can deal with the extra pressures while others are struggling. A majority of these governments are coalition governments in parliamentary systems. Unlike the U.S. presidential system, parliamentary systems are not based on checks and balances but on a more literal interpretation of representation. Turnouts are much higher in elections, more parties represent more shades of individual preferences, and the polity is much more politicized in paying daily attention to daily

politics. But in the end, the coalition government is endowed with remarkable power to make decisions about allocations of scarce resources that are rarely challenged by any other serious political player in the polity. In short, the future of globalization depends on a very specific set of rules in predominantly parliamentary systems that govern most of the national constituents of the emerging new global order (Przeworski et al., 2000). These sets of rules that constrain and determine how the voice of the people is translated to economic allocations of scarce resources are the subject of our book.

Over the last four decades, inspired by the seminal work of the late William H. Riker—*The Theory of Political Coalitions* (1962)—much theoretical work has been done that leads to a fair amount of accumulated knowledge on the subject. This book is aimed at three parallel goals. First, we enhance this fairly developed body of theory with new theoretical insights. Second, we confront our theoretical results with empirical evidence we have been collecting and analyzing with students and colleagues in the past decade, introducing, in the process, the new Bayesian statistical approach of empirical research to the field of study of parliamentary systems. Finally, we want to make what we know, in regards to both theory and empirical analysis, available to those who study the new democracies in Eastern Europe, South America, Africa, and Asia.

Since the collapse of the Soviet Union in the early 1990s, many countries in Eastern Europe, and even Russia itself, have become democratic. Most of these newcomers to the family of democratic regimes have fashioned their government structures after the model of Western European multiparty parliamentary systems. In doing so, they hoped to emulate the success of their western brethren. However, recent events suggest that even those more mature democratic polities can be prone to radicalism, as indicated by the recent surprising success of Le Pen in France, or the popularity of radical right parties in Austria (led by Haider) and Netherlands (led by Fortuyn).

In Eastern Europe, the use of proportional representative electoral systems has often made it difficult for centrist parties to cooperate and succeed in government. Proportional representation (PR) has also led to difficulties in countries with relatively long-established democratic systems. In Turkey, for example, a fairly radical fundamentalist party gained control of the government. In Israel, PR led to a degree of parliamentary fragmentation and government instability. These problems have greatly contributed to the particular difficulties presently facing any attempt at peace negotiations between Israelis and Palestinians.

In Russia, the fragmentation of political support in the Duma is a consequence of the peculiar mixed PR electoral system in use. Finally, in Argentina, and possibly Mexico, a multiparty system and presidential power may have contributed to populist politics and economic collapse in the former and disorder in the latter.

In all of the above cases, the interplay of electoral politics and the complexities of coalitional bargaining have induced puzzling outcomes. In general, scholars study these different countries under the rubric of "comparative" politics. In fact, however, there is very little that is truly "comparative," in the sense of being based on generalized inductive or deductive reasoning.

Starting in the early 1970s, scholars used Riker's theoretical insights in an empirical context, focusing mostly on West European coalition governments. This early mix of empirical and theoretical work on Europe by Browne and Franklin (1973), Laver and Taylor (1973), and Schofield (1976) provided some insights into political coalition governments. However, by the early 1980s it became clear that, to succeed, this research program needed to be extended to incorporate both empirical work on elections and more sophisticated work on political bargaining (Schofield and Laver, 1985).

The considerable amount of work done during the last few decades on election analysis, party identification, and institutional analysis has tended to focus on the United States, a unique two-party, presidential system. Unfortunately, most of this research has not been integrated with a theoretical framework that is applicable to multiparty systems. In two-party systems such as the United States, if the "policy space" comprises a single dimension, then a standard result known as the *median voter* theorem indicates that parties will converge to the median, centrist voter ideal point. It can be shown that even when there are more than two parties, then as long as politics is "unidimensional," then all candidates will converge to the median (Feddersen, Sened, and Wright, 1990). It is well known, however, that in multiparty proportional-rule electoral systems, parties do not converge to the political center (Cox, 1990). Part of the explanation for this difference may come from the fact that a standard assumption of models of two-party elections is that the parties or candidates adopt policies to maximize votes (or seats). In multiparty proportional-rule elections (that is, with three or more parties), it is not obvious that a party should rationally try to maximize votes. Indeed, small parties that are centrally located may be assured of joining government. In fact, in multiparty systems another phenomenon occurs. Small parties often

adopt radical positions, ensure enough votes to gain parliamentary representation, and bargain aggressively in an attempt to affect government policy from the sidelines (Schofield and Sened, 2002). Thus, many of the assumptions of theorists that appear plausible in a two-party context, are implausible in a multiparty context.

In 1987, the National Science Foundation (under Grant SES 8521151) funded a conference with 18 participants at the European University Institute in Fiesole near Florence. The purpose of the conference was to bring together rational choice theorists and scholars with an empirical focus, in an effort to make clear to theorists that their models, while applicable to two-party situations, needed generalization to multiparty situations. At the same time, it was hoped that new theoretical ideas would be of use to the empirical scholars in their attempt to understand the complexities of West European multiparty politics. This was in anticipation of, but prior to, the collapse of the communist regimes in Eastern Europe. A book edited by Budge, Robertson, and Hearl (1987) analyzed party manifestos in West European polities and these data provided the raw material for discussion among the participants in the Fiesole Conference. The conference led to a number of original theoretical papers (Austen-Smith and Banks, 1988, 1990; Baron and Ferejohn, 1989; Schofield, Grofman and Feld, 1989; Laver and Shepsle, 1990; Schofield, 1993; Sened, 1995, 1996), three books (Laver and Schofield, 1990; Shepsle, 1990; Laver and Shepsle, 1996) and several edited volumes (Laver and Budge, 1992; Barnett, Hinich and Schofield, 1993; Laver and Shepsle, 1994; Barnett et al., 1995; Schofield, 1996).

Just as these works were being published in the mid-1990s, new statistical techniques began to revolutionize the field of empirical research in political science. This school of *Bayesian* statistics allows for the construction of a new generation of much more refined statistical models of electoral competition (Schofield et al., 1998; Quinn, Martin, and Whitford, 1999). These new techniques, and much-improved computer hardware and software, allowed, in turn, the study of more refined theoretical models (Schofield, Sened, and Nixon, 1998; Schofield and Sened, 2002). We are only in the beginning of this new era of the study of multiparty political systems.

The collapse of the Soviet Union and its satellite communist regimes and democratization trends in South America, Eastern Europe, and Africa create an urgency and a wealth of new cases and data to feed this research program with new challenges of immediate and obvious practical

relevance. In particular, the domain of empirical concerns has grown considerably to cover new substantive areas including:

1. The rise of radical parties in Western Europe;
2. Cooperation and coalition formation in East European politics;
3. Fragmentation in politics in the Middle East and Russia;
4. Presidentialism and multiparty politics in Latin America; and
5. Policy implications of parliamentary and coalition politics.

Our book is motivated and guided by the vision of the late William H. Riker who believed that the process of forming coalitions was at the core of all politics, whether in presidential systems, such as in the United States, or in the multiparty systems common in Europe. In his writings, he argued that it was possible to create a theoretically sound, deductively structured, and empirically relevant science of politics. We hope this book will carry forward the research program Riker (1953) first envisioned more than fifty years ago.

On the practical side, we want our work to help developed and developing countries to better structure their institutions to benefit the communities they serve. In the end, stable democracies, even more so in a global order, are a necessary condition for popular benefits. And it is quite astonishing how directly relevant and how important is the set of rules that govern the conduct of government in democratic systems. It is this set of rules that will be at the center of attention in this book.

The particular cases we study are established democratic systems in Israel, Italy, the Netherlands, Britain, and the United States. This focus has allowed us to obtain electoral information and interpret it in a historical context. Given the theoretical framework developed in Chapter 3, we believe that our findings also apply to the new members of the family of democratic systems and can be used in these new environments. Only such new tests can genuinely establish the validity of our theoretical claims and empirical observations.

In pure parliamentary systems, parties run for elections, citizens elect members of these parties to fill seats in Parliaments, members of the Parliament form coalition governments, and these governments make the decisions on the distribution of resource allocations and the implementation of alternative policies. Even in the United States, there is the necessity for coordination or coalition between members of Congress and the President.

Once a government is in power, constituents have little, if any, influence on the allocation of scarce resources. Thus, much of the bargaining process takes place prior to and during the electoral campaign. Candidates who run for office promise to implement different policies. Voters supposedly guard against electing candidates unless they have promised policy positions to their liking. When candidates fail to deliver, voters have the next election to reconstruct the bargain with the same or new candidates.

Preferences are not easily aggregated from the individual level to the collective level of Parliament and transformed into social choices. There exists no mechanism that can aggregate individual preferences into well-behaved social preference orders without violating one or another well-established requirements of democratic choice mechanisms (Arrow, 1951). Individuals' preferences are present mostly inasmuch as they motivate social agents to act in the bargaining game set up by the institutional constraints and rules that define the parliamentary system. Members of Parliament or of Congress take the preferences of their constituents into account if they want to be elected or re-elected. Government thus consists of parliamentary or congressional members who are bound by their pre-electoral commitment to their voters.

The difficulty in detecting a clear relationship between promises made to voters and actual distributions of national resources is a result of the complexity of the process. At each level, agents are engaged in a bargaining process that yields results that are then carried to the next stage. Each layer of the bargaining process is, in large degree, obscure to us, and the interconnections between the multiple layers makes the outcome even more difficult to understand.

In this book we study the mechanism that requires government officials to take into account the preferences of their constituents in the political process. Democracy is representative inasmuch as it is based on institutions that make elected officials accountable to their constituents and responsible for their actions in the public domain. This accountability and responsibility are routinely tested in every electoral campaign. The purpose of this book is to clarify how voter preferences come to matter in a democracy-through the bargaining that takes place before and after each electoral campaign, then during the formation of government, and then within the tenure of each Parliament.

According to common wisdom, the essence of democracy is embedded in legislators representing the preferences of their constituents when

Table 1.1. *Political Systems Determined by the Electoral Rule and Party Discipline.*

	Electoral Rule		
	Proportional Rule		Plurality Rule
Party Discipline	Strong	West European Parliamentary Systems	English Westminster
	Weak	Factional	U.S. Presidential

making decisions over how to allocate scarce resources. Schofield et al. (1998: 257) distinguish four generic democratic systems based on two defining features: the electoral rule used and the culture of party discipline. Their observations are summarized in Table 1.1.

The two most common of these four types are the U.S. presidential and the West European parliamentary systems. Our book gives an analysis of the multiparty parliamentary systems of Israel, Italy, and the Netherlands based on PR. We also examine the "plurality" parliamentary system of Britain and the presidential system of the United States. The remarkable quality of studies in this field notwithstanding, our contribution is intended mainly to provide a comprehensive theoretical framework for organizing current and future research in this field.

Austen-Smith and Banks (1988, 2005) have suggested that the essence of a multiparty representative system (MP) is that it is characterized by a social choice mechanism intended to aggregate individual preferences into social choices in four consecutive stages:

1. The pre-electoral stage: Parties position themselves in the relevant policy space by choosing a leader and declaring a manifesto.
2. The election game: Voters choose whether and for whom to vote.
3. Coalition formation: Several parties may need to reach a contract as to how to participate in coalition government.
4. The legislative stage: Policy is implemented as the social choice outcome.

A comprehensive model of an MP game must include all four stages. A good way to think about it is to use the notion of backward induction: To study the outcome of a game with multiple sequential stages we start the analysis at the last stage. We figure out what contingencies may be favored at the last stage of the game and then go back to the previous stage to see if agents can choose their strategies at that earlier stage of the game to obtain a more favorable outcome at the following stage. In the

context of the four-stage MP game, to play the coalition bargaining game, parties must have relatively clear expectations about what will happen at the legislative stage. To vote, voters must have expectations about the coalition formation game and the policy outcome of the coalition bargaining game. Finally, to position themselves so as to maximize their expected utility, parties must have clear expectations about voting behavior.

1.2 THE STRUCTURE OF THE BOOK

Chapter 2 introduces the basic concepts of the spatial theory of electoral competition. This is the theoretical framework that we utilize throughout the book. The chapter goes on to characterize the last stage of the MP game or the process by which Parliament determines future policies to implement by offering instances of how party leaders' beliefs about the electoral process and the nature of coalition bargaining will influence the policy choices prior to the election. We provide a nontechnical illustration of the logic of coalition bargaining in Section 2.8. Sections 2.9 and 2.10 provide an outline of the various electoral models we use.

Chapter 3 gives the technical details of the theoretical model we deploy. Unfortunately, the formal aspects of the model are quite daunting. Since the essence of the model is described in Chapter 2, we suggest that the reader pass over Chapter 3 in first reading, perhaps checking back on occasion to get the gist of the principal theorem.

The first part of the chapter gives the formal theory of vote maximization under differing stochastic assumptions. For the various models, the *electoral* theorem shows that there are differing conditions on the parameters of the model which are necessary and sufficient for convergence to the electoral mean. We essentially update Madison's perspective from *Federalist 10,* in which he argues that elections involve judgment, rather than just interests or preferences. We model these electoral judgments by a stochastic variable that we term *valence*. When the electorally perceived valences vary sufficiently among the parties, then low-valence parties have an electoral incentive to adopt radical policy positions. The electoral calculus in the model is then extended to a more general case in which party "principals," or decision makers, have policy preferences.

Chapter 4 begins the empirical modelling of the interaction of parties and voters. We provide an empirical estimation of the elections in 1988, 1992, and 1996 in Israel. The electoral theorem is used to determine where the vote-maximizing equilibria are located. It is shown that the location

of the major parties, Labour and Likud, closely match the theoretical prediction of the theorem. We use the mismatch between the theory and estimated location of the low-valence parties to argue that they positioned themselves to gain advantage in coalition negotiation.

In Chapters 5, 6, and 7, we discuss in more detail elections in Italy, the Netherlands, and Britain. In Italy, we observe that the collapse of the political system after 1992 led to the destruction of the "core" location of the dominant Christian Democrat Party. The electoral model effectively predicts party positions, except possibly for the Northern League. In the Netherlands and Britain, the electoral theorem suggests that all parties should have converged to the electoral center. We propose an extension of the electoral theorem to include the effect of activists on electoral judgments. In Britain in particular, the model suggests that the effect of the "exogenous" valence is "centripetal," tending to pull the two major parties toward the electoral center. In contrast, we argue that the effect of party activists on the party's valence generates a "centrifugal" tendency toward the electoral periphery.

Chapter 8 considers the 1964 and 1980 elections in the United States to give a theoretical account, based on activist support, of the transformation that has been observed in the locations of the Republican and Democratic Parties. We suggest that this is an aspect of a *dynamic equilibrium* that has continually affected U.S. politics.

Throughout the book we draw conclusions from the empirical evidence to show how the basic electoral model can be extended to include coalition bargaining and activist support. These empirical chapters are based on work undertaken with our colleagues over the last ten years. The theoretical argument in Chapter 3 is drawn from Schofield and Sened (2002) and Schofield (2004, 2006b). Chapter 4 is adapted from Schofield and Sened (2005a) as well as earlier work in Schofield, Sened, and Nixon (1998). The analysis of Italy in Chapter 5 is based on Giannetti and Sened (2004). The study of elections in the Netherlands, given in Chapter 6, is based on Schofield et al. (1998), Quinn, Martin, and Whitford (1999), and Schofield and Sened (2005b). The work on the British election of 1979 in Chapter 7 uses the data and probit analysis of Quinn, Martin, and Whitford (1999), and the analysis of the 1992 and 1997 elections comes from Schofield (2005a,b). Chapter 8 discusses U.S. elections using a model introduced in Miller and Schofield (2003) and Schofield, Miller, and Martin (2003). In a companion volume, Schofield (2006a) presents a more detailed narrative of these events in U.S. political history.

1.3 ACKNOWLEDGMENTS

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2

Elections and Democracy

2.1 ELECTORAL COMPETITION

> [I]t may be concluded that a pure democracy, by which I mean a society, consisting of a small number of citizens, who assemble and administer the government in person, can admit of no cure for the mischiefs of faction. . . . Hence it is that such democracies have ever been spectacles of turbulence and contention; have ever been found incompatible with personal security . . . and have in general been as short in their lives as they have been violent in their deaths.
>
> A republic, by which I mean a government in which the scheme of representation takes place, opens a different prospect. . . .
>
> [I]f the proportion of fit characters be not less in the large than in the small republic, the former will present a greater option, and consequently a greater probability of a fit choice. (Madison, 1787).

It was James Madison's hope that the voters in the Republic would base their choices on judgments about the fitness of the Chief Magistrate. Madison's argument to this effect in *Federalist 10* may very well have been influenced by a book published by Condorcet in Paris in 1785, extracts of which were sent by Jefferson from France with other materials to help Madison in his deliberation about the proper form of government. While Madison and Hamilton agreed about the necessity of leadership in the Republic, there was also reason to fear the exercise of tyranny by the Chief Magistrate as well as the turbulence or mutability of decision making both in a direct democracy and in the legislature. Although passions and interests may sway the electorate, and operate against fit choices, Madison argued that the heterogeneity of the large electorate would cause judgment to be the basis of elections. The form of the electoral college as the method of choosing the Chief Magistrate led to a system of representation that we may label "first past the post" by majority choice. It is intuitively obvious

that such a method tends to oblige the various groups in the Republic to form electoral coalitions, usually resulting in two opposed presidential candidates. Of course, many elections have been highly contentious, with three or four contenders. The election of 1800, for example, had Jefferson, Burr, John Adams, and Pinckney in competition. In 1824, John Quincy Adams won the election against Andrew Jackson, William Crawford, and Henry Clay by the majority decision of Congress. In that election, Jackson had the greatest number (a plurality) of electoral college votes (99 out of 261) and a plurality of the popular vote, but not a majority. Perhaps the most contentious of elections was in 1860, when Lincoln won with 40 percent of the popular vote, and 180 electoral college votes out of 303, against Steven Douglas, Breckinridge, and Bell. See Schofield (2006a) for a discussion of this election.

Even though the use of this electoral method for the choice of President may be unsatisfactory from the point of view of direct democracy, it does appear, in general, to "force" a choice on the electorate. Proportional representation (PR), on the other hand, is a very different method rule. In such an electoral method, there is usually a high correlation between the share of the popular vote that a party receives and its representation in Parliament. Depending on the precise nature of the electoral method, there may be little incentive for parliamentary groups to form pre-election political coalitions. As a result, it is usually the case that no party gains a majority of the seats, so that post-election governmental coalitions are necessary. A consequence of this may be a high degree of governmental instability.

Although formal models of elections have been available for many decades, most of them were concerned with constructing a theoretical framework applicable to the United States. The models naturally concentrated on two-party competition, where the assumed motivation of each of the contenders was to gain a majority of the votes. As the remarks just made suggest, even such a framework is unable to deal with a number of the most interesting elections in U.S. history, where there are more than two candidates, and "winning" is not the same as vote maximization. More importantly, from our perspective, these models did not easily generalize to the situation of PR, where no party could expect to win.

The work presented here is an attempt to present an integrated theory of multiparty competition that can be applied, at least in principle, to polities with differing electoral systems.

2.2 TWO-PARTY COMPETITION UNDER PLURALITY RULE

The early formal models of two-party competition leave much to be desired. It seems self-evident that presidential candidates offer very different policies to the electorate. Although the members of Congress of the same party differ widely in the policies they individually espouse, there is an obvious difference in the general policy characteristics of the two parties. The Republican Party Manifesto that was intended to herald a new era of Republican dominance in 1994 could not be mistaken for the declaration of the Democrat Party. The variety of results known as the *median voter* theorem (Hotelling, 1929; Downs, 1957; Black, 1958; Riker and Ordeshook, 1973) were all based on the "deterministic" assumption that each voter picked the party with the nearest policy position. Assuming that policies necessarily resided in a single dimension, the effort by each contender to win a majority would oblige them to choose the policy position of the median voter. Such a voter's preferred policy is characterized by the feature that half the voters lie on the left of the position, and half on the right. This result can be generalized to the case with multiple candidates and costly campaigns (Fedderson, Sened, and Wright, 1990) or uncertainty in party location (McKelvey and Ordeshook, 1985), but it is crucial to the argument that there be only one dimension.

A corrective to this formal result was what became known as the *chaos* theorem. This was the conclusion of a long research effort from Plott (1967) to Saari (1996) and Austen-Smith and Banks (1999). An illustration of this theorem is given below. It was valid for two-party competition only, and assumed that the motivation of candidates was to gain a majority of the popular vote. Whether or not candidates had intrinsic policy preferences, these were assumed irrelevant to the desire to win. One variety of the theorem showed that in two dimensions, it was generally the case that no matter what position the first candidate took, there was a position available to the second that was winning. One way of expressing this is that there would be no two-party equilibrium, or so-called *core* (Schofield, 1983). As a consequence, candidates could, in principle, adopt indeterminate positions (McKelvey, 1976). In three dimensions, candidate positions could end up at the electoral periphery (McKelvey and Schofield, 1987).

Figure 2.1 gives an illustration with just three voters and preferred positions A, B, and C. The sequence of positions $\{x, a, b, c, d, e, f, g, h, y,\}$ is a majority trajectory, from x to y, with y beating h beating g, etc., and a beating x.

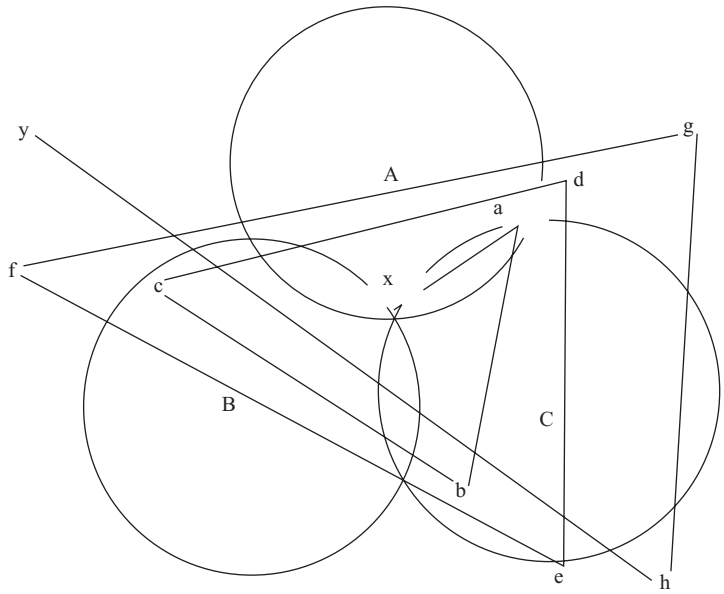


Figure 2.1. An illustration of instabiliy under deterministic voting with three voters with preferred points A, B, and C.

A third class of results assumed that candidates deal with this chaos through ambiguity in their policies, by "mixing" their declarations. The results by Kramer (1978) and Banks, Duggan, and Le Breton (2002) suggest again that candidate policies will lie close to the electoral center.

Yet another set of results weakened the assumption that voters were "deterministic" and instead allowed for a *stochastic* component in voter choice (Hinich, 1977). The recent work by McKelvey and Patty (2005) and Banks and Duggan (2005) has formalized the model of voter choice in two-party elections, where each candidate attempts to maximize expected plurality (the difference between the candidate's expected share and the opposition's) and has shown, essentially that the equilibrium is one where both candidates converge to the mean of the voter distribution.

Although Madison may have feared for the incoherence of voter choice, and his fears are, in essence reflected in the chaos theorem, there seems little evidence of the strong conclusion that may be drawn, that "anything can happen in politics" (Riker, 1980, 1982a). What does appear to be true, however, is that policy is mutable: One party wins and tries to implement its declared policy, and then later the opposition party wins, tries to undo the previous policies, and implement its own. If this is at all close to the

nature of politics, then neither the median voter theorem, nor its stochastic variant has much to say about real politics.

2.3 MULTIPARTY REPRESENTATIVE DEMOCRACY

We consider that the aforementioned formal results, purporting to show the predominance of a centripetal tendency toward the electoral center in representative democracy, are fundamentally flawed. The reason is that they do not pay heed to Madison's belief that elections involve judgments as well as interests. We show by empirical studies of elections from five polities that judgments *do* form part of the utility calculus of voters. The weight given to judgment, rather than to preference in the stochastic vote model, we call *valence*. The studies show that adding valence to the empirical model enhances the statistical significance, as indicated by the so-called Bayes' factor. When these valence terms are included in the formal model, then convergence to the electoral mean depends on an easily computed "convergence coefficient." When the necessary conditions, given in our Theorems 3.1 and 3.2, are violated, then not all parties will locate at the electoral center. In fact, low-valence parties will find that their vote-maximizing positions are at the electoral periphery. We show that this prediction from the formal model accords quite well with the actual positioning of parties in Israel and Italy. We draw from this our main hypothesis.

Hypothesis 2.1: *A primary objective of all parties in a representative democracy is to adopt policy positions that maximize electoral support.*

We can test this hypothesis by using the parameter estimates of the empirical models to determine whether the actual locations of parties accord with the estimated equilibrium positions as indicated by the formal model. Our analyses indicate that for Israel and Italy there is a degree of concordance between empirical and formal analyses. The formal analysis indicates that the high-valence parties in Israel—Labour and Likud—should adopt positions relatively close to, but not precisely at, the electoral mean, but that the low-valence parties, such as Shas, should position themselves at the electoral periphery. The concordance is close, but not exact. The model we propose to account for the discrepancy between theory and fact in multiparty polities takes account of the policy preferences of parties in the sense that they are concerned to position themselves in

the pre-election situation, so as to better their chances of membership in governing coalition.

Hypothesis 2.2: *Any discrepancy between the estimated equilibrium positions of parties obtained from the application of Hypothesis 2.1 in polities based on proportional electoral methods arises because of the requirement of party leaders to consider post-election coalition negotiation.*

To evaluate this hypothesis in a formal fashion it is necessary to attempt to model how party leaders form beliefs about the effect their policy declarations have on the formation of post-election coalition government.

Obviously, considerations about coalition negotiation cannot be used to account for discrepancies between the theory derived from Hypothesis 2.1 and the location of parties in plurality polities such as Britain and the United States, if only because coalition formation, if it occurs, would be a pre-election phenomenon.

One way to adapt Hypothesis 2.1 is to extend the idea of valence, so that it is not exogenously determined, but is, instead, the consequence of the actions of activists who contribute time and resources to enhance the perceived valence of the party, or party candidate, in the electorate. This gives us our third hypothesis.

Hypothesis 2.3: *Any discrepancy between the estimated equilibrium positions of parties obtained from the application of Hypothesis 2.1 in polities based on plurality electoral systems arises because the valence of each party is a function of activist support. When the model is transformed to account for activist valence, then the positions of parties should be in equilibrium with respect to vote maximization.*

Because of our ambition to present a unified theory of political choice, we are obliged to construct a theory for an arbitrary number, p, of parties (where p may be 2 or more) competing in a policy space X of dimension w. We hope to relate the theory that we present to empirical analyses drawn from five polities. Two of these (Israel and the Netherlands) use electoral systems for the Parliament that are based on PR. Israel in particular has a large number of parties. In addition, it used a plurality method for the selection of the Prime Minister in 1996. A third polity, Italy, used PR until 1992, but then adopted a mixed PR/plurality electoral method. The fourth polity, Britain, uses plurality rule, but has more than two parties.

The last polity we consider is the United States, but we start the discussion with the four-candidate election of 1860.

We suppose that the set of parties $P = \{1, \ldots, j, \ldots, p\}$ is exogenously determined. In fact, the number of parties competing with each other can vary from election to election. In principle, it should be possible to model the formation of new parties from activist groups. Our discussion of the United States in Chapter 8 suggests how this might be done.

Similarly, we use $N = \{1, \ldots, i, \ldots, n\}$ to denote the set of voters. Obviously, the set of voters varies from election to election so we should perhaps use a suffix to denote the various elections. As above, we assume that the policy space, X, has dimension w. We do not restrict w in an a priori fashion. There are many ways to determine the nature of X, but our preference is for a methodology based on some large-number electoral sample, by which we can ascertain the basic beliefs or concerns of the members of the voting public. The empirical analyses that we use suggest that only two dimensions are sufficient in each polity to obtain statistically significant models of voter choice.

Because we consider that Hypothesis 2.1 will not be entirely adequate, we shall work back from the post-election legislative phase to the election, and then consider the pre-election selection of party leader and the formation of party policy.

2.4 THE LEGISLATIVE STAGE

In this phase, the party positions are given by an array

$$\mathbb{Z} = (z_1, \ldots, z_j, \ldots, z_p),$$

where each z_j is a policy position in X that is representative of the party. The election that has just occurred has given a vector $V = (V_1, \ldots, V_p)$ of vote shares that has been turned by the electoral system into a vector $S = (S_1, \ldots, S_p)$ of parliamentary seat shares. This vector generates a family $\mathcal{D}$ of winning or decisive coalitions. It is usual, but not absolutely necessary, that $\mathcal{D}$ comprises the family of subsets of P that control at least half the parliamentary seats. Given the set P of parties, and all possible vectors of seat shares we let $\mathbb{D} = \{\mathcal{D}_t : t = 1, \ldots, T\}$ be the set of all possible families of winning coalitions. We regard $\mathbb{D}$ as one way to represent the set of possible election outcomes. We are generally most interested in the situation where *multiparty* refers to the feature that there are at least three parties, so that, in general, each $\mathcal{D}$ will consist of a number of disjoint coalitions. However, we can use some aspects of the model we

propose to examine two-party competition. This suggests the following categorization:

2.4.1 *Two-Party Competition with Weakly Disciplined Parties*

This is essentially the situation in the U.S. Congress. From this perspective, every member of the House and Senate could be regarded as a single party, with a policy position representative in some fashion of the member's district or state. Similarly, the President's policy position would be some position made known in the course of the election. The decisive coalition structure, $\mathcal{D}$, is the set of possible decisive coalitions, involving the veto capacity of the President against Congress, and Congress's counterveto capacity (Hammond and Miller, 1987). Analyzing the legislative behavior of Congress is the basis for an extensive literature, but this is not our concern here. However, some aspects of the model we present here may be relevant to the selection of the President through the method of the electoral college. Instead of supposing that every member of Congress is a single party, it could also be supposed that members coalesced into factions, based on policy similarities. Coalition formation involving relatively disciplined factions could then be examined in the context of our model.

2.4.2 *Party Competition under Plurality Rule*

It is well known that plurality rule, or "first past the post," induces a distortion in the translation of vote shares to seat shares, sufficient usually to guarantee that one party or the other gains a majority of the seats. In this case, the decisive coalition, $\mathcal{D}$, can be assumed to be a single party. Under this assumption, the family of all possible government coalitions may be taken to be $\mathbb{D} = \{\mathcal{D}_j : j = 1, \ldots, p\}$, where each $\mathcal{D}_j$ comprises a single party, j. However, even in the case of the British Parliament it is in principle possible for no party to gain a majority. Thus, a more general formulation would be to allow $\mathbb{D}$ to include possible coalitions of parties. In the simpler models of legislative behavior in such a Parliament it is presumed that the majority party leader can control government policy making, with the cooperation of the Cabinet, and through the operation of the Whip. If party j controls a majority, and the policy position of the party leader is z_j, the policy outcome could be assumed to be z_j. However, there will always be some uncertainty in the willingness of the parliamentary members to support a particular position. Consequently,

a more general formulation is to suppose that the post-election policy outcome is a "lottery," $\tilde{g}_t$, across various policy positions of different activist groups for the party. We characterize the various activist groups as being led by *party principals*. Chapter 7 on Britain develops this notion.

2.4.3 *Party Competition under Proportional Representation*

It is usual that no party controls a majority of the seats. In such a situation it is natural to assume that bargaining between the parties will be determined by the particular set, $\mathcal{D}_t$, of decisive coalitions that is created by the election. Assuming that the parties are strongly disciplined, so that each party, j, is represented by the policy position, z_j, of its leader, then the policy outcome will also be a lottery—that is, some combination of $\{z_j\}$ and probabilities. In this case, however, the precise lottery will depend on the positions of all parties. Moreover, this lottery will depend on the seat shares of the parties, and thus ultimately on the particular decisive structure, $\mathcal{D}_t$, holding after the election. Since $\mathcal{D}_t$ depends on the election result, and this depends on the vector $\mathbf{z}$ of party positions, we can show this dependence by writing $\tilde{g}_t(\mathbf{z})$ for this lottery.

2.4.4 *Coalition Bargaining*

Sened (1995, 1996) and Banks and Duggan (2000) have modeled bargaining between parties in the post-election phase and have shown that there are essentially two different situations. One situation is where a party, absent a majority, is nonetheless in such a commanding position because of its central position and seat share that it can essentially control policy. In this case, the dominant party, j, is termed a *core party*. The lottery can then be identified with z_j. The second situation is when there is no core party. In this case, bargaining theory suggests that any one of a number of possible coalition governments can come into being. As indicated by the notation, the policy positions and the probabilities associated with each of the governments will depend on $\mathcal{D}_t$ and z. We say *coalitional risk* is associated with the formation of government. In addition there will be bargaining over nonpolicy governmental perquisites. Empirical analyses of portfolio distribution have shown a relation between seat proportions in governing coalitions and portfolio shares (Browne and Franklin, 1973; Laver and Schofield, 1990). If we extend the idea of a post-election lottery to include government perquisites (such as cabinet positions), we can also denote this lottery by $\tilde{g}_t^{\alpha}(\mathbf{z})$, where α denotes a parameter that governs

the tradeoff between policy preferences and perquisites. Obviously, party discipline may be only partial, and the uncertainty associated with the ability of party leaders to control their members will affect the lottery $\tilde{g}_t^{\alpha}(\mathbf{z})$. We therefore use this symbol to refer to the political agents' beliefs about the outcomes of coalition bargaining when political strength is given by the structure $\mathcal{D}_t$ and party locations are given by $\mathbf{z}$.

2.5 THE ELECTION

We use $\mathbf{L} = (L_1, \ldots, L_j, \ldots, L_p)$ to denote the set of leaders of the various parties at election time. An important component of the electoral models that we consider is that they incorporate the effect of valence.

Stokes (1963, 1992) first introduced this concept many years ago. Valence relates to voters' judgments about positively or negatively evaluated conditions that they associate with particular parties or candidates. These judgments could refer to party leaders' competence, integrity, moral stance, or "charisma" over issues such as the ability to deal with the economy, foreign threat, and so forth. The important point to note is that these individual judgments are independent of the positions of the voter and the party. Estimates of these judgments can be obtained from survey data (see, for example, the work on Britain by Clarke, Stewart, and Whiteley, 1995, 1997, 1998; and Clarke et al., 2004). However, from such surveys it is difficult to determine the "weight" an individual voter attaches to the judgment in comparison to the weight of the policy difference between the voter and the party. As a consequence, the empirical models usually estimate valence for a party or party leader as a constant or intercept term in the voter-utility function. The party valence variate can then be assumed to be distributed throughout the electorate in some appropriate fashion. This stochastic variation is expressed in terms of a vector of disturbances, which, in the most general model, is assumed to be distributed multivariate normal with covariance matrix, Ω. This formal assumption parallels that of multinomial probit (MNP) estimation. The more common assumption is that the errors satisfy a "Type I extreme value distribution," and this induces multinomial logit (MNL) estimation. To model the election in this way requires knowledge of the set of preferred points of voters $\{x_i\}$ together with the vector $(z_1, \ldots, z_j, \ldots, z_p)$ of party positions. In addition, the effects of sociodemographic characteristics of voters can be incorporated into the model. The model then assumes that the implicit utility of voter i for party j is increasing in the valence λ_j, of party j, and decreasing in the weighted quadratic distance between

the voter's position and that of the party. In addition, it is possible to incorporate the influence that the sociodemographic characteristics η_i of voter i may have on the voter's political choice. The model is stochastic because of the implicit assumption that the valence λ_{ij} that voter i assigns to j is a combination of the expectation λ_j and a random disturbance ε_j, with appropriate distribution. Formal definitions of the various models are set out at the end of this chapter. Because voter utility is stochastic, it is impossible to assert with precision which party a voter will choose. However, it is possible in empirical models to estimate the probability matrix $[\rho^*_{ij}(\mathbf{z})]$. Here we use $\rho^*_{ij}(\mathbf{z})$ to denote the probability that voter i chooses party j. Note that because of uncertainty in estimation, $\rho^*_{ij}(\mathbf{z})$ will also be a stochastic variable with expectation $\bar{\rho}_{ij}(\mathbf{z})$. Taking the mean value gives the expected vote share, $\mathcal{E}_j(\mathbf{z})$, of party j. For the baseline formal model we use $V_j(\mathbf{z})$ to denote the expected vote share.

The results of empirical estimation give rise to estimates for the valences, represented by $\lambda = (\lambda_1, \ldots, \lambda_j, \ldots, \lambda_p)$. Obviously these valence values will depend on the characteristics $\mathbf{L} = (L_1, \ldots, L_j, \ldots, L_p)$ of the various leaders.

In this formulation, given the choice of leaders

$$\mathbf{L} = (L_1, \ldots, L_j, \ldots, L_p)$$

and policy positions $\mathbf{z} = (z_1, \ldots, z_j, \ldots, z_p)$ then the "outcome" of the election is a stochastic variable, which we represent by the symbol $\Pi(\mathbf{z})$. By this we mean to emphasize that $\Pi(\mathbf{z})$ describes the common beliefs, or estimated probabilities, associated with all possible relevant features of the election that will occur as result of the set of declarations given by $\mathbf{z}$.

The "electoral game" revolves around the decision of each party to select a policy position or "manifesto" to declare to the electorate at the time of the election. There are a number of possible modelling strategies that ignore the uncertainty inherent in the election and focus on electoral expectations.

2.6 EXPECTED VOTE MAXIMIZATION

2.6.1 *Exogenous Valence*

In this formulation, the valence terms of the parties are fixed, or *exogenous,* and the leader and the other members of the party are agreed that the party's policy position should be one which maximizes the party's vote share. Since party share depends on other party positions, it is natural to

deploy the Nash equilibrium concept (Nash, 1950a,b, 1951). In this case, a vector of party positions $\mathbf{z}^*$ is a pure Nash equilibrium (PNE) if no party may unilaterally change z_j so as to increase its vote share. In our analyses of Israel and Italy, we compare the formal model of voting, with exogenous valence, with empirical models based on MNL estimation to determine the degree of fit between the models. The results of the formal model presented in Chapter 3 make it evident that the conditions for existence of PNE are very restrictive. Instead, we focus on what we call a "local Nash equilibrium," denoted LNE. The conditions for existence of LNE can be computed from the parameters obtained by the estimation. Theorems 3.1 and 3.2 show that the necessary and sufficient conditions for convergence to the electoral mean for both logit (MNL) and probit (MNP) models depends on a *convergence coefficient*, c, given essentially by the expression

$$c = 2Av^2.$$

Here v^2 is the total electoral variance while A is a function of the parameters (β, λ) and is increasing in β and in the difference in valence between high- and low-valence parties. For the multinomial probit model based on the normal distribution, c is decreasing in the measure of total error variance. In two dimensions, the necessary condition is that $c \leq 2$. This result has a clear interpretation. If the "spatial effect," βv^2, is large, then a party with a low enough valence, λ_1, say, will find that its vote share increases as the party vacates the electoral mean. This immediately implies that the LNE will consist of party positions strung along a principal electoral axis. This necessary condition is violated in Israel, and we therefore obtain a theoretical reason why convergence does not occur. Because of a discrepancy between the prediction of the formal model and the estimated party positions, we deploy Hypothesis 2.2.

2.6.2 *Activist Valence*

Since parties require activist support, for resources of time and money, and this support will depend on the actual position adopted by the party, we may modify the voter utility equation to be dependent on the valence, $\mu_j(z_j)$, of the party and attributable to the contributions of the party members. This is intended to model the additional valence induced by the availability of activist resources which are used to carry the party message to the electorate. Although activists respond to the declared position, and thus indirectly affect the party choice, they do not directly control

policy. The party leader must still choose a policy position to maximize the expected vote share, $\mathcal{E}_j(\mathbf{z})$. Notice however, that the party's choice of leader will affect the valence, or electoral perception, of the party. To keep distinct the leader's position and that of representative members of the party, we assume that the preferences of the members of the party are represented by an agent whom we call the *party principal*. The application of the formal model to empirical estimations for elections in Britain in 1979, 1992, and 1997, in Chapter 7, indicates that, under the exogenous valence model, the high-valence Labour and Conservative parties should have converged to the electoral mean. Simulation of an empirical model for the Netherlands for electoral data from 1979 also indicate that vote-maximizing parties should have converged to the center. Nonconvergence in these two polities leads us to a model of activist valence.

2.6.3 *Activist Influence on Policy*

Under the two earlier formulations, the leader's role is simply to implement the policy position chosen by the party principal. If the leader has no interest in the policy position, then it is obvious that there will be no credible commitment to the declared policy, except possibly because of the threat of activist revolt. In our analysis in Chapter 6 of the Netherlands in the elections of 1977 and 1981, we essentially suppose that each party position is chosen by the party principal.

A more general model includes the policy concerns of activists as well as party members in the formulation of the party manifesto.

2.7 SELECTION OF THE PARTY LEADER

The party comprises parliamentary members, party members, and activists. In principle, all members are interested in the policy proposed by the party and in the final governmental outcome. We can represent a delegate's utility by an additive expression involving perquisites and the quadratic loss given by the distance between the government's chosen policy and the delegate's preferred point.

Assume now that the leaders of each of the parties have been chosen, so that the valences are known. If the vector of positions of the other parties is also known, then a delegate of party j can, in principle, compute the stochastic result of the election to follow. That is to say, for any policy position, z_j, chosen by the party, we assume that the delegate has consistent beliefs about the nature of the electoral response. We represent these beliefs by the "belief operator" Π. Thus, when parties have chosen

their strategies, $\mathbf{z}$, we assume they hold common beliefs, $\Pi(\mathbf{z})$, about the election. In particular, $\Pi(\mathbf{z})$ encodes information on the probability, $\pi_t(\mathbf{z})$, that the coalition structure, $\mathcal{D}_t$, occurs after the election. We have argued that when the coalition structure, $\mathcal{D}_t$, occurs, then the consequences of interparty bargaining can be represented by the lottery $\tilde{g}_t^{\alpha}(\mathbf{z})$. By taking expectations across all possible coalition structures, the delegate can compute the expected utility from a choice, z_j, and can therefore determine which choice of party position is the best response to the positions, $\mathbf{z}_{-j} = (\ldots z_{j-1}, z_{j+1}, \ldots)$, of the other parties.

The delegates may very well disagree in their computation of their party's best response. We have suggested that one way to overcome this intraparty conflict is for the party to choose a principal for the party, who in some fashion has typical policy preferences of the party elite. There are a number of obvious strategies for modelling the choice of the party manifesto.

(i) The principal computes the best response to the other party principals' choices, and writes the party manifesto, based on personal policy preferences. The leader of the party then presents the manifesto to the electorate.

(ii) The principal attempts to find a party leader whose own known policy preferences are a compromise between the heterogeneous preferences of the various activist and delegate subgroups within the party. Picking a party leader whose sincere policy position the party can endorse as its strategic policy declaration thus solves the problem of the credible commitment of the party leader to the declared policy of the party (Banks, 1990). Notice that this choice of the party leader may be one of extreme complexity, since it involves a long chain of reasoning, including guessing at the leader's likely electoral valence, the effect on the stochastic electoral operator, and the effect of the election outcome on coalition bargaining.

(iii) It is obviously an oversimplification to assume that the choice of party leader can be left to a party principal. The degree of policy conflict may be so extreme that different subgroups within the party elect their own principals to compete with each other over the choice of party leader. Miller and Schofield (2003) suggest that this is likely to be a characteristic of plurality electoral systems such as the United States and Britain. As a consequence, one can expect severely contested leadership elections after a party has performed poorly at the election. However, if the party succeeds at the election, then we can

assume that the party leader will stay in power after the election, and can be credibly expected to implement his or her position.

The choice of the set of party leaders' policy positions, or party manifestos, can be expressed as an equilibrium to the very complex game just presented. While the usual equilibrium concept utilized to examine such games is Pure Nash equilibrium (PNE), the conditions known to be sufficient for existence of this equilibrium are unlikely to hold. We therefore use what we have called a "local Nash equilibrium" (LNE). The conditions for existence for an LNE are much less stringent than those for a PNE. Indeed, a PNE by definition must be an LNE, so that if an LNE of a particular kind fails to exist, then the PNE will also fail to exist. This local equilibrium concept essentially supposes that political protagonists consider "small" changes in strategy, rather than the "global" changes envisaged in the Nash equilibrium notion. Most importantly we give reasons to believe that the set of LNE is nonempty. Determining conditions for existence of LNE at the electoral mean is accomplished in Theorems 3.1 and 3.2, but the determination of this set analytically for general electoral models is very difficult. Nonetheless, once an empirical model has been constructed, then it is possible to estimate the set of LNE by simulation.

2.8 EXAMPLE: ISRAEL

To illustrate the framework just presented, we borrow some of our empirical findings from Chapter 4, in which we discuss in detail the case of Israel. We return to this illustration in Section 3.5. Table 2.1 gives the election results between 1988 and 2003, while Figure 2.2 presents our estimates of the party positions in 1992. The background to this figure is an estimate of the electoral distribution of voter ideal points, derived from Arian and Shamir (1995). We discuss estimation techniques and data in Chapter 4, where more details on the two policy dimensions are given. As in all our electoral figures, the outer contour line contains 95 percent of the voter ideal points, whereas the inner contours contain 75 percent, 50 percent, and 10 percent of the ideal points. We assume Euclidean loss functions based on the party points given in Figure 2.2, and ignore the additional complexity induced by governmental perquisites. (See Section 2.9 for a sketch of this electoral model.) We can show that Labour was a "core party" after the election of 1992. To see this, consider the obvious coalition based on the leadership of Likud. A coalition of Likud

Table 2.1. *Elections in Israel, 1998–2003.*

	Party	Knesset Seats				
		1988	1992	1996	1999	2003
Left	Labour (LAB)	39	44	34	28	19[a]
	Democrat (ADL)	1	2	4	5	2[a]
	Meretz (MRZ)	–	12	9	10	6
	CRM, MPM, PLP	9	–	–	–	3
	Communist (HS)	4	3	5	3	3
	Balad	–	–	–	2	3
Subtotal		*53*	*61*	*52*	*48*	*36*
Center	Olim	–	–	7	6	2[b]
	III Way	–	–	4	–	–
	Center	–	–	–	6	–
	Shinui (S)	2	–	–	6	15
Subtotal		*2*		*11*	*18*	*17*[b]
Right	Likud (LIK)	40	32	30	19	38[b]
	Gesher	–	–	2	–	–
	Tzomet (TZ)	2	8	–	–	–
	Israel beiteinu	–	–	–	4	7
Subtotal		*42*	*40*	*32*	*23*	*45*
Religious	Shas (SHAS)	6	6	10	17	11
	Yahadut (AI, DH)	7	4	4	5	5
	Mafdal (NRP)	5	6	9	5	6
	Moledet (MO)	2	3	2	4	–
	Techiya, Thia (TY)	3	–	–	–	–
Subtotal		*23*	*19*	*25*	*31*	*22*
TOTAL		*120*	*120*	*120*	*120*	*120*

[a] Am Ehad or ADL, under Peretz, combined with Labour, to give the party $19 + 2 = 21$ seats.
[b] Olim joined Likud to form one party giving Likud $38 + 2 = 40$ seats, and the right $40 + 7 = 47$ seats.

with Tzomet and the four religious parties controls only 59 seats out of 120. To be decisive, this coalition needs 61 seats and so must add either Meretz or Labour. If Meretz is added to the coalition, then the set of policies that this decisive coalition can implement can be identified with the convex hull of the points associated with the members of the coalition. However, the policy point representing Labour lies within this set. Consequently, if Labour proposes its ideal point, then no decisive coalition can propose another that it prefers. Thus, the Labour position cannot be defeated by another policy position supported by a decisive coalition. As

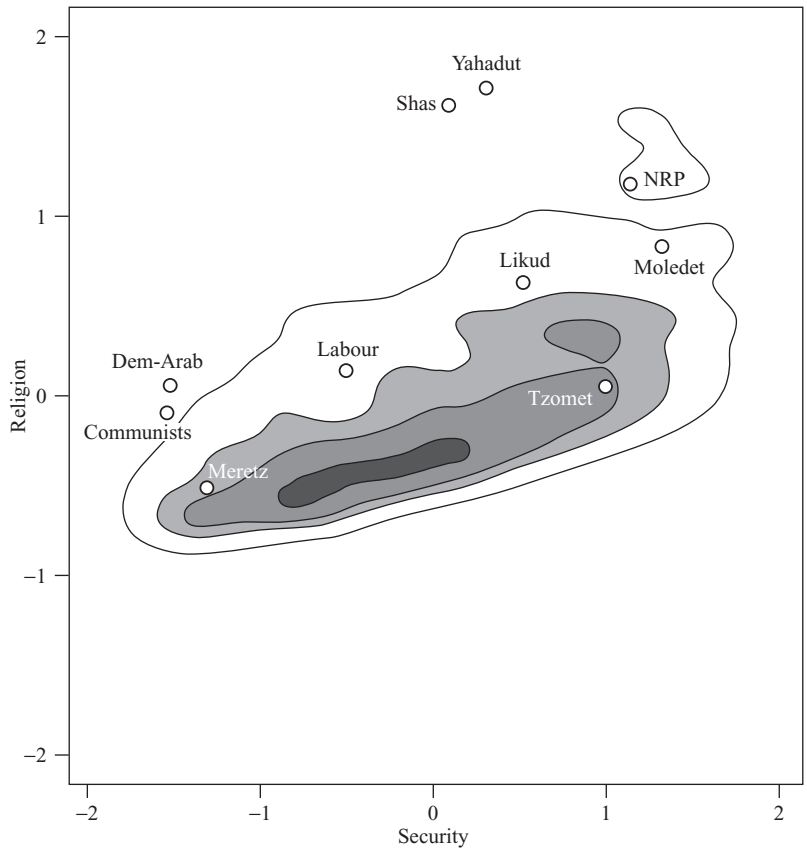


Figure 2.2. Estimated party positions in the Knesset at the 1992 election.

a consequence, we call this point the *core* of the coalition game, given the set of winning coalitions, $\mathcal{D}_{1992}$. Another way to show that Labour is at the core is to construct the median lines in the figure, where a median line through two party positions cuts the policy space in two, so that coalition majorities lie on either side of the line. For example, in Figure 2.3, the line through Shas and Labour (with 50 seats) has more than 10 seats on either side, thus demonstrating that it is a median. Three different median lines are drawn in Figure 2.3, all intersecting in the Labour position. The intersection of these lines guarantees that the Labour position is a core. This technique involving medians is one method of determining whether a party position is a core (see also McKelvey and Schofield, 1987).

All versions of coalition bargaining theory suggest that the core point will be the outcome (Sened, 1996; Banks and Duggan, 2000). Note also

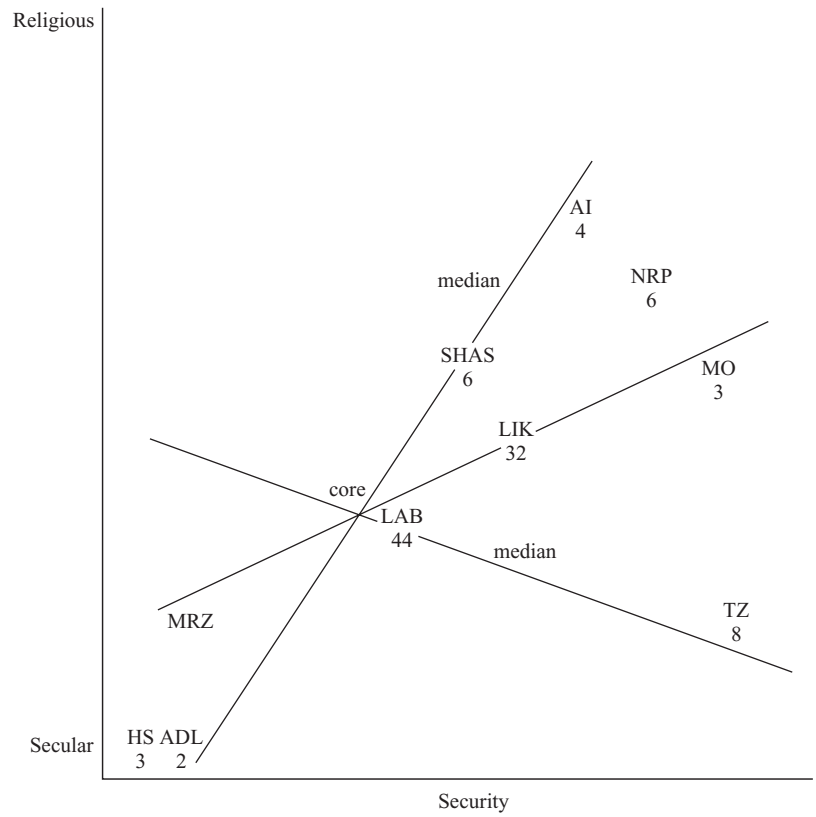


Figure 2.3. Estimated median lines and core in the Knesset after the 1992 election.

that this core point is "structurally stable," in the sense that a small perturbation of the preferred policy point of the parties does not change the core property. We denote the structurally stable core by $\mathbb{SC}_1(\mathbf{z})$. Notice that this concept depends on both the vector of party positions and the particular set of winning coalitions, $\mathcal{D}_{1992}$. We call $\mathcal{D}_{1992}$ the *decisive structure*. Since the core outcome is associated with a single party, even though that party lacks a majority of the seats, we expect the Labour Party to form a minority government (Laver and Schofield, 1990; Sened, 1996). As we discuss in Chapter 4, this is precisely what happened. We use the notation, $\mathbb{D}_1$, for the family of decisive structures, including $\mathcal{D}_{1992}$, under which Labour could be located at the core. We also say that this decisive structure implies that Labour is the strongest party and that its position implies that it is also *dominant*. Since Labour appears to have

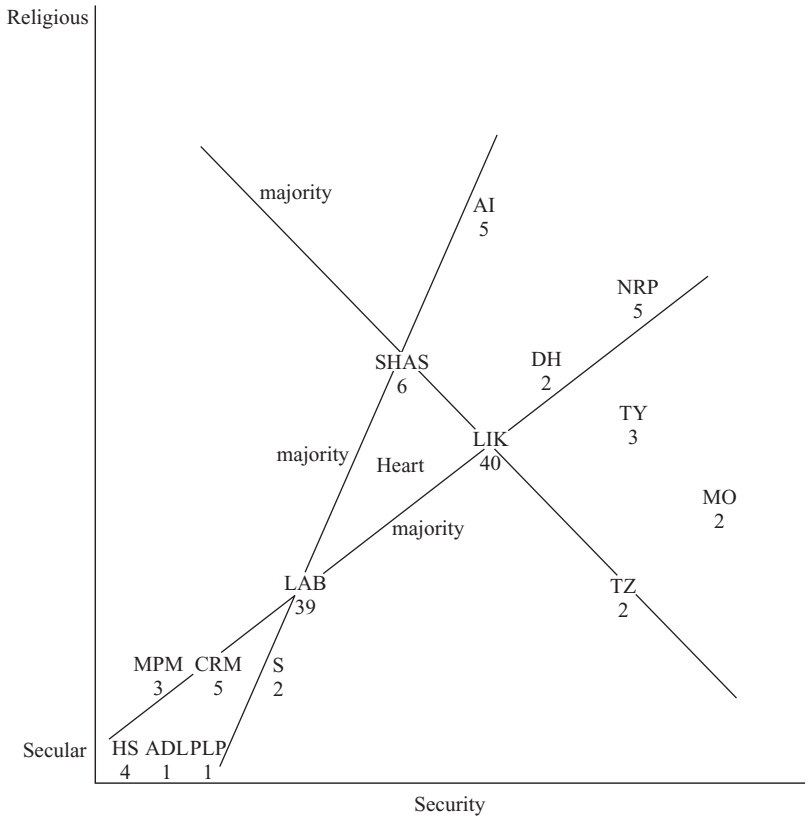


Figure 2.4. Estimated median lines and empty core in the Knesset after the 1988 election.

occupied the core position in 1992 we also say, for the post-election environment determined by $\mathbb{D}_1$ and **z**, that Labour was the *core party*.

However, for the coalition structure, $\mathcal{D}_{1988}$, that occurred in 1988, the coalition of the religious parties (with 23 seats) and Likud (with 40 seats) controlled 63 seats altogether. This gave the coalition a majority, even without Meretz or Labour. More generally, in this Parliament, there was no core policy. To see this, consider the Likud preferred point in Figure 2.4. Since Labour, the left, and Shinui together with Shas control 61 seats (a majority of the seats), they could potentially form a government coalition. Moreover, the declared position of Likud does not belong to the convex hull of the positions of this new potential coalition. Thus, the coalition can in principle agree to a policy point that each member prefers to the Likud policy, and on the basis of this new policy force through a vote of no confidence against the Likud-led government. Even if Likud

agreed to a different policy point that Shas would find acceptable, there would always be a position that the new coalition can offer to Shas to overturn the government policy point. Clearly, the Likud position cannot be a core point. To form a government, whether based on the leadership of Labour or Likud, it is necessary to include other parties. The obvious party to include is Shas, which can be regarded as pivotal between coalitions based on Likud or Labour. Bargaining over government formation will then involve, at the least, Likud, Shas, and Labour. We suggest that the policy positions that can occur as a result of bargaining in the absence of a core party lie inside a subset of policies known as the *heart*. The formal definition of this set is provided in Chapter 3, but we can provide an informal definition using Figure 2.4. The median lines in this figure do not intersect, demonstrating that the core is empty. The results of McKelvey and Schofield (1987) show that, with the decisive structure, $\mathcal{D}_{1988}$, voting cycles can occur inside the set bounded by the positions of Likud, Labour, and Shas. Indeed, bargaining between the parties over policy will lead them into this set.

Figure 2.5 shows the estimated positions of the parties at the election of 1996. Precisely, as in 1988, and using Table 2.1 to compute $\mathcal{D}_{1996}$, we can assert that the core for 1996 is empty. A schematic representaion of the Knesset, together with the location of the heart after the election of 2003, is also given in Figure 2.6. We return to these figures in the following chapters.

We denote the family of coalition structures, including $\mathcal{D}_{1988}$, $\mathcal{D}_{1996}$, and $\mathcal{D}_{2003}$, with an empty core, by the symbol $\mathbb{D}_0$. Here 0 is taken to mean that the core is empty. Since the heart depends both on the location of the parties, $\mathbf{z}$, as well as the decisive structure, we use the symbol $\mathcal{H}_0(\mathbf{z})$ for the heart associated with $\mathbb{D}_0$.

The formal bargaining model proposed by Banks and Duggan (2000) gives a lottery or randomization across the convex set generated by the ideal points of all parties. The heart instead is based on the idea that the protagonists believe that, in the situation given by this election, there will be no minority government, but that a limited set of possible coalitions can occur. Although Labour was the strongest party (with 34 seats) under the decisive structure, $\mathcal{D}_{1996}$, it was no longer *dominant*. The key idea underlying the notion of the heart is that in the 1988 and 1996 situations, there are essentially three different possible governments: Likud, Shas, and parties on the "right"; Labour, Shas, and parties on the "left"; and the Labour, Likud coalition. From 1996 to the present, one or other of the first two coalition governments have been the norm, but Sharon and

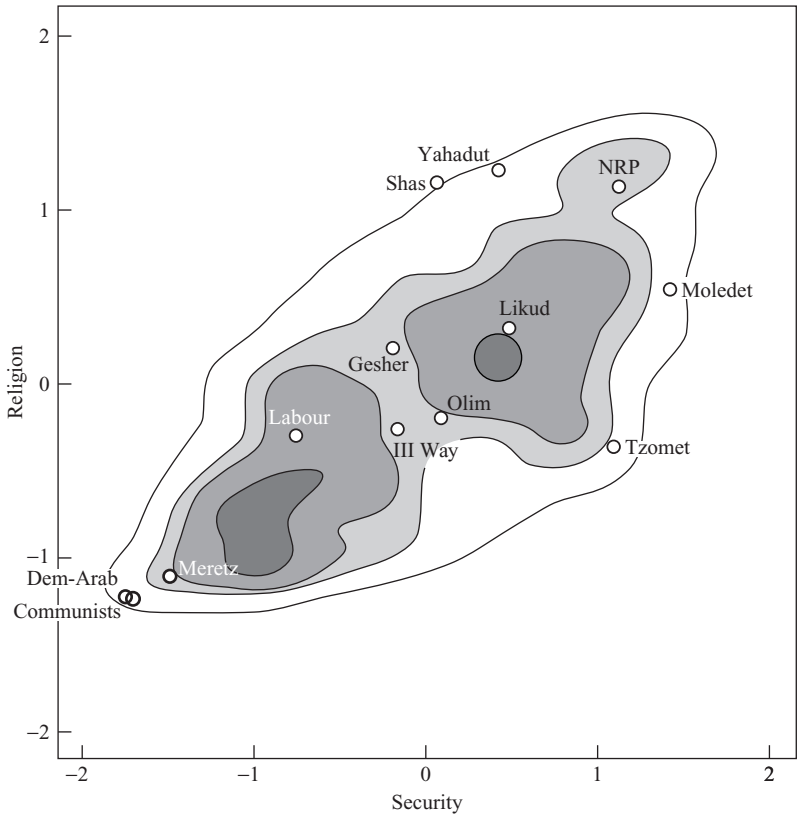


Figure 2.5. Estimated party positions in the Knesset in 1996.

Peres, leaders of Likud and Labour respectively, agreed to form this third coalition in January 2005. We regard the difference between the $\mathbb{D}_0$ structure holding in 1988 and 1996 and the $\mathbb{D}_1$ structure holding in 1992 to be crucial in understanding coalition bargaining. Because Labour benefits substantially when it is a core party, we expect Labour to adopt a position that increases the probability that $\mathbb{D}_1$ occurs. Conversely, Likud should attempt to maximize the probability that $\mathbb{D}_0$ occurs. Since these probabilities will depend on the beliefs of the party principals about the electoral outcome, and these beliefs depend on the vector of party positions, we can write

$$\pi_0(\mathbf{z}) = \Pr[\mathbb{D}_0 \text{ occurs at } \mathbf{z}] \text{ and } \pi_1(\mathbf{z}) = \Pr[\mathbb{D}_1 \text{ occurs at } \mathbf{z}].$$

In principle, these probabilities can be derived from the "belief operator" Π.

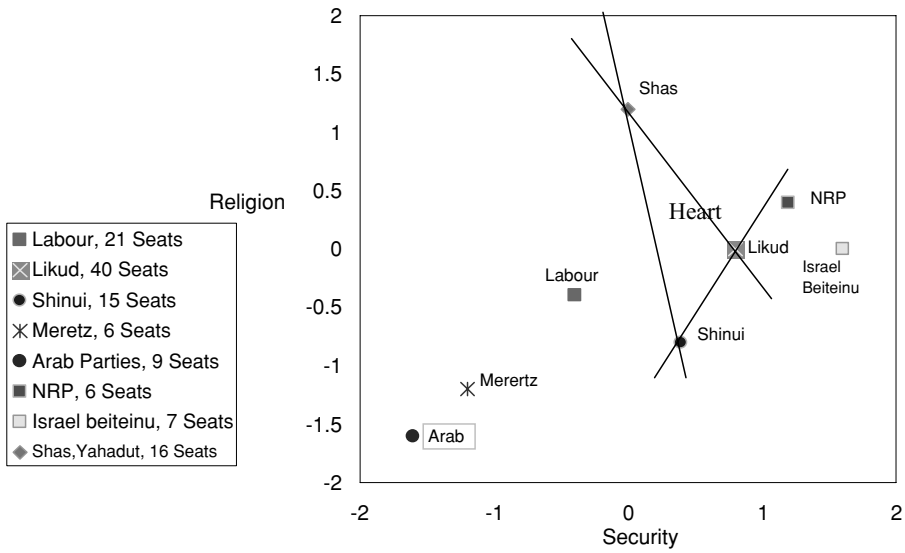


Figure 2.6. A schematic representation of the configuration of parties in the Knesset after 2003.

Thus, we can restate the conclusion of this argument.

Hypothesis 2.4: *Any potential core party, j, should adopt a position in an attempt to maximize the probability, $\pi_j(z)$, associated with the coalition structure $\mathbb{D}_j$, which allows j to be at a core position.*

In the example from Israel, this hypothesis would indicate that since Likud cannot expect to be a core party, then it should attempt to minimize $\pi_1(\mathbf{z})$, or alternatively, to maximize $\pi_0(\mathbf{z})$.

2.9 ELECTORAL MODELS WITH VALENCE

The spatial model with exogenous valence:

The empirical model assumes that the implicit utility of voter i for party j has the form

$$u_{ij}(x_i, z_j) = \lambda_{ij} - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i. \tag{2.1}$$

Here $\theta_j^{\mathrm{T}}\eta_i$ models the effect of the sociodemographic characteristics η_i of voter i in making a political choice. That is, θ_j is a k-vector specifying how the various sociodemographic variables appear to influence the choice for party j. The term $\beta\|x_i - z_j\|^2$ is the Euclidean quadratic loss as-

sociated with the difference between the declared policy of party j and preferred position x_i of voter i. The model is stochastic because of the implicit assumption that $\lambda_{ij} = \lambda_j + \varepsilon_j$, where the disturbances $\{\varepsilon_j : j = 1, \ldots, p\}$ have some multivariate distribution Ψ. The definition of voter probability is

$$\begin{aligned}\rho_{ij}(\mathbf{z}) &= \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j] \\ &= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j],\end{aligned}$$

where

$$u^*_{ij}(x_i, z_j) = \lambda_j - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i \tag{2.2}$$

is the observable component of utility. Particular assumptions on the distribution of the disturbances then allows estimation of the voter probabilities. Because the various parameters are estimated, we use $\rho^*_{ij}(\mathbf{z})$ to denote the stochastic variable, with expectation

$$\mathcal{E}xp(\rho^*_{ij}(\mathbf{z})) = \bar{\rho}_{ij}(\mathbf{z}). \tag{2.3}$$

Taking the mean value gives the empirical expected vote share,

$$\mathcal{E}_j(\mathbf{z}) = \frac{1}{n}\sum_i \bar{\rho}_{ij}(\mathbf{z}). \tag{2.4}$$

The baseline formal model is based on the parallel assumption that

$$u_{ij}(x_i, z_j) = \lambda_j - \beta\|x_i - z_j\|^2 + \varepsilon_j. \tag{2.5}$$

Here again $\{\varepsilon_j : j = 1, \ldots, p\}$ is distributed by Ψ. The probability $\rho_{ij}(\mathbf{z})$ is then defined in analogous fashion and the formal vote share is defined by

$$V_j(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n} \rho_{ij}(\mathbf{z}). \tag{2.6}$$

Notice that we differentiate between the vote share $\mathcal{E}_j(\mathbf{z})$ for the empirical model and $V_j(\mathbf{z})$ for the baseline formal model. In particular, the formal model does not incorporate sociodemographic variables. Since the sociodemographic component of the empirical model is assumed not to be dependent on party position, the PNE and the LNE of the two models should coincide (when the parameters of the model coincide). We say the two models are compatible. The simplest distribution assumption to use is that Ψ is the Type I extreme value distribution. This parallels what

is known as *multinomial condition logit* (MNL) estimation (Dow and Endersby, 2004).

When the valences are given by the vector $\lambda = (\lambda_1, \ldots, \lambda_j, \ldots, \lambda_p)$ and ranked $\lambda_1 \leq \ldots \lambda_j \leq \ldots \leq \lambda_p$, and the extreme value distribution is used, then the convergence coefficient is given by the expression

$$c = 2\beta[1 - 2\rho_1]v^2 = 2Av^2. \tag{2.7}$$

Here ρ_1 is the common probability that a voter will choose the lowest valence party when all parties are at the electoral mean.

The spatial model with activist valence: In this case, the valence is partly a function of party position and is written $\mu_j(z_j)$ so that voter utility is given by the expression

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta\|x_i - z_j\|^2 + \varepsilon_j. \tag{2.8}$$

Electoral models based on exogenous valence and activist valence provide the basis for estimation of the electoral operator Π.

2.10 THE GENERAL MODEL OF MULTIPARTY POLITICS

2.10.1 *Policy Preferences of Party Principals*

In this model, principals are "policy motivated" but also benefit from government perquisites.

Consider a party delegate of party j who has a most-preferred policy point x_j. If the party joins a governing coalition after the election, and receives perquisites of office, denoted δ_j, then we can represent that delegate's utility by the expression

$$U_j((x_j, \alpha_j) : (y, \delta_j)) = U_j^*(y, \delta_j) = -\|y - x_j\|^2 + \alpha_j\delta_j, \tag{2.9}$$

where y is the policy implemented by government, and again

$$-\|y - x_j\|^2$$

is a measure of the quadratic loss associated with the difference between the delegate's preferred point and y. The coefficient α_j gives the relative value of policy over perquisite.

2.10.2 *Coalition and Electoral Risk*

(i) We now consider the set of all possible decisive structures, say, $\{\mathbb{D}_0, \mathbb{D}_1, \mathbb{D}_t, \ldots, \mathbb{D}_p\}$, where $\mathbb{D}_t$, for $t = 1, \ldots, p$, is a possible coalition

structure where party t can be a core party, and $\mathbb{D}_0$ is the family of coalition structures lacking a core. We let $\mathcal{H}_t(\mathbf{z})$ be the heart defined by $\mathbb{D}_t$ and the vector $\mathbf{z}$. We let Π denote the stochastic electoral operator, which defines *inter alia* the probabilities $\{\pi_t(\mathbf{z}) : t = 0, \ldots, p\}$. These probability functions model the *electoral risk* associated with the polity. We implicitly assume that the operator Π is compatible with, and can be deduced from, the above electoral models.

(ii) Given a post-election coalition structure $\mathbb{D}_t$, and the vector of party positions $\mathbf{z}$, the beliefs of the parties regarding policy outcomes in the legislative stage can be expressed as a lottery $\tilde{g}_t(\mathbf{z})$ defined over the set of policy outcomes in the heart $\mathcal{H}_t(\mathbf{z})$. In particular, if the structurally stable core, $\mathbb{SC}_t(\mathbf{z})$, is nonempty at $\mathbf{z}$, then $\mathbb{SC}_t(\mathbf{z}) = \mathcal{H}_t(\mathbf{z})$ and so $\tilde{g}_t(\mathbf{z}) = \mathbb{SC}_t(\mathbf{z})$. These lottery or coalition functions model the *coalition risk* associated with the polity.

(iii) Given Π, then the beliefs of the party principals can be described by the *game form* $\tilde{g}(\mathbf{z}) = \{(\tilde{g}_t(\mathbf{z}), \pi_t(\mathbf{z})); t = 0, \ldots, p\}$.

(iv) Each principal for party j attempts to maximize the expected utility function

$$U_j(\mathbf{z}) = \sum_{t=0}^{p} \pi_t(\mathbf{z}) \mathbf{U_j}(\tilde{g}_t(\mathbf{z})). \tag{2.10}$$

Here $\mathbf{U_j}(\tilde{g}_t(\mathbf{z}))$ is the expected utility derived from the lottery $\tilde{g}_t(\mathbf{z})$ and determined by the policy preferences held by the principal of party j.

Hypothesis 2.5: *The outcome of the political game is a local equilibrium for the game given by the utility profile* $U = (U_1, \ldots, U_p)$.

Comment: It follows from this hypothesis that any party j that has a reasonable expectation of locating at the core position will also be obliged to attempt to maximize π_j, the probability associated with the coalition structure through which it may be the core party. Calculation of π_j may be difficult, but a proxy for maximizing π_j for a party like Labour, in the example above, may be to maximize its expected vote share $\mathcal{E}_j$. In our analyses of Israel and Italy in Chapters 4 and 5, we find that there is a close correspondence between the estimated location of high-valence parties, and the positions computed to be local equilibria of the vote-maximizing game. This suggests that the unknown utility functions in Hypothesis 2.5 for at least some of the parties can be approximated by

vote-share functions. Moreover, discrepancies found between the estimated positions and the equilibrium positions under vote maximization for the low-valence parties may be explained by the more general theory underlying Hypothesis 2.5. Combining the model of vote maximization with that of coalition bargaining is the topic of the next chapter.

3

A Theory of Political Competition

The spatial model of politics initially focused on the analysis of two agents, j and k, competing in a policy space X for electoral votes. The two agents (whether candidates or party leaders) are assumed to pick policy positions z_j, z_k, both in X, which they present as manifestos to a large electorate. Suppose that each member of the electorate votes for the agent that the voter truly prefers. When X involves two or more dimensions, then under conditions developed by Plott (1967), Kramer (1973), McKelvey (1976, 1979), Schofield (1978, 1983, 1985), and many others,* there will generically exist no Condorcet or *core* point unbeaten under majority rule. That is to say, whatever position, z_j, is picked by j there always exists a point z_k that will give agent k a majority over agent j.

However, the existence of a Condorcet point has been established in those situations where the policy space is one-dimensional. In this case, the agents can be expected to converge to the position of the median voter (Downs, 1957). When X has two or more dimensions, it is known that a Condorcet point exists when electoral preferences are represented by a spherically symmetric distribution of voter ideal points. Even when the distribution is not spherically symmetric, a Condorcet point can be guaranteed as long as the decision rule requires a sufficiently large majority (Caplin and Nalebuff, 1988). Although a PNE generically fails to exist in competition between two agents under majority rule, there will exist mixed strategy equilibria whose support lies within a central electoral domain called the *uncovered set*.†

* See also Cohen and Matthews (1980), McKelvey and Schofield (1986, 1987), Banks (1995), Saari (1996), and Austen-Smith and Banks (1999).

† See (Kramer, 1978; Miller, 1980; McKelvey, 1986; Cox, 1987a; Banks et al., 2002; Bianco and Sened, 2003; Bianco et al., 2006).

One problem with the application of these two types of models, in real-world politics, has been the extreme nature of the predictions. The instability results seem to suggest that the outcome of two-party political competition is dependent essentially on random events. The results for mixed strategy equilibria suggest a strong form of convergence in the positions of political agents. Attempts to extend these deterministic models to the situation with more than two parties have also shown instability, or nonexistence of pure strategy vote-maximizing equilibria (Eaton and Lipsey, 1975) or have had to impose additional conditions to deal with discontinuities in the pay-off functions of the agents (Dasgupta and Maskin, 1986).

A way of avoiding the intrinsic failure of continuity in the pay-off functions of agents in these deterministic models is to allow for a stochastic component in voter choice. Hinich (1977) argued that vote-maximizing candidates would adopt a position at the mean of the voter distribution when they faced a stochastic electorate. His argument for two-party competition has been extended by Enelow and Hinich (1982, 1984, 1989), Coughlin (1992), and most recently by McKelvey and Patty (2005) and Banks and Duggan (2005). Lin, Enelow, and Dorussen (1999) have also obtained a *mean voter* theorem for the general case of many candidates.

Applying a stochastic model of voting is the standard technique for estimating voter response in empirical analyses (Alvarez and Nagler, 1998; Alvarez, Nagler, and Bowler, 2000). In an early application it was noted by Poole and Rosenthal (1984) that there was no evidence of convergence to the electoral mean in U.S. presidential elections. As acknowledged in Chapter 1, we earlier completed stochastic models of elections with our collaborators for Britain, Germany, the Netherlands, Israel, Italy, and the United States. Simulation of these models has led to contradictory results. Sometimes the simulation resulted in convergence to the electoral mean (Netherlands and Britain) and sometimes divergence (Israel and Italy). In all cases however, there was no indication that the parties did indeed converge. In later chapters we review these empirical models.

These earlier models entailed the addition of heterogeneous intercept terms for each party. One interpretation of these intercept or constant terms is that they are valences or party biases. As we noted above, valence refers to voters' judgments about positively or negatively evaluated aspects of candidates, or party leaders, which cannot be ascribed to the policy choice of the party or candidate (Stokes, 1992). One may conceive of the valence that a voter ascribes to a candidate as a judgment of the

candidate's quality or competence. This idea of valence has been utilized in a number of recent formal models of voting (Ansolabehere and Snyder, 2000; Groseclose, 2001; Aragones and Palfrey, 2002). To date, a full characterization of the effect of valence on the stochastic model has not been obtained for the case with an arbitrary number of parties.

The next section of this chapter presents such a characterization in terms of the Hessian of the vote-share function of the party leader or candidate who has the lowest valence. The empirical models typically assume that the stochastic component of the model is multinomial logit, derived from the Type I extreme value distribution on the errors. Theorem 3.1 makes this assumption, and shows that there exists a convergence coefficient, which is a function of all the parameters of the model and which classifies the model in the following sense: when the policy space is of dimension w, then the necessary condition for existence of a PNE at the electoral mean, and thus for the validity of the mean voter theorem, is that the coefficient is bounded above by w. Theorem 3.1 also shows that a weaker condition, that the convergence coefficient be bounded above by 1, is sufficient for an LNE at the mean. In the two-dimensional case, the eigenvalues of the Hessian can be readily computed. It is shown that the convergence coefficient is (i) an increasing function of the maximum valence difference, (ii) an increasing function of the number of parties or candidates, and (iii) an increasing function of the electoral variance of the voter-preferred points. In the more complex case, when the stochastic errors are multivariate normal, and therefore covariate, Theorem 3.2 asserts that a different convergence coefficient also classifies the model in precisely the same sense.

When the necessary convergence condition fails, then the origin will be a saddlepoint or minimum of the vote-share function for the lowest valence party. By changing position in the major electoral axis (or eigenspace of the vote function) this party can increase its vote share. It follows that in equilibrium, all parties will adopt positions on this principal axis, with the lowest-valence parties the furthest from the origin. No party will adopt a position at the electoral mean. Chapter 4 presents empirical electoral models for the elections of 1988, 1992, and 1996 in Israel. Chapter 5 follows with an analysis of the 1996 election in Italy. The results indicate that the necessary condition failed. Simulation of the empirical model for Israel found that the vote-maximizing positions of the parties were indeed not at the electoral mean. Our results show that there was a close correspondence between the estimated actual positions of the parties and the equilibrium positions obtained by simulation.

Both these stochastic models (based on the Type I extreme value distribution) assume that the party leaders are motivated simply to maximize vote shares in order to gain office. Moreover, because the model focuses on expected vote share, it ignores the possibility of uncertainty in electoral response. One way to introduce uncertainty, at least in two-party models, is to focus instead on the "probability of victory." Implicitly, such a model acknowledges that the vote-share functions are stochastic variables. To extend such a model to the multiparty case (where there are three or more parties) requires a modification of the notion of the "probability of winning." An obvious extension is to model electoral uncertainty in terms of the probabilities associated with different collections of decisive coalitions. The natural way to construct such a model is to allow party policy decisions to be made by party principals who have policy preferences. In the later part of this chapter, we model such policy-motivated choices using concepts from social choice theory.

3.1 LOCAL EQUILIBRIA IN THE STOCHASTIC MODEL

The purpose of this section is to construct a model of positioning of parties in electoral competition so as to account for the generally observed phenomenon of nonconvergence. The model adopted is an extension of the multiparty stochastic model of Lin, Enelow, and Dorussen (1999), constructed by inducing asymmetries in terms of valence. The basis for this extension is the extensive empirical evidence that valence is a significant component of the judgments made by voters of party leaders. There are a number of possible choices for the appropriate game form for multiparty competition. The simplest one, which is used here, is that the utility function for agent j is proportional to the vote share, V_j, of the agent. With this assumption, we can examine the conditions on the parameters of the stochastic model that are necessary for the existence of a PNE for this particular game form. Because the vote-share functions are differentiable, we use calculus techniques to estimate optimal positions. As usual with this form of analysis, we can obtain sufficient conditions for the existence of local optima, or LNE. Clearly, any PNE will be an LNE, but not conversely. Additional conditions of concavity or quasi-concavity are sufficient to guarantee existence of PNE. However, in the models we consider, it is evident that these sufficient conditions will fail, leading to the inference that PNE are typically nonexistent. Existence of mixed-strategy Nash equilibria is an open question in such games. It is of course true that the true utility functions of party leaders are unknown.

However, comparison of LNE, obtained by simulation of empirical models, with the estimated positions of parties in the various polities that have been studied can provide insight into the true nature of the game form of political competition.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of party declarations, or manifestos, and choose their own positions as best responses to other party declarations in order to maximize their own vote share. The stochastic model essentially assumes that party leaders cannot predict vote response precisely. In the model with exogenous valence, the stochastic element is associated with the weight given by each voter, i, to the average perceived quality or valence of the party leader.

Definition 3.1 ***The Formal Stochastic Vote Model***

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, N, of size n. As usual, we assume that X is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with w finite. Each of the parties, or agents, in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare. Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of agent policy positions. Given $\mathbf{z}$, each voter i is described by a vector $\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$, where

$$u_{ij}(x_i, z_j) = \lambda_j - \beta ||x_i - z_j||^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j.$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The term λ_j is the exogenous valence of agent j, β is a positive constant and $||\cdot||$ is the usual Euclidean norm on X. The terms $\{\epsilon_j\}$ are the stochastic errors, whose cumulative distribution will be denoted by Ψ.

We consider various distribution functions. The most common assumption in empirical analyses is that Ψ is the "Type I extreme value distribution" (sometimes called *log Weibull*). Our principal theorem is based on this assumption. However, we also consider the situation where the errors are independently and identically distributed by the normal distribution (iind), with zero expectation, each with stochastic variance σ^2. A more general assumption is that the stochastic error vector $\epsilon = (\epsilon_1, \ldots, \epsilon_p)$ is multivariate normal with general variance/covariance matrix Ω.

It is natural to suppose that the valence of party j as perceived by voter i is the stochastic variate $\lambda_{ij} = \lambda_j + \epsilon_j$, where λ_j is simply the expectation $\mathcal{E}xp(\lambda_{ij})$ of λ_{ij}. We assume in this chapter that the valence vector

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_p) \text{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \ldots \geq \lambda_2 \geq \lambda_1.$$

Because of the stochastic assumption, voter behavior is modelled by a probability vector. The probability that a voter i chooses party j is

$$\begin{aligned}\rho_{ij}(\mathbf{z}) &= \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j] \\ &= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j].\end{aligned}$$

Here Pr stands for the probability operator generated by the distribution assumption on ϵ. The *expected vote share* of agent j is

$$V_j(\mathbf{z}) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(\mathbf{z}). \tag{3.1}$$

We shall use the notation $V : X^p \to \mathbb{R}^p$ and call V the *party profile function*. In the vote model it is assumed that each agent j chooses z_j to maximize V_j, conditional on $\mathbf{z}_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p)$.

Because of the differentiability of the cumulative distribution function, the individual probability functions $\{\rho_{ij}\}$ are C^2-differentiable in the strategies $\{z_j\}$. Thus, the vote share functions will also be C^2-differentiable. Let $x^* = (1/n)\Sigma_i x_i$. Then the mean voter theorem for the stochastic model asserts that the "joint mean vector" $\mathbf{z}_0 = (x^*, \ldots, x^*)$ is a PNE. Lin, Enelow, and Dorussen (1999) used C^2-differentiability of the expected vote-share functions, in the situation with zero valence, to show that the validity of the theorem depended on the concavity of the vote-share functions. They asserted that a sufficient condition for this was that the variance, σ^2, of each error term was "sufficiently large." Because concavity cannot in general be assured, we utilize a weaker equilibrium concept, that of "local strict Nash equilibrium" (LSNE). A strategy vector $\mathbf{z}^*$ is an LSNE if, for each j, z^*_j is a critical point of the vote function $V_j(z^*_1, \ldots, z^*_{j-1}, z_j, \ldots, z^*_{j+1}, \ldots, z^*_p)$ and the eigenvalues of the Hessian of this function (with respect to z_j) are negative. Definition 3.2 gives the various definitions of the equilibrium concepts used throughout this book.

Definition 3.2 ***Equilibrium Concepts for the Formal Model***

(i) A strategy vector $\mathbf{z}^* = (z^*_1, \ldots, z^*_{j-1}, z^*_j, z^*_{j+1}, \ldots, z^*_p) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile function $V : X^p \to \mathbb{R}^p$ iff, for each agent $j \in P$, there exists a neighborhood X_j of z^*_j in

X such that

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) > V_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$
$$\text{for all } z_j \in X_j - \{z_j^*\}.$$

(ii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *local weak Nash equilibrium* (LNE) iff, for each agent j, there exists a neighborhood X_j of z_j^* in X such that

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \geq V_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$
$$\text{for all } z_j \in X_j.$$

(iii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *strict*, respectively, *weak*, *pure strategy Nash equilibrium* (PSNE, respectively, PNE) iff X_j can be replaced by X in (i), (ii), respectively.

(iv) The strategy z_j^* is termed a *local strict best response*, a *local weak best response*, a *global strict best response*, a *global weak best response*, respectively, to $\mathbf{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_{j+1}^*, \ldots, z_p^*)$, depending on whether z_j^* locally or globally, strictly or weakly, maximizes V_j at $\mathbf{z}_{-j}^*$.

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE, it must be an LNE, while if it is a PSNE, then it must be an LSNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}_0^* = (x^*, \ldots, x^*)$ to be an LNE and thus a PNE, without having to invoke concavity. Theorem 3.1, below, also gives a sufficient condition for the joint mean vector $\mathbf{z}_0^*$ to be an LSNE. A corollary of the theorem shows, in situations where the valences differ, that the necessary condition is likely to fail. In dimension w, the theorem can be used to show that for $\mathbf{z}_0^*$ to be an LSNE, the necessary condition is that a convergence coefficient, defined in terms of the parameters of the model, must be strictly bounded above by w. Similarly, for $\mathbf{z}_0^*$ to be an LNE, then the convergence coefficient must be weakly bounded above by w. When this condition fails, then the joint mean vector $\mathbf{z}_0^*$ cannot be an LNE and therefore cannot be a PNE. Of course, even if the sufficient condition is satisfied, and $\mathbf{z}_0^* = (x^*, \ldots, x^*)$ is an LSNE, it need not be a PNE.

To state the theorem, we first transform coordinates so that in the new coordinates, $x^* = 0$. We refer to $\mathbf{z}_0 = (0, \ldots, 0)$ as the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends

on the distribution of voter ideal points. These are encoded in the voter covariation matrix. We first define this and then use it to characterize the vote-share Hessians.

Definition 3.3 ***The Voter Covariance Matrix,*** $\frac{1}{n}\nabla$

To characterize the variation in voter preferences, we represent in a simple form the covariation matrix (or data matrix), ∇, given by the distribution of voter ideal points. Let X have dimension w and be endowed with a system of coordinate axes $(1, \ldots, r, s, \ldots, w)$. For each coordinate axis let $\xi_r = (x_{1r}, x_{2r}, \ldots, x_{nr})$ be the vector of the rth coordinates of the set of n voter ideal points. We use (ξ_r, ξ_s) to denote scalar product.

The symmetric $w \times w$ voter covariation matrix ∇ is then defined to be

$$\nabla = \begin{pmatrix} (\xi_1, \xi_1) & . & . & (\xi_1, \xi_w) \\ . & (\xi_r, \xi_r) & . & . \\ . & . & (\xi_s, \xi_s) & . \\ (\xi_w, \xi_1) & . & . & (\xi_w, \xi_w) \end{pmatrix}.$$

The covariance matrix is defined to be $\frac{1}{n}\nabla$.

We write $v_s^2 = \frac{1}{n}(\xi_s, \xi_s)$ for the electoral variance on the sth axis and

$$v^2 = \sum_{r=1}^{w} v_r^2 = \frac{1}{n}\sum_{r=1}^{w}(\xi_r, \xi_r) = trace\left(\frac{1}{n}\nabla\right)$$

for the total electoral variance. Here $trace(\frac{1}{n}\nabla)$ is the sum of diagonal terms in $\frac{1}{n}\nabla$. The electoral covariance between the rth and s^{th} axes is $(v_r, v_s) = \frac{1}{n}(\xi_r, \xi_s)$.

Definition 3.4 ***The Extreme Value Distribution,*** Ψ

(i) The cumulative distribution has the closed form

$$\Psi(h) = \exp[-\exp[-h]],$$

with probability density function

$$\Psi(h) = \exp[-h]\exp[-\exp[-h]],$$

and variance $\frac{1}{6}\pi^2$.

(ii) With this distribution it follows from Definition 3.1 for each voter i, and party j, that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{p} \exp u^*_{ik}(x_i, z_k)}. \tag{3.2}$$

Note that (ii) implies that the model satisfies the independence of irrelevant alternative property (IIA): For each individual i, and each pair, j, k, the ratio

$$\frac{\rho_{ij}(\mathbf{z})}{\rho_{ik}(\mathbf{z})}$$

is independent of a third party l (See Train, 2003: 79).

While this distribution assumption facilitates estimation, the IIA property may be violated. In the empirical work on the Netherlands given in Chapter 6 we consider the case of covariant errors, thus allowing for violation of IIA.

The formal model just presented, and based on Ψ is denoted $M(\boldsymbol{\lambda}, \beta; \Psi)$.

Definition 3.5 ***The Convergence Coefficient of the Model $M(\boldsymbol{\lambda}, \beta; \Psi)$***

(i) At the vector $\mathbf{z}_0 = (0, \ldots, 0)$ the probability $\rho_{ij}(\mathbf{z}_0)$ that i votes for party j is

$$\rho_j = \left[1 + \sum_{k \neq j} \exp\left[\lambda_k - \lambda_j\right]\right]^{-1}. \tag{3.3}$$

(ii) The coefficient A_j for party j is

$$A_j = \beta(1 - 2\rho_j).$$

(iii) The Hessian for party j at $\mathbf{z}_0$ is

$$C_j = \left[2[A_j]\left(\frac{1}{n}\nabla\right) - I\right], \tag{3.4}$$

where I is the w by w identity matrix.

(iv) The *convergence coefficient of the model* $M(\boldsymbol{\lambda}, \beta; \Psi)$ is

$$c(\boldsymbol{\lambda}, \beta; \Psi) = 2\beta[1 - 2\rho_1]v^2 = 2A_1 v^2. \tag{3.5}$$

The definition of ρ_j follows directly from the definition of the extreme value distribution. Obviously if all valences are identical, then $\rho_1 = \frac{1}{p}$, as expected. The effect of increasing λ_j, for $j \neq 1$, is clearly to decrease ρ_1, and therefore to increase A_1, and thus $c(\boldsymbol{\lambda}, \beta; \Psi)$.

Theorem 3.1 *The condition for the joint origin to be an LSNE in the model $M(\lambda, \beta; \Psi)$ is that the Hessian*

$$C_1 = \left[2[A_1]\left(\frac{1}{n}\nabla\right) - I\right]$$

of the party 1, with lowest valence, has negative eigenvalues.

Comment on the Theorem. The proof of Theorem 3.1 is given in the Appendix to this chapter. It depends on considering the first- and second-order conditions at $\mathbf{z}_0$ for each vote-share function. The first-order condition is obtained by setting $dV_j/dz_j = 0$ where we use this notation for full differentiation, keeping $\{z_k : k \neq j\}$ constant. This allows us to show that $\mathbf{z}_0$ satisfies the first-order condition. The second-order condition is that the Hessian d^2V_j/dz_j^2 be negative definite at the joint origin. (A presentation of these standard results is given in Schofield, 2003b). If this holds for all j at $\mathbf{z}_0$, then $\mathbf{z}_0$ is an LSNE. However, we need only examine this condition for the vote function V_1 for the lowest-valence party. As we shall show, this condition on the Hessian of V_1 is equivalent to the condition on C_1, and if the condition holds for V_1, then the Hessians for $V_2, \ldots, V_p$ are all negative definite at $\mathbf{z}_0$. As usual, conditions on C_1 for the eigenvalues to be negative depend on the trace, trace(C_1), and determinant, det(C_1), of C_1. These depend on the value of A_1 and on the electoral variance/covariance matrix, $\frac{1}{n}\nabla$. Using the determinant of C_1, we can show that $2A_1v^2 < 1$ is a sufficient condition for the eigenvalues to be negative. In terms of the convergence coefficient $c(\boldsymbol{\lambda}, \beta; \Psi)$ we can write this as $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$. In a policy space of dimension w, the necessary condition on C_1, induced from the condition on the Hessian of V_1, is that $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$. This condition is obtained from examining the trace of C_1. If this necessary condition for V_1 fails, then $\mathbf{z}_0$ can be neither an LNE nor an LSNE.

Ceteris paribus, an LNE at the joint origin is "less likely" the greater are the parameters β, $\lambda_p - \lambda_1$, $\lambda_{p-1} - \lambda_1, \ldots, \lambda_2 - \lambda_1$ and v^2.

Note that for a general spatial model with an arbitrary, non-Euclidean but differentiable metric $d(x_i, z_j) = ||x_i - z_j||$, a similar expression for A_1 can be obtained, but in this case the covariance term $\frac{1}{n}\nabla$ will not have such a ready interpretation. Note also that if the nondifferentiable Cartesian metric $d(x_i, z_j) = \Sigma_{k=1}^{w}|x_{ik} - z_{jk}|$ were used, then the first-order condition would be satisfied at the median rather than at the mean.

Even when the sufficient condition is satisfied, so the joint origin is an LSNE, the concavity condition (equivalent to the negative semi-definiteness of all Hessians *everywhere*) is so strong that there is no good reason to expect it to hold. The empirical analyses of Israel and of Italy, presented in Chapters 4 and 5, show that the necessary condition fails. In these polities, a PNE, even if it exists, will generally not occur at the origin.

The theorem immediately gives the following corollaries.

Corollary 3.1 *Assume X is two-dimensional. Then, in the model $M = M(\boldsymbol{\lambda}, \beta; \Psi)$, the sufficient condition for the joint origin to be an LSNE is that $c(\lambda, \beta; \Psi)$ be strictly less than 1. The necessary condition for the joint origin to be an LNE is that $c(\lambda, \beta; \Psi)$ be no greater than 2.*

Proof. The condition that both eigenvalues of C_1 be negative is equivalent to the condition that $\det(C_1)$ is positive and $\text{trace}(C_1)$ is negative. Now

$$\det(C_1) = (2A_1)^2\left[(v_1, v_1)\cdot(v_2, v_2) - (v_1, v_2)^2\right] + 1 - (2A_1)\left[(v_1, v_1) + (v_2, v_2)\right].$$

By the triangle inequality, the term $\left[(v_1, v_1)\cdot(v_2, v_2) - (v_1, v_2)^2\right]$ is non-negative. Thus, $\det(C_1)$ is positive if

$$2\beta[1 - 2\rho_1]v^2 < 1.$$

This gives the sufficient condition that $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$ for an LSNE at the joint origin, $\mathbf{z}_0^*$.

The necessary condition for $\mathbf{z}_0^*$ to be an LNE is that the eigenvalues be nonpositive. Since $\text{trace}(C_1)$ equals the sum of the eigenvalues, we can use the fact that $\text{trace}(C_1) = (2A_1)[(v_1, v_1) + (v_2, v_2)] - 2$, to obtain the necessary condition

$$2\beta[1 - 2\rho_1]v^2 - 2 \leq 0 \text{ or } c(\boldsymbol{\lambda}, \beta; \Psi) \leq 2.$$

Thus, $c(\boldsymbol{\lambda}, \beta; \Psi) \leq 2$ gives the necessary condition. □

Corollary 3.2 *In the two-dimensional case, the two eigenvalues of* C_1, *for the model* $M(\lambda, \beta; \Psi)$, *are*

$$a_1 = A_1\{\left[v_1^2 + v_2^2\right] + \left[[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2\right]^{\frac{1}{2}}\} - 1$$

$$a_2 = A_1\{\left[v_1^2 + v_2^2\right] - \left[[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2\right]^{\frac{1}{2}}\} - 1.$$

Proof. This follows immediately from the fact that $a_1 + a_2 = \text{trace}(C_1) = c(\boldsymbol{\lambda}, \beta; \Psi) - 2$. □

Corollary 3.3 *In the case that* X *is* w*-dimensional, then the sufficient condition for the joint origin to be an LSNE for the model* $M(\boldsymbol{\lambda}, \beta; \Psi)$ *is that* $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$, *while the necessary condition for the joint origin to be an LNE is that* $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$.

Proof. This follows immediately by the same proof technique as Corollary 3.1. □

We now consider the model $M(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi)$ where the errors are independently and identically, normally distributed (iind), given by a covariance matrix $\sigma^2 I$, and with probability density function (pdf)

$$\varphi(h) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left[\frac{h}{\sigma}\right]^2\right].$$

Definition 3.6 ***The Convergence Coefficient of the Model*** $\boldsymbol{M(\lambda, \beta; \sigma^2 I, \varphi)}$.

(i) For each agent j, define

$$\lambda_{av(j)} = \frac{1}{p-1} \sum_{k \in P - \{j\}} \lambda_k.$$

(ii) Define the coefficient A_j for the contest of agent j against the competing agents to be

$$A_j(\varphi) = \left[\frac{(p-1)\beta}{p\sigma^2}\right]\left[\lambda_{av(j)} - \lambda_j\right].$$

(iii) The Hessian matrix C_j associated with agent j is defined to be

$$C_j(\varphi) = \left[2A_j\left(\frac{1}{n}\nabla\right) - I\right].$$

(iv) The convergence coefficient of the model $M(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi)$ is given by

$$c(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi) = 2A_1(\varphi)v^2. \tag{3.6}$$

We now state the result on the model $M(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi)$.

Theorem 3.2 *The necessary and sufficient condition for the joint origin to be an LSNE for the model $M = M(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi)$ is that the eigenvalues of the Hessian matrix $C_1(\varphi)$ all be negative.*

The proof of this theorem is given in Schofield (2004) and follows in similar fashion to the proof of Theorem 3.1. As a corollary, necessary and sufficient conditions for convergence can be obtained for the model based on the normal distribution.

Note that the case $\lambda_p = \lambda_1$ was studied by Lin, Enelow, and Dorussen (1999). In this case, the convergence coefficient $c(\boldsymbol{\lambda}, \beta; \sigma^2 I, \varphi)$ is zero so the joint origin, $\mathbf{z}_0$, is an LSNE. Theorem 3.2 makes clear why Lin et al. argued that if σ^2 were sufficiently large, then a PNE would occur at the joint origin.

Train (2003: 39) comments that the "difference between extreme value and independent normal errors is indistinguishable empirically." For this reason, in examining whether convergence can be expected in the empirical logit model, we use the result for the formal model, $M(\lambda, \beta; \Psi)$. Corollary 3.2 and the obvious extension can be used to determine the eigenvalues of the appropriate Hessians in dimension n.

Recent work by Banks and Duggan (2005) has examined two-party competition for the probabilistic vote model. Instead of vote maximization, they assume each party j attempts to maximize the *plurality function* $U_j(z_j, z_k) = V_j(z_j, z_k) - V_k(z_j, z_k)$. To demonstrate that the joint mean (x^*, x^*) is a PNE of the plurality maximization game they use the concavity of the plurality vote functions. It is obvious, however, that if the eigenvalues of the Hessians just considered are not all nonpositive, then concavity will fail. Obviously analogues of Theorems 3.1 and 3.2 can be developed to obtain necessary conditions for existence of LNE and PNE in the plurality two-party game, depending on the distribution assumptions on the errors.

3.2 LOCAL EQUILIBRIA UNDER ELECTORAL UNCERTAINTY

Using the expected vote-share functions as the maxim and for the electoral game has its attraction. As we have seen, the expected vote-share functions can be readily computed because they are linear functions of the entries in the voter probability matrix $(\rho_{ij}(\mathbf{z}))$. At least for two-party competition, more natural payoff functions to use are the parties' *probability of victory.* To develop this idea, we can introduce the idea of the stochastic vote-share functions $\{V_j^*(\mathbf{z}) : j = 1, \ldots, p\}$. Then the expected vote-share functions used above are simply the expectations $\{\mathcal{E}xp(V_j^*(\mathbf{z}))\}$ of these stochastic variables. In the two-party case, the probability of victory for agents 1 and 2 can be written

$$\pi_1(\mathbf{z}) = \Pr[V_1^*(\mathbf{z}) > V_2^*(\mathbf{z})] \text{ and } \pi_2(\mathbf{z}) = \Pr[V_2^*(\mathbf{z}) > V_1^*(\mathbf{z})].$$

As Patty (2006) has commented, an agent's probability of victory is a complicated nonlinear expression of the voters' behavior as described by the vote matrix $(\rho_{ij}(\mathbf{z}))$. Just as we can define LNE and PNE for the game given by the profile function $V : X^p \to \mathbb{R}^p$, we can also define LNE and PNE for the two-party profile function $\pi = (\pi_1, \pi_2) : X^2 \to \mathbb{R}^2$. Duggan (2000, 2006), Duggan and Fey (2006), and Patty (2001, 2005) have explored those conditions under which equilibria for expected vote-share functions and probability of victory are identical. As might be expected these equilibria are generically different (Patty, 2006).

We now develop a model based on electoral uncertainty, which we consider to be a generalization of the Duggan/Patty models of two-party competition. To do this we introduce the idea of a party principal.

The strategy, z_j, of party j corresponds to the position of the party leader and is chosen by the party principal, j, whose preferred position is x_j. We first develop the model with only two parties. If party j wins the election with a leader at position $z_j \in X$, while party j receives a nonpolicy perquisite δ_j, then the payoff to the principal, j, is

$$U_j((x_j, \alpha_j) : (z_j, \delta_j)) = U_j^*(z_j, \delta_j) = - \| z_j - x_j \|^2 + \alpha_j \delta_j \, .$$

Thus, the profile function $U = (U_1, U_2) : X^2 \to \mathbb{R}^2$ can be taken to be given by the expected payoffs

$$U_1(z_1, z_2) = \pi_1(z_1, z_2) U_1^*(z_1, \delta_1) + \pi_2(z_1, z_2) U_1^*(z_2, 0)$$
$$U_2(z_1, z_2) = \pi_2(z_1, z_2) U_2^*(z_2, \delta_2) + \pi_1(z_1, z_2) U_2^*(z_1, 0).$$

This expression ignores the probability of a draw. In the case of a draw, the outcome can be assumed to be lottery between the party positions

z_1 and z_2. The multiparty model we propose is a natural extension of the two-party model and is built as follows. As before, we can examine conditions sufficient for existence of LNE or PNE for such a two-party profile function (see Cox, 1984, 1997 for an example). To extend this to a model of *multiparty* competition with $p \geq 3$, we must deal with the fact that it is possible that no party gains a majority of the parliamentary seats (or in the case of U.S. presidential elections, a majority of the electoral college). We argue that in multiparty competition the possible outcomes of the election correspond to the family of all decisive coalition structures

$$\mathbb{D} = \{\mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_t, \ldots, \mathcal{D}_T\},$$

which can be obtained from the set P of parties. For convenience, we may assume that the subfamily $\{\mathcal{D}_1, \ldots, \mathcal{D}_p\}$, with $p < T$, corresponds to the subfamily of coalition structures where the parties $\{1, \ldots, p\}$, respectively, win the election with a majority of the seats in the Parliament. Notice that the outcomes $\{\mathcal{D}_1, \ldots, \mathcal{D}_T\}$ are defined in terms of the distribution of seat shares $(S_1, S_2, \ldots, S_p)$ in the Parliament, and not simply vote shares. The more interesting cases are given by $t > p$, and for convenience we can assume that for such a t, the coalition structure $\mathcal{D}_t = \{M \subset N : \Sigma_{j \in M} S_j > 1/2\}$. Decisive coalition structures can of course be defined in more complex ways. Since there is an intrinsic uncertainty in the way votes are translated into seats, it makes sense to focus on the probabilities associated with these decisive structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability that $\mathcal{D}_t$ occurs is denoted $\pi_t(\mathbf{z})$. We also assume that the vector

$$\pi(\mathbf{z}) = (\pi_1(\mathbf{z}), \ldots, \pi_p(\mathbf{z}))$$

corresponds to the probabilities that parties $1, \ldots, p$, respectively, win the election. This vector is generated by the belief operator Π introduced earlier. When party j wins then the outcome, of course, is the situation $(z_j, 1)$. That is, party j implements the position z_j of its party leader and takes a share 1 of nonpolicy perquisites. When no party wins, but a decisive coalition $\mathcal{D}_t$ occurs, for $t \geq p + 1$, then the outcome is a lottery that we denote by $\tilde{g}_t^{\alpha}(\mathbf{z})$. We assume

$$\tilde{g}_t^{\alpha}(\mathbf{z}) \in \tilde{W} = Bor(X \times \Delta_P).$$

Here Δ_P is the set of possible distributions of government perquisites among the parties, and $W = (X \times \Delta_P)$ while $Bor(X \times \Delta_P)$ is the space of Borel probability measures over $X \times \Delta_P$ endowed with the weak topology (Parthasathy, 1967). Thus, $\tilde{g}_t^{\alpha}(\mathbf{z})$ specifies a finite lottery of points in X

coupled with a lottery of distributions of perquisites among the parties belonging to the decisive structure $\mathcal{D}_t$ (see Banks and Duggan, 2000, for a method of deriving this lottery). We implicitly assume that the utility function of the principal of party j, given by the expression U_j^* above, defines the function

$$U_j : (X \times \Delta_P) \to \mathbb{R},$$

where

$$U_j(z, (\delta_1, \ldots, \delta_p)) = U_j^*(z, \delta_j) = -\|z - x_j\|^2 + \alpha_j \delta_j.$$

Further, we assume each U_j be extended to a function

$$\mathbf{U_j} : (Bor(X \times \Delta_P)) \to \mathbb{R},$$

measurable with respect to the sigma-algebra on $Bor(X \times \Delta_P)$. Note that if $g \in \tilde{W}$, then it is a measure on the Borel sigma-algebra of W. Since $\mathbf{U_j} : W \to \mathbb{R}$ is assumed measurable, the integral $\int \mathbf{U_j} dg$ is well defined and can be identified with $\mathbf{U_j}(g) \in \mathbb{R}$. Note, also, that in the weak topology, a sequence $\{g_k\}$ of measures converges to g if and only if $\int \mathbf{U} dg_k$ converges to $\int \mathbf{U} dg$ for *every* bounded, continuous utility function $\mathbf{U}$ with domain W. We further assume that $\tilde{g}_t^\alpha : X^P \to \tilde{W}$ is C^2-differentiable as well as continuous. This means that for all j the induced function $U_j^t : X^P \to \mathbb{R}$, given by $U_j^t(\mathbf{z}) = \mathbf{U_j}\tilde{g}_t^\alpha(\mathbf{z})$, is also C^2-differentiable, so its Hessian with respect to z_j is everywhere defined and continuous. Observe that $\tilde{g}_t^\alpha$ is used to model the common beliefs of the principals concerning the outcome of political bargaining in the post-election situation given by $\mathcal{D}_t$. The common beliefs of the principals concerning electoral outcomes are given by a C^2-differentiable function $\pi : X^p \to \Delta_T$ from X^p to the simplex Δ_T (of dimension T-1) where T is the cardinality of the set of all possible coalition structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability is $\pi_t(\mathbf{z})$ that the distribution of parliamentary seats among the parties gives the decisive structure $\mathcal{D}_t$. The *electoral probability function* π, induced from Π, models the uncertainty associated with the election. Note that this uncertainty also includes the uncertainty over the valences of the various party leaders. We now provide the formal definitions for the multiparty political game.

Definition 3.7 ***The Game Form with Policy Preferences***

(i) The *electoral probability function* $\pi = (\pi_1, \ldots, \pi_T) : X^p \to \Delta_T$ is a smooth function from X^p to the simplex Δ_T (of dimension T-1),

where $\mathbb{D} = \{\mathcal{D}_1, \ldots, \mathcal{D}_T\}$ is the set of all possible decisive coalition structures. This function captures the notion of *electoral risk*.

(ii) For fixed $\mathcal{D}_t$, the outcome of bargaining at the parameter $\alpha = (\alpha_1, \ldots, \alpha_p)$ and at the strategy vector $\mathbf{z}$ is a lottery

$$\tilde{g}_t^{\alpha}(\mathbf{z}) \in (Bor(X \times \Delta_P)).$$

This captures the notion of *coalition risk at* $\mathcal{D}_t$.

(iii) At the fixed decisive structure $\mathcal{D}_t$ and strategy vector $\mathbf{z}$, the payoff to the principal of party j is

$$U_j^t(\mathbf{z}) = \mathbf{U_j}\big(\tilde{g}_t^{\alpha}(\mathbf{z})\big).$$

(iv) The *game form* $\{\tilde{g}_t^{\alpha}, \pi_t\}$ at the parameter α is denoted $\tilde{g}^{\alpha}$. At the strategy vector $\mathbf{z}$, the payoff to the principal j is given by the von Neumann-Morgenstern utility function

$$U_j^g(\mathbf{z}) = \mathbf{U_j}(\tilde{g}^{\alpha}(\mathbf{z})) = \sum_{t=1,\ldots,T} \pi_t(\mathbf{z}) U_j^t(\mathbf{z}). \tag{3.7}$$

(v) The *game profile* derived from the game form $\tilde{g}^{\alpha}$ at the utility profile $\{U_j\}$ is denoted

$$U^g = (\mathbf{U_1} \circ \tilde{g}^{\alpha}, \ldots, \mathbf{U_p} \circ \tilde{g}^{\alpha}) = \big(..U_j^g..\big) : X^p \to \mathbb{R}^p.$$

(vi) The game form $\tilde{g}^{\alpha}$ is *smooth* iff the function $U^g : X^p \to \mathbb{R}^p$ is C^2-differentiable. Let $\mathbb{U}(X^p, \mathbb{R}^p)$ be the set of C^2-differentiable utility profiles $\{U : X^p \to \mathbb{R}^p\}$ endowed with the C^2 topology. (Roughly speaking, two profiles are close in this topology if all values and first and second derivatives of each U_j are close).

(vii) A *generic property in* $\mathbb{U}(X^p, \mathbb{R}^p)$ is one that is true for a set of profiles that is open dense in the C^2 topology. (See Hirsch, 1976 and Schofield, 2003a for the definition of the C^2-topology and the notion of generic property.)

(viii) For the fixed smooth game form $\tilde{g}^{\alpha}$, let $\{U : X^p \to \mathbb{R}^p\} \subset \mathbb{U}(X^p, \mathbb{R}^p)$ be the set of utility profiles induced as the parameters of voter ideal points and electoral beliefs are allowed to vary.

(ix) Let $\mathbb{G}$ be the set of smooth game forms. The transformation $\tilde{g} \to U^g : \mathbb{G} \to \mathbb{U}(X^p, \mathbb{R}^p)$ induces a topology on the set $\mathbb{G}$, where this topology is obtained by taking the coarsest topology such that this transformation is continuous.

(x) The vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, \ z_j^*, \ z_{j+1}^*, \ldots, z_p^*) \in X^p$ is an LSNE for the profile $U \in \mathbb{U}(X^p, \ \mathbb{R}^p)$ iff for each j there is a

neighborhood X_j of z_j^* in X, with the property that

$$U_j(z_1^*, \ldots, z_j^*, z_{j+1}^*, \ldots, z_p^*) > U_j(z_1^*, \ldots, z_j, z_{i+1}^*, \ldots, z_p^*)$$

for all $z_j \in X_j - \{z_j^*\}$.

(xi) $\mathbf{z}^* \in X^p$ is a *critical Nash equilibrium* (CNE) for the profile U iff, for each j, the first order condition $\frac{dU_j}{dz_j} = 0$ is satisfied at $\mathbf{z}^*$.

(xii) A PSNE for U is an LSNE for U with the additional requirement that each X_j is in fact X.

(xiii) For a fixed profile $x \in X^n$ of voter ideal points, fixed electoral beliefs π, and fixed game form g, the vector $\mathbf{z}^*$ is called the LSNE, PSNE, or CNE if it satisfies the appropriate condition for the game profile $U^g : X^p \to \mathbb{R}^p$.

(xiv) An LSNE $\mathbf{z}^* \in X^p$ for the profile U is *locally isolated* iff there is a neighborhood Z^* of $\mathbf{z}^*$ in X^p that contains no LSNE for U other than $\mathbf{z}^*$.

Schofield (2001) and Schofield and Sened (2002) have shown that, for each parameter, α, there is an open dense set of smooth game forms, with the property that each form $\tilde{g}^\alpha$ in the set exhibits a locally isolated LSNE. In principle, this result suggests that if the electoral function is smooth, and if the outcome of coalition bargaining is differentiable in the location of parties, then there will exist local equilibria that can be used to deduce party positions. Of course, this model is very much more complex than the vote maximizing version presented in the previous section.

For the Theorem to be valid, we require that the strategy space X^p is a compact, convex subset of a finite dimensional topological vector space. We shall call such a space a *Fan space* (Fan, 1964). We also require the following boundary condition on the profile. Say a profile $U \in \mathbb{U}(X^p, \mathbb{R}^p)$ satisfies the *boundary condition* if for every point $\mathbf{z}$ on the boundary of the Fan space, X^p, the induced gradient $(\frac{dU_1}{dz_1}, \ldots, \frac{dU_p}{dz_p})$ points toward the interior of X^p. Let $\mathbb{U}_b(X^p, \mathbb{R}^p)$ be the subspace of profiles satisfying the boundary condition.

Theorem 3.3 *Assume X is a Fan space and p is finite. Then the property that the LSNE exists and is locally isolated is generic in the topological space $\mathbb{U}_b(X^p, \mathbb{R}^p)$.*

Sketch of Proof. For each j, consider the set $T_j = \{\mathbf{z} \in X^p : \frac{dU_j}{dz_j} = 0\}$. By the inverse function theorem, T_j is generically a smooth manifold of dimension $(p-1)\dim(X)$. By transversality theory, the intersection $\cap_{j \in P} T_j$

is of codimension $p\dim(X)$ in X^p. But X^p has dimension $p\dim(X) = pw$. Since the set of CNE $\equiv \cap_{j\in P} T_j$, this shows that there is an open dense set $\mathbb{U}_b^*(X^p, \mathbb{R}^p)$ such that for each $U \in \mathbb{U}_b^*(X^p, \mathbb{R}^p)$, the set of CNE of U is of dimension 0; that is, it consists of locally isolated points. Now for each such U, construct a gradient field $\mu(U)$ on X^p whose zeros consist precisely of the CNE of U (see Schofield, 2001, for this construction). Since X is assumed to be compact, convex, it is homeomorphic to the ball. Because of the boundary assumption on profiles, the field $\mu(U)$ points inward on the boundary of X^p. The Morse inequalities (Milnor, 1963; Dierker, 1976) imply that there must be at least one critical point $\mathbf{z}^*$of $\mu(U)$ whose index is maximal. Thus, the Hessian of each U_j at $\mathbf{z}^*$ must be negative definite, and $\mathbf{z}^*$ corresponds to a locally isolated LSNE of the profile U. □

This theorem suggests that if we consider any fixed game form $\tilde{g}$, then existence of locally isolated LSNE is a generic property in the space $U : X^p \to \mathbb{R}^p\} \subset \mathbb{U}(X^p, \mathbb{R}^p)$. Moreover, if the transformation $\mathbb{G} \to \mathbb{U}(X^p, \mathbb{R}^p)$ is well behaved, in the sense that open sets are transformed to open sets, then continuity of the transformation would imply that existence of LSNE is a generic property in the space $\mathbb{G}$.

3.3 THE CORE AND THE HEART

In the previous section we assumed that the outcome of bargaining between the party leaders could be described by a lottery $\tilde{g}_t^\alpha(\mathbf{z})$, determined by the vector $\mathbf{z}$ of positions of party leaders. The analysis of Banks and Duggan indicated that in general this outcome would coincide with the *core* of the coalition game determined by the post-election decisive structure $\mathcal{D}_t$ and the vector $\mathbf{z}$. To develop this idea further we now give the formal definitions of the core and other solution concepts based on social choice theory.

Definition 3.8 ***Concepts of Social Choice Theory***

(i) A (strict) *preference* Q on a set, or space, W is a correspondence $Q : W \to 2(W)$, where $2(W)$ stands for the family of all subsets of W (including the empty set ϕ). Again, we assume W is a *Fan space*.

(ii) Let $Q : W \to 2(W)$ be a preference correspondence on the space W. The *choice* of Q is

$$\mathbb{C}(Q) = \{x \in W : Q(x) = \phi\}.$$

(iii) The covering correspondence Q^* of Q is defined by $y \in Q^*(x)$ iff $y \in Q(x)$ and $Q(y) \subset Q(x)$. Say y *covers* x. The *uncovered set*, $\mathbb{C}^*(Q)$ of Q, is

$$\mathbb{C}^*(Q) = \mathbb{C}(Q^*) = \{x \in W : Q^*(x) = \phi\}.$$

(iv) If W is a topological space, then $x \in W$ is *locally covered* (under Q) iff for any neighborhood Y of x in W, there exists $y \in Y$ such that

$$y \in Q(x) \text{ and } Y \cap Q(y) \subset Y \cap Q(x).$$

If x is not locally covered, then write $Q^{**}(x) = \phi$.

(v) The *heart* of Q, written $\mathcal{H}(Q)$, is defined by

$$\mathcal{H}(Q) = \{x \in W : Q^{**}(x) = \phi\}.$$

A preference Q is *convex* iff for all x, the preferred set $Q(x)$ of x is strictly convex. In general, if $\mathbb{C}(Q)$ is nonempty, then it is contained in both $\mathbb{C}^*(Q)$ and $\mathcal{H}(Q)$. It can be shown that if $\mathbb{C}(Q) \neq \phi$ and $Q' \to Q$ in an appropriate topological sense, then it is possible to find a sequence $\{z^s \in \mathcal{H}(Q')\}$ such that $\{z^s\}$ converges to some point in the core, $\mathbb{C}(Q)$.

Now let $CON(W)^P$ stand for all "smooth" convex preference profiles for the set of political agents $P = \{1, \ldots, p\}$. Thus $q \in CON(W)^P$ means $q = (q_1, \ldots, q_p)$, where each $q_j : W \to 2(W)$ is a convex preference whose indifference surfaces are smooth. In particular, this means we can represent the preference profile q by a C^2-utility profile $U \in \mathbb{U}(W, \mathbb{R}^p)$. Let rep: $CON(W)^P \to \mathbb{U}(X, \mathbb{R}^p)$ be the representation map.

Definition 3.9 ***The Heart and the Uncovered Set***

(i) Let $\mathcal{D}$ be a fixed set of decisive coalitions and W be a Fan space. Let $q \in CON(W)^P$ be a smooth preference profile. Define

$$\sigma_{\mathcal{D}}(q) = \cup_{M \in \mathcal{D}}\{\cap_{i \in M} q_i\} : W \to 2(W)$$

to be the preference correspondence induced by $\mathcal{D}$ at the profile q. The *core* of the political game given by $\mathcal{D}$ at q, written $\mathbb{C}_{\mathcal{D}}(q)$, is $\mathbb{C}(\sigma_{\mathcal{D}}(q))$.

(ii) The *heart* of $\mathcal{D}$ at q, written $\mathcal{H}_{\mathcal{D}}(q)$, is defined to be $\mathcal{H}(\sigma_{\mathcal{D}}(q))$. The *uncovered set* of $\mathcal{D}$ at q, written $\mathbb{C}^*_{\mathcal{D}}(q)$, is $\mathbb{C}^*(\sigma_{\mathcal{D}}(q))$.

(iii) The *Pareto set* of the profile q is $\mathbb{C}_P(q) = \mathbb{C}(\sigma_P(q))$, where

$$\sigma_P(q) : \{\cap_{i \in P} q_i\} : W \to 2(W)$$

is the Pareto, or strict unanimity, preference correspondence.

(iv) A correspondence $Q: W \to Z$ is *lower hemi continuous* (lhc) with respect to topologies on W, Z iff for any open set $Y \subset Z$ the set

$$\{x \in W : Q(x) \cap Y \neq \phi\}$$

is open in W.

(v) A *continuous selection* g for Q is a function $g : W \to Z$, continuous with respect to the topologies on W, Z such that $g(x) \in Q(x), \forall x \in W$, whenever $Q(x) \neq \phi$.

(vi) A correspondence $\mathcal{H} : CON(W)^P \to 2(W)$ is called C^2-*lower hemi continuous* (C^2-lhc) if the map

$$\mathcal{H} \circ rep^{-1} : \mathbb{U}(X, \mathbb{R}^p) \to CON(W)^P \to 2(W)$$

is also lhc with respect to the C^2-topology on $\mathbb{U}(X, \mathbb{R}^p)$.

Schofield (1996, 1998, 1999a,b) has shown that the heart is nonempty, Paretian, and C^2-lhc. The heart correspondence can then be shown to admit a continuous selection (Michael, 1956).

Theorem 3.4 summarizes the technical properties of the heart correspondence.

Theorem 3.4 *Let W be a Fan space, and $\mathcal{D}$ any voting rule. Then $H_{\mathcal{D}} : CON(W)^P \to 2(W)$ is C^2-lhc. Moreover, for any $q \in CON(W)^P$, the set $H_{\mathcal{D}}(q)$ is closed, nonempty, and is a subset of the Pareto set $C_P(q)$. Moreover, $H_{\mathcal{D}}$ admits a continuous selection $g_{\mathcal{D}} : CON(W)^P \to W$ such that $g_{\mathcal{D}}(q) \in \mathbb{C}(\sigma_{\mathcal{D}}(q))$ whenever $C(\sigma_{\mathcal{D}}(q))$ is nonempty. Indeed, $g_{\mathcal{D}}$ can be factored to give a C^2-differentiable map*

$$g_{\mathcal{D}} \circ rep^{-1} : \mathbb{U}(X, \mathbb{R}^p) \to CON(W)^P \to W.$$

This last property means that if U is a C^2-differentiable profile then the induced profile $U \circ g_{\mathcal{D}}$ is also C^2-differentiable.

For convenience, we say $g_{\mathcal{D}}$ is a smooth *Paretian selection which converges to the core*.

To use Theorem 3.4 to model coalition bargaining, we assume as before that the preferred position of the *leader* (or *agent*) for party j determines the declaration z_j of the party. We assume that the outcome of bargaining is an element of $W = (X \times \Delta_P)$, namely a policy choice x and a distribution $(\delta_1, \ldots, \delta_p)$ of the total perquisites. Thus, the leader of party j receives

utility

$$U_j((z_j, \alpha_j) : (x, (\delta_1, \ldots, \delta_p))) = U_j^*(x, \delta_j) = - \| z_j - x \|^2 + \alpha_j \delta_j.$$

This implies that the leader can be described by a smooth, strictly convex preference correspondence $q_j^{\alpha_j}(z_i) : X \times \Delta_P \to X \times \Delta_P$. Let $\alpha = (\alpha_1, \ldots, \alpha_p)$, $\mathbf{z} = (z_1, \ldots, z_p)$, and $q^\alpha(\mathbf{z})$ denote the profile of leader preferences. The Pareto set $\mathbb{C}_P(q^\alpha(\mathbf{z}))$ in $X \times \Delta_P$ is the unanimity choice of this preference profile. As in the previous section, we now consider a family $\mathbb{D} = \{\mathcal{D}_1, \ldots, \mathcal{D}_t, \ldots, \mathcal{D}_T\}$ of decisive coalitions. We call each set $\mathcal{D}_t$ the *voting rule induced by the election*. For each $\mathcal{D}_t$, we can define the heart of the voting rule on the space $W = X \times \Delta_P$ as $\mathcal{H}_{\mathcal{D}_t}(q^\alpha(\mathbf{z}))$. This set we write as $\mathcal{H}_t^\alpha(\mathbf{z})$. We write the core $\mathbb{C}(\sigma_{\mathcal{D}_t}(q^\alpha(\mathbf{z})))$ as $\mathbb{C}_t^\alpha(\mathbf{z})$. Theorem 3.4 can then be applied to show that each correspondence $\mathcal{H}_t^\alpha$ is C^2-lhc and admits a C^2-selection which converges to the core $\mathbb{C}_t^\alpha(\mathbf{z})$. The family of correspondences $\{\mathcal{H}_t^\alpha\}$ we write as $\mathcal{H}_{\mathbb{D}}^\alpha$.

To extend these concepts to the situation where the electoral outcome is a lottery, we again use the definition of $\tilde{W} = Bor(X \times \Delta_P)$, the set of all lotteries over $X \times \Delta_P$, endowed with the weak topology. Now let $\tilde{\mathcal{H}}_t^\alpha : X^p \to 2(\tilde{W})$ be the extension of the heart correspondence to this space, so $\tilde{\mathcal{H}}_t^\alpha(\mathbf{z})$ is the set of lotteries over the set $\mathcal{H}_t^\alpha(\mathbf{z})$ with the induced topology. Then lhc of $\mathcal{H}_t^\alpha$ implies lhc of $\tilde{\mathcal{H}}_t^\alpha$ (Schofield, 1999a,b).

Theorem 3.5 *For a fixed voting rule, $\mathcal{D}_t$, there exists a smooth selection $\tilde{g}_t^\alpha : X^p \to \tilde{W}$ of the correspondence $\tilde{\mathcal{H}}_t^\alpha : X^p \to 2(\tilde{W})$, which converges to the core.*

As in the previous section, $\tilde{g}_t^\alpha$ is meant to capture the notion of *coalition risk* at the vector $\mathbf{z}$ of party positions and at the decisive structure $\mathcal{D}_t$. Convergence to the core is intended to capture the following logic. If the core $\mathbb{C}_t^\alpha(\mathbf{z})$ is nonempty, then the selection $\tilde{g}_t^\alpha(\mathbf{z})$ must put all probability weight on this set, guaranteeing that this is the outcome. In such a situation there is no coalition risk.

We can now repeat the analysis of the previous section for the case of a game form $\tilde{g}^\alpha = \{\tilde{g}_t^\alpha, \pi_t\}$ obtained as a selection from the heart correspondence. First, let K be some compact convex subset of $\mathbb{R}^p$ for the parameters α, and let $\tilde{g}$ be a general game form that specifies the game form $\tilde{g}^\alpha = \tilde{g}_t^\alpha, \pi_t\}$ for each $\alpha \in K$.

Definition 3.10 *The* game form $\tilde{g}$, *which specifies* $\{\tilde{g}_t^\alpha, \pi_t\}$ *at* $\alpha \in K$ *is* heart compatible over K *iff each component* $\tilde{g}_t^\alpha : X^p \to \tilde{W}$ *is a smooth selection of the heart correspondence* $\tilde{\mathcal{H}}_t^\alpha : X^p \to 2(\tilde{W})$.

Theorem 3.6 *There exists a game form $\tilde{g}$ which is heart compatible and has the following property: If the induced utility profiles are given by $\{U^g : X^p \to R^p\}$ then there is an open dense set in*

$$\{U^g : X^p \to \mathbb{R}^p\} \cap \mathbb{U}_b(X^p, \mathbb{R}^p)$$

such that each profile in this set exhibits a locally isolated LSNE.

In applying this Theorem, it will prove useful to consider the notion of a *structurally stable core* for the particular case when nonpolicy perquisites are zero.

Definition 3.11 *Consider the case $\alpha = (\alpha_1, \ldots, \alpha_p) = (0, \ldots, 0)$. If the core $\mathbb{C}_t^0(z)$ at $\mathbf{z}$ and $\mathcal{D}_t$ is nonempty then it is said to be* structurally stable *if, for any $x \in \mathbb{C}_t^0(z)$, there exists a neighborhood Z^* of $\mathbf{z}$ in X^p and a neighborhood X^* of x in X such that $X^* \cap \mathbb{C}_t^0(z^*) \neq \phi$ for all $\mathbf{z}^* \in Z^*$.*

When the core at z and $\mathcal{D}_t$ is structurally stable then it is denoted $\mathbb{SC}_t^0(z)$.

In other words, the *policy core* $\mathbb{C}_t^0(\mathbf{z})$ is structurally stable if a small arbitrary perturbation of the profile $\mathbf{z}$ simply perturbs the location of the core. The symmetry conditions developed by McKelvey and Schofield (1986, 1987) allow us to determine when a policy core is structurally stable. In general, these symmetry conditions are easiest to use when the policy core coincides with the position of a party.

Definition 3.12 *A party j is said to be a* core party *at the profile $\mathbf{z} = (z_1, \ldots, z_p)$ and with the decisive structure $\mathcal{D}_t$ iff it is the case that $\mathbb{C}_t^0(\mathbf{z}) = z_j$ and there exists a neighborhood Z^* of $\mathbf{z}$ in X^p such that $\mathbb{C}_t^0(\mathbf{z}^*) = z_j^*$ for all $\mathbf{z}^* \in Z^*$.*

Notice that if j is a core party, then the core at z_j must also be structurally stable. Laver and Schofield (1990) argue that if j is a (nonmajority) core party at $\mathbf{z}$ and $\mathcal{D}_t$, then the party should be able to implement the policy position z_j by constructing a minority coalition government including party j, but not necessarily comprising a majority coalition. This follows because no majority coalition $M \in \mathcal{D}_t$ can propose some counter policy $z \in X$ that all parties in the coalition M prefer to z_j.

We earlier defined the decisive structures $\{\mathcal{D}_1, \ldots, \mathcal{D}_p\}$ to be those where party $1, \ldots, p$ respectively obtains a majority of the seats. Obviously a party with a majority can implement its position, so it must also be a core party. But this is also true for a nonmajority core party in the case

that $\mathbb{SC}_t^0(\mathbf{z}) = z_j$. This allows us to partition $\mathbb{D}$ into equivalence classes. First, we use the term *feasible profile* to refer to a profile $\mathbf{z}$ that belongs to a subset X_0^p of profiles that are considered by the party principals. The following definitions depend on this restriction to such a subset of the joint strategy space.

Definition 3.13 *For each $j \in P$, let $\mathbb{D}_j$ denote the subfamily of $\mathbb{D}$ with the property that for each $\mathcal{D}_t \in \mathbb{D}_j$, two conditions hold: (i) there exists a feasible profile $\mathbf{z} = (z_1, \ldots, z_p)$ such that j is a core party at $\mathbf{z}$ and $\mathcal{D}_t$, and (ii) there is no feasible profile $\mathbf{z}'$ such that party $k \neq j$ is a core party at $\mathbf{z}'$ and $\mathcal{D}_t$.*

Note that party j will have a majority in the structure $\mathcal{D}_j$ so necessarily it will be the unique core party for any profile. As a result, $\mathcal{D}_j \in \mathbb{D}_j$. As Schofield (1995) has shown, for j to be a core party it is necessary that the vector of seat shares satisfies certain restrictions. The $\frac{3}{4}$ case where each of the four parties has exactly $\frac{1}{4}$ of the seat share is "exceptional" because then each of the parties is a core party in two dimensions. The restrictions that characterize $\mathbb{D}_j$ require that the jth seat share necessarily satisfies the condition $S_j > S_k$, for $k \neq j$. In the elections we examine below in Britain and in the United States, it is typical that one party k, say, gains a majority seat share so $S_k > \frac{1}{2}$. However, in the multiparty systems in Israel, Italy, and the Netherlands, based on variants of proportional electoral laws, no party gains a majority seat share. We argue that the crucial characteristic of the election is whether there exists a core party. For empirical applications we somewhat modify the definition of $\mathbb{D}_0$, made previously in Section 2.8.

Definition 3.14

(i) Let $\mathbb{D}_0$ denote the subfamily of $\mathbb{D} - \cup_{j=1}^p \{\mathbb{D}_j\}$ such that for each $\mathcal{D}_t \in \mathbb{D}_0$ and any feasible profile $\mathbf{z} = (z_1, \ldots, z_p)$ the policy core $\mathbb{C}_t^0(\mathbf{z})$ is either empty or not structurally stable.

(ii) Let Δ_{p+1} be the simplex of dimension p. Then the *modified electoral probability function* $\pi^* = (\pi_0^*, \ldots, \pi_p^*) : X^p \to \Delta_{\mathrm{pH}}$ is defined by

$$\pi_0^*(\mathbf{z}) = \Pr[\mathbb{D}_0 \text{ occurs at } \mathbf{z}].$$

$$\text{For } j = 1, \ldots, p, \pi_j^*(\mathbf{z}) = \Pr[\mathbb{D}_j \text{ occurs at } \mathbf{z}].$$

$$\pi_{p+1}^*(\mathbf{z}) = \Pr[\text{not } \mathbb{D}_0 \text{ or not } \cup_{j=1}^p \{\mathbb{D}_j\} \text{ occurs at } \mathbf{z}.$$

The $(p+2)$ different states distinguished in this definition provide a qualitative characterization of the electoral outcomes.

3.4 EXAMPLE: THE NETHERLANDS

To illustrate the idea of the heart and coalition risk, consider the following example for the Netherlands.

Chapter 6 examines the elections of 1977 and 1981 in the Netherlands. There are four main parties: Labour (PvdA), Christain Democratic Appeal (CDA), Liberals (VVD), and Democrats (D'66), with approximately 40 percent, 35 percent, 20 percent, and 5 percent of the popular vote. Given uncertainty about the elections, there are two relevant coalition structures:

$$\mathbb{D}_0 = \{\text{PvdA,CDA}\},\{\text{PvdA,VVD}\},\{\text{CDA,VVD}\}$$

$$\mathbb{D}_{PvdA} = \{\text{PvdA,CDA}\},\{\text{PvdA,VVD, D'66}\},\{\text{CDA,VVD,D'66}\}.$$

The second structure is denoted $\mathbb{D}_{PvdA}$ because it is evident that a structurally stable policy core can occur at a profile

$$\mathbf{z} = (z_{PvdA}, z_{CDA}, z_{VVD}, z_{D'66})$$

whenever z_{PvdA} lies in the interior of the convex hull of the three positions $z_{CDA}, z_{VVD}, z_{D'66}$. To see this, note that although {CDA,VVD, D'66} is a decisive coalition, its members cannot agree over a policy position that they all prefer to z_{PvdA}. It is also the case that this situation is insensitive to small perturbations of party positions, and so the core at z_{PvdA} is structurally stable. Thus, with this configuration PvdA is a core party.

On the other hand, with the decisive structure $\mathbb{D}_0$ there is no vector of party positions that gives a structurally stable core outcome. This situation is typical of the multiparty situations that we examine in Israel, Italy, and the Netherlands. Table 3.1 gives the election results for 1977 and 1981 in the Netherlands. It is immediately obvious that the coalition {CDA,VVD} had 77 seats in 1977, and thus comprised a majority. Consequently, the coalition structure $\mathbb{D}_0$ was in place. However, in 1981, this coalition won only 74 seats, so the coalition structure was $\mathbb{D}_{PvdA}$ or $\mathbb{D}_1$. Figure 3.1 shows the electoral distribution together with the estimated party positions, based on survey data for 1979. These estimates are discussed in Chapter 6. We wish to emphasize here that optimal party positioning for the 1981 election depends on party estimates of the functions $\pi_0^*(\mathbf{z})$ and $\pi_1^*(\mathbf{z})$.

Table 3.1. *Election Results in the Netherlands, 1977–1981.*

Party (acronym)	1977	1981
	(Seats)	
Labour (PvdA)	53	44
Democrats '66 (D'66)	8	17
Liberals (VVD)	28	26
Christian Dem Appeal (CDA)	49	48
	(138)	(135)
Communists (CPN)	2	3
Dem '70 (D'70)	1	0
Radicals (PPR)	3	3
Pacific Socialists (PSP)	–	3
Reform Federation (RPF)	–	2
Reform Pol Ass (GDV)	1	1
Farmers Party (BP)	1	0
State Reform Party (SGP)	3	3
	(11)	(15)
Total	149	150

To apply the model presented above, consider the question of optimal position for the CDA prior to the 1981 election. To simplify the analysis, let us concentrate on the situation where the CDA expects the coalition structure $\mathbb{D}_0$. Thus, we may suppose that $\pi_1^*(\mathbf{z}) = 0$ for all feasible vectors $\mathbf{z}$. In a situation where perquisites are zero (so $\alpha = 0$), consider $\{g_0^0, \pi_0\}$ with $\pi_0 = 1$. Since D'66 plays no role under this coalition structure, we may ignore it, and suppose that the sincere positions of the principals of the three parties {PvdA,CDA,VVD} are given, as in Figure 3.2, by

$$\mathbf{z}_{prin} = (z_{PvdA}, z_{VVD}, z_{CDA}) = (-\surd 3, 0), (\surd 3, 0), (0, 1).$$

The heart $\mathcal{H}_0^0(\mathbf{z})$ associated with any vector $\mathbf{z}$ of party positions and the coalition structure $\mathbb{D}_0$ can easily be seen to be the convex hull of the party positions. For purposes of illustration, for any profile $\mathbf{z}$, let $\tilde{g}_0^0(\mathbf{z})$ be the lottery that specifies the uniform distribution across $\mathcal{H}_0^0(\mathbf{z})$. Obviously $\tilde{g}_0^0$ is a smooth selection of the heart correspondence. To illustrate the best response of the CDA, suppose the positions of PvdA and VVD are given by (z_{PvdA}, z_{VVD}) as in the figure and let us compare the utilities for the CDA at the positions $z_{CDA}{}^* = (0, 3)$ and $z_{CDA} = (0, 1)$. From the symmetry of the figure it follows that the von Neumann-Morgenstern utility function

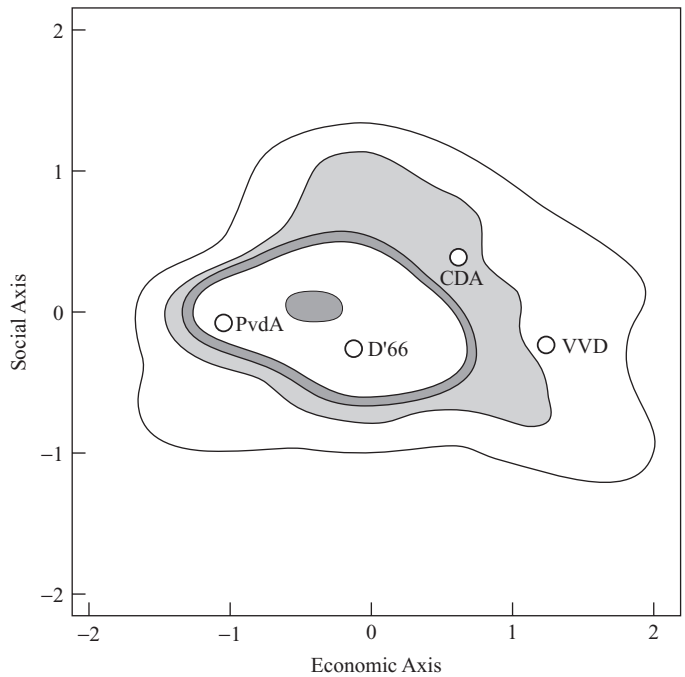


Figure 3.1. Estimated party positions in the Netherlands, based on 1979 data.

U_{CDA} satisfies the equation

$$\begin{aligned} U_{CDA}\big(\tilde{g}_0^0(z_{PvdA}, z_{CDA}{}^*, z_{VVD})\big) &= \frac{1}{3}U_{CDA}\big(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{VVD})\big) \\ &\quad + \frac{1}{3}U_{CDA}\big(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{CDA}{}^*)\big) \\ &\quad + \frac{1}{3}U_{CDA}\big(\tilde{g}_0^0(z_{CDA}{}^*, z_{CDA}, z_{VVD})\big) \\ &= U_{CDA}\big(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{VVD})\big). \end{aligned}$$

By continuity, there is a position denoted y_{CDA} on the arc [(0,1),(0,3)] which gives the best response of the CDA to (z_{PvdA}, z_{VVD}). The analysis of the example is developed further in Schofield and Parks (2000), where they show that there exist LSNE for this fixed coalition structure such that some parties adopt "radical" positions.

This example suggests that party principals may choose more radical positions for their leaders in order to influence coalition bargaining in their favor. We may call this phenomenon *the centrifugal effect of coalition risk*.

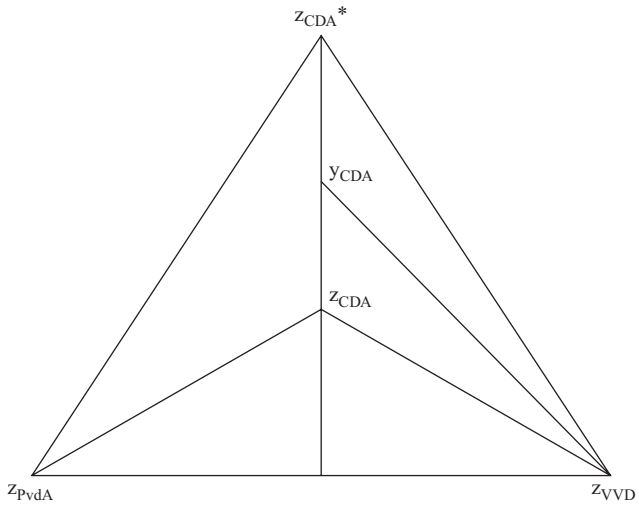


Figure 3.2. Coalition risk in the Netherlands at the 1981 election.

3.5 EXAMPLE: ISRAEL

To further illustrate the theory, consider again the Israeli case briefly discussed in Chapter 2. Figure 3.3 reproduces Figure 2.3 to show the estimated positions of the parties at the time of the 1992 election. Table 2.1 in Chapter 2 shows that, after this election in 1992, the coalition $M_1 = \{$Labour, Meretz, ADL, HS$\}$ controlled 61 seats while the coalition M_2 of the religious parties and the right, including Likud, controlled only 59 seats out of 120. Thus, the 1992 decisive structure may be written $\mathcal{D}_{1992}$ and has the form

$$\{M_1, M_2 \cup \textit{Labour}, M_2 \cup \textit{Meretz}\}$$

Since the Labour position z_{labour} in Figure 3.3 obviously lies inside the convex hull of the positions of parties in any winning coalition, we observe that $z_{labour} = \mathbb{SC}^0_{1992}(\mathbf{z})$ is the structurally stable core. Now it is possible to find a profile z with z_{likud} lying inside the convex hull of the positions of the parties in M_1. Such a profile we regard as empirically infeasible. It therefore follows that Labour would be the uniquely feasible core party under $\mathcal{D}_{1992}$. Thus, $\mathcal{D}_{1992} \in \mathbb{D}_{labour}$. Moreover, Labour is *dominant* under $\mathcal{D}_{1992}$ with the party positions similar to those given in Figure 3.3. As above we refer to this family of coalition structures as $\mathbb{D}_1$.

Again, using Table 2.1, we note that after the 1988 election, the coalition M_2 controlled 65 seats and so belonged to $\mathcal{D}_{1988}$. Clearly, there is a

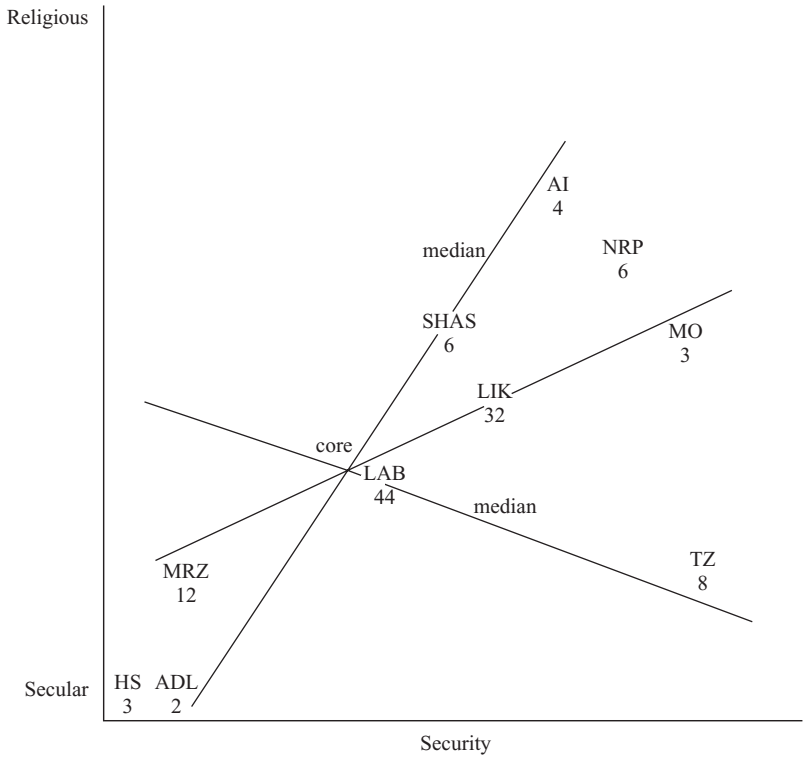


Figure 3.3. Estimated party positions and core in the Knesset after the 1992 election.

profile $\mathbf{z}$ with z_{labour} lying inside the convex hull of the positions of the parties in M_2, but again this can be regarded as infeasible. We can therefore assert that there is no feasible $\mathbf{z}$ such that $\mathbb{SC}^0_{1988}(\mathbf{z})$ is nonempty, which leads us to infer that $\mathcal{D}_{1988} \in \mathbb{D}_0$. Again, Figure 3.4 shows the heart $\mathcal{H}^0_0(\mathbf{z})$ given by the decisive structure $\mathbb{D}_0$ and profile $\mathbf{z}$ as given in the figure.

Prior to the 1996 election, there are therefore two qualitatively distinct possible outcomes, namely $\{\mathbb{D}_0, \mathbb{D}_1\}$. To examine optimal party positions prior to the election of 1996, first consider the outcomes under the assumption that $\mathbb{D}_1$ occurs. Without perquisites, the outcome will be $\mathbb{SC}^0_1(\mathbf{z}) = z_{labour}$. Since we assume party principals have policy preferences, the principal of Likud should choose a position to minimize $\pi^*_1(\mathbf{z}) = \Pr[\mathbb{D}_1]$. One obvious way to do this is to choose z_{likud} as a best response in order to maximize its expected vote share. In contrast, Labour should attempt to maximize $\pi^*_1(\mathbf{z}) = \Pr[\mathbb{D}_1]$. The principal of Shas cannot affect policy outcomes under this eventuality.

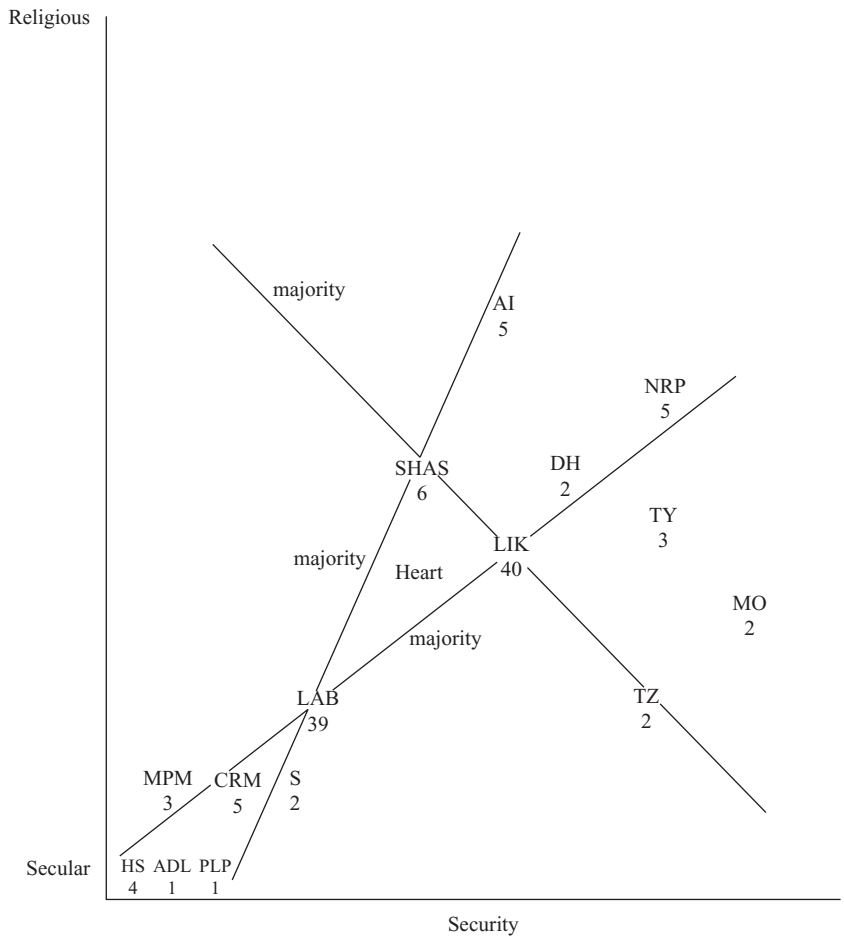


Figure 3.4. Estimated party positions and heart in the Knesset after the 1988 election.

Now consider the situation under $\mathbb{D}_0$. As indicated in Figure 3.4, the heart will be a subset of the convex hull of the positions in the coalition $M_3 = \{$Likud, Labour, Shas$\}$. As in the previous example, this suggests that Shas should adopt a "radical" position in order to influence coalition outcomes.

To summarize: Labour should adopt a position as a best response in order to maximize $\pi_1^*(\mathbf{z})$ while Likud should minimize $\pi_1^*(\mathbf{z})$. As a first approximation, these strategies can be interpreted as maximizing the vote-share functions V_{labour}, V_{likud}, respectively. For Shas, and other small religious parties, optimal strategies will depend on their estimates of π_0^* and π_1^*. Since these probabilities will be little affected by the Shas position,

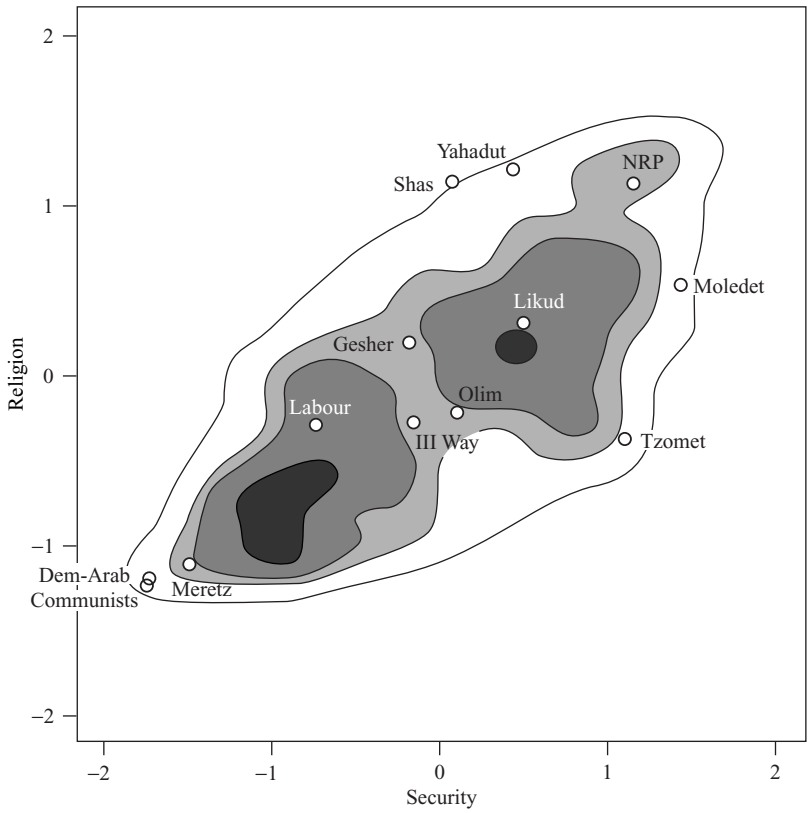


Figure 3.5. Estimated party positions in the Knesset after the 1996 election.

we can assert that the larger is the estimate of $\pi_0^*(\mathbf{z})$, then the further will the optimal Shas position be from the axis drawn between the Labour and Likud. Figure 3.5 shows the estimated positions of the parties at the election of 1996.

As we show in the next chapter, the position adopted by Shas in this figure is compatible with this interpretation of the motivations of the party principals.

3.6 APPENDIX: PROOF OF THEOREM 3.1

Proof of the Theorem (See Schofield, 2006b).

At $\mathbf{z}_{-1} = (0, \ldots, 0)$, let $\rho_{i1}(z_1)$ be the probability that i votes for 1. Then

$$\rho_{i1}(z_1) = \Pr[[\lambda_1 - \beta||x_i - z_1||^2 - \lambda_j + \beta||x_i - z_j||^2 > \epsilon_j - \epsilon_1],\ \ j \neq 1].$$

Using Definition 3.4 for the extreme value distribution Ψ we obtain

$$\rho_{i1}(\mathbf{z}) = \frac{\exp[\lambda_1 - \beta||x_i - z_1||^2]}{\sum_{j=1}^{p} \exp[\lambda_j - \beta||x_i - z_j||^2]}.$$

Thus,

$$\rho_{i1}(z_1) = [1 + \Sigma_{j=2}[\exp(f_j)]]^{-1},$$
$$\text{where } f_j = \lambda_j - \lambda_1 + \beta||x_i - z_1||^2 - \beta||x_i||^2,$$
$$\text{and } \frac{d\rho_{i1}}{dz_1} = 2(\beta(z_1 - x_i)\left[\rho_{i1}^2 - \rho_{i1}\right].$$

At $z_1 = 0$, $\rho_{i1} = \rho_1$ is independent of i, so we obtain

$$\frac{d\rho_{i1}}{dz_1} = 2(\beta(z_1 - x_i)\left[\rho_1^2 - \rho_1\right]$$
$$\text{and } \frac{dV_1}{dz_1} = \frac{1}{n}\sum_i \frac{d\rho_{i1}}{dz_1} = 0 \text{ at } z_1 = \frac{1}{n}\sum_i x_i.$$

This gives the first-order condition $z_1 = 0$. Obviously, the condition $\frac{dV_i}{dz_j} = 0$ is satisfied at $z_1 = \frac{1}{n}\sum_i x_i = 0$. Thus, $\mathbf{z}_0 = (0, \ldots, 0)$ satisfies the first-order condition.

At $z_{-1} = (0, \ldots, 0)$, the Hessian of ρ_{i1} is

$$\frac{d^2\rho_{i1}}{dz_1^2} = \left\{\rho_{i1} - \rho_{i1}^2\right\}\{[1 - 2\rho_{i1}][\nabla_{i1}(z_1)] - 2\beta I\}.$$

Here $[\nabla_{i1}(z_1)] = 4\beta^2[(x_i - z_1)(x_i - z_1)^{\mathrm{T}}$ is the w by w matrix of cross-product terms. Now $\sum_i[\nabla_{i1}(0)] = 4\beta^2\nabla$, where ∇ is the electoral covariation matrix given in Definition 3.3. Then, the Hessian of V_1 at $z_1 = 0$ is given by

$$\frac{1}{n}\sum_i \frac{d^2\rho_i}{dz_1^2} = \left\{\rho_1 - \rho_1^2\right\}\left\{[1 - 2\rho_1][4\beta^2]\left[\frac{1}{n}\nabla\right] - 2\beta I\right\}.$$

Because the first term $\{\rho_1 - \rho_1^2\}$ is positive, the eigenvalues of this matrix will be determined by the eigenvalues of

$$C_1 = \left[2[A_1]\left(\frac{1}{n}\nabla\right) - I\right],$$
$$\text{where } A_1 = \beta[1 - 2\rho_1],$$

as required. Moreover,

$$\lambda_p \geq \lambda_{p-1} \geq \ldots \geq \lambda_2 \geq \lambda_1$$

$$\text{implies that } \rho_p \geq \rho_{p-1} \geq \ldots \geq \rho_2 \geq \rho_1$$

$$\text{so that } A_1 \geq A_2 \geq \ldots \geq A_p.$$

$$\text{This implies that } \mathit{trace}(C_1) \geq \mathit{trace}(C_2) \geq \ldots \geq \mathit{trace}(C_p)$$

$$\text{and } \det(C_1) \geq \det(C_2) \geq \ldots \geq \det(C_p)\,.$$

Thus, if C_1 has negative eigenvalues, then so do $C_2, \ldots, C_p$, and this implies that $z_1 = z_2 = \ldots = z_p = 0$ will all be mutual local strict best responses. This shows that the stated condition is sufficient for $\mathbf{z}_0 = (0, \ldots, 0)$ to be an LSNE.

Conversely, if C_1 does not have negative eigenvalues, then $\mathbf{z}_0$ cannot be an LSNE. This gives the necessary condition. □

4

Elections in Israel, 1988–1996

As discussed in Chapter 3, formal models of voting usually make the assumption that political agents, whether parties or candidates, attempt to maximize expected vote shares. Stochastic models typically derive the mean voter theorem—that each agent will adopt a convergent policy strategy at the mean of the electoral distribution. This conclusion, however, is contradicted by some of the empirical evidence.

In this chapter we emphasize the competitive dynamics of the electoral process in order to examine the inconsistency between theory and evidence. In particular, we argue that to fully elucidate vote motivations of the parties, it is necessary to incorporate valence terms in the statistical model and, therefore, in the theoretical model as well.

The *valence* of each party derives from the average weight given, by members of the electorate, to the overall competence of the particular party leader. In empirical models, a party's valence is independent of current policy declarations, and can be shown to be statistically significant in the estimation. As Theorem 3.1 has shown, when valence terms are incorporated in the formal model, then the convergent vote-maximizing equilibrium can fail to exist. We contend that the empirical evidence is consistent with a formal stochastic model of voting in which valence terms are included. *Low-valence parties,* in equilibrium, will tend to adopt positions at the electoral periphery. *High-valence parties* will contest the electoral center, but will not, in fact, occupy the electoral mean. We use evidence from the Israeli case to support and illustrate our theoretical argument.

Empirical and theoretical models of representative democracy typically have two distinct components. At the microlevel, individual voting behavior is modelled as a function of the preferences, or beliefs, of the voters and the policy positions or declarations of political candidates (or

agents). It is commonly assumed that agents adopt strategies to maximize a utility function defined in terms of the overall vote share of the agent. Other possibilities include maximizing seat share, or some combination of policy consequences with seat or vote share, or probability of winning a majority (Duggan, 2000; Patty, 2005, 2006).

The natural formal concept to use in examining political agent strategies is that of *Nash equilibrium*—the vector of agent strategies with the property that no agent may deviate from the Nash equilibrium strategy and gain anything by doing so. Almost all formal models of agent strategy suggest that political agents, in equilibrium, will adopt convergent strategies; that is, they will adopt strategies that are located in some central domain of the space, as defined by voter preferences or beliefs (Calvert, 1985; Banks, Duggan, and Le Breton, 2002).

Arguments and evidence that parties do not adopt centrist strategies have been commonplace for decades (Duverger, 1954; Robertson, 1976; Daalder, 1984; Budge, et al., 1987). Theoretical models have been devised to account for policy divergence. These include theories based on activist support, (Aldrich, 1983a,b; 1995; Aldrich and McGinnis, 1989), directional voting (Adams and Merrill III, 1999; Merrill III and Grofman, 1999; Merrill III, Grofman, and Feld, 1999; Adams, 2001; Warwick, 2004; Adams, Merrill, and Grofman, 2005 and valence (Stokes, 1963, 1992).

Incorporating valence, or the perception in the electorate of a candidate's competence, is a plausible way to modify the usual vote models. Recent models incorporating valence have concentrated on adopting the basic Downsian model (Downs, 1957), where the voters "know with certainty" the location of the candidates (Ansolabehere and Snyder, 2000). Empirical models of voting make the implicit assumption that there is a degree of uncertainty (or more properly, risk) in the individual voter choice (Poole and Rosenthal, 1984). Therefore, it is appropriate to use, as a benchmark for such empirical studies, a formal model of voting that also incorporates risk.

The stochastic or probabilistic formal vote model has been developed to extend the early work of Hinich (1977). Initially focusing on two-candidate competition (Cahoon, Hinich, and Ordeshook, 1978; Enelow and Hinich, 1984; Coughlin, 1992), it has recently been extended to the case of multiparty competition with three or more candidates (Adams, 1999a,b; Lin, Enelow, and Dorussen, 1999). This work has indicated that parties will adopt convergent strategies at the mean of the electoral distribution. This conclusion is subject to a constraint that the stochastic

component is "sufficiently" important. To date, the relevance of this result to empirical analysis of voting behavior has not been evaluated because the constraint has not been formulated in a precise enough fashion to be applied to empirical work. This chapter provides a re-evaluation of voting behavior in multiparty elections, in the light of Theorem 3.1 in the previous chapter.

For the discussion and analysis of the case of Israel we combine available and original survey data for Israel for 1988 to 1996 that allows us to construct an empirical model of voter choice in Knesset elections. We use expert evaluations to estimate party positions and then construct an empirical vote model that we show is statistically significant. Using the parameter estimates of this model, we developed a "hill-climbing" algorithm to determine the empirical equilibria of the vote-maximizing political game. Contrary to the conclusions of the formal stochastic vote model, the "mean voter" equilibrium (where all parties adopt the same position at the electoral mean) did not appear as one of the simulated equilibria. Since the voter model that we developed predicts voter choice in a statistically significant fashion, we infer that the assumptions of the formal stochastic vote model are compatible with actual voter choice. Moreover, equilibria determined by the simulation were "close" to the estimated configuration of party positions for the three elections of 1988, 1992, and 1996. We infer from this that the assumption of vote-share maximization on the part of parties is a realistic assumption to make about party motivation.

The usual assumption to make to ensure existence of a Nash equilibrium at the mean voter position depends on showing that all party vote-share functions are *concave* in some domain of the party strategy spaces (Banks and Duggan, 2005). Concavity of these functions depends on the parameters of the model. Because the appropriate empirical model for Israel incorporated valence parameters, these were part of the concavity condition for the baseline formal model. Concavity is a global property of the vote-share functions, and is generally difficult to empirically test. As in the formal analysis in the previous chapter, we focus on a weaker property known as *local concavity,* given by appropriate conditions on the second derivative (the Hessian) of the vote-share functions. If local concavity fails, then so must concavity. The constraints required for local concavity in the formal vote model are shown to be violated by the estimated values of the parameters in the empirical model. Consequently, our empirical model of vote-maximizing parties could not lead us to expect convergent strategies at the mean electoral position.

The formal result presented in Chapter 3 is valid in a policy space of unrestricted dimension, but has a particularly simple expression in the two-dimensional case.

Theorem 3.1 allows us to determine whether a low-valence party would in fact maximize its vote shares at the electoral mean. More precisely, we can determine whether the mean voter position is a best response for a low-valence party when all other parties are at the mean. In the empirical model, we estimate that low-valence parties would, in fact, minimize their vote share if they chose the mean electoral position. This inference leads us to the following conclusions: (i) some of the low-valence parties, in maximizing vote shares, should adopt positions at the periphery of the electoral distribution; (ii) if this does occur, then the first-order conditions for equilibrium, associated with high-valence parties at the mean, will be violated. Consequently, for the sequence of elections in Israel, we should expect that it is a nongeneric property for any party to occupy the electoral mean in any vote-maximizing equilibrium (Schofield and Sened, 2005b).

There may be constraints on policy choice because of activist party members, and ideological commitment by party elite. However, vote and seat shares are measures of party success, and are an obvious basis for party motivation. A formal model that does not give this due regard is unlikely to be particularly relevant. As we further elaborate in the next chapter, we infer from our results that vote maximization is the key factor in party policy choice. Clearly, optimal party location depends on the valence by which the electorate, on average, judges party competence.

Our simulations suggest that if a single party has a significantly high valence, for whatever reason, then it has the opportunity to locate itself near the electoral center. On the other hand, if two parties have high but comparable valences, then our simulation suggests that neither will closely contest the center. We observe that the estimated positions of the two high-valence parties, Labour and Likud, are almost precisely identical to the simulated positions under expected vote maximization. The positions of the low-valence parties are, as predicted, close to the periphery of the electoral distribution. However, they are not identical to simulated vote-maximizing positions. This suggests that the perturbation away from vote-maximizing equilibria is either due to policy preferences on the part of party principals or to the effect of party activists (Aldrich, 1983a,b; Miller and Schofield, 2003). We argue that this perturbation is best accounted for in terms of coalitional risk, as discussed in Chapter 3.

The formal and empirical analyses presented here are applicable to any polity using an electoral system based on PR. The underlying formal

model is compatible with a wide variety of different theoretical political equilibria. The theory is also compatible with the considerable variation of party political configurations in multiparty systems (Laver and Schofield, 1990).

As in our discussion in the previous chapter, our analysis of the formal vote model emphasizes the notion of a *local* Nash equilibrium in contrast to the notion of a *global* Nash equilibrium usually employed in the technical literature. One reason for this emphasis is that we deploy the tools of calculus and simulation via hill-climbing algorithms to locate equilibria. As in calculus, the set of LNE must include the set of PNE. Sufficient conditions for existence of a PNE are therefore more stringent than for a LNE. In fact, the necessary and sufficient condition for LNE at the electoral center, in the vote-maximizing game with valence, is so stringent that we regard it to be unlikely to obtain in polities with numerous parties and varied valences. We therefore infer that existence of a PNE at the electoral center is very unlikely in such polities. In contrast, the sufficient condition for a local, noncentrist equilibrium is much less stringent. Indeed, in each polity there may well be multiple LNE. This suggests that the particular configuration of party positions in any polity can be a matter of historical contingency.

4.1 AN EMPIRICAL VOTE MODEL

As discussed in Chapter 3, we assume that the political preferences (or beliefs) of voter i can be described by a "latent" utility vector of the form

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p)) \in \mathbb{R}^p. \tag{4.1}$$

Here $\mathbf{z} = (z_1, \ldots, z_p)$ is the vector of strategies of the set, P, of political agents (candidates, parties, etc.). The point z_j is a vector in a policy space X that we use to characterize party j. (For the formal theory, it is convenient to assume X is a compact, convex subset of Euclidean space of dimension w, but this is not an absolutely necessary assumption. We make no prior assumption that $w = 1$.) Each voter, i, is also described by a vector x_i, in the same space X, where x_i is used to denote the beliefs or "ideal point" of the voter. We assume

$$u_{ij}(x_i, z_j) = \lambda_j - A_{ij}(x_i, z_j) + \theta_j^{\mathrm{T}} \eta_i + \varepsilon_j. \tag{4.2}$$

We use $A_{ij}(x_i, z_j)$ to denote some measure of the distance between the vectors x_i and z_j. In the usual Euclidean model presented in Chapter 3 it is assumed that $A_{ij}(x_i, z_j) = \beta \|x_i - z_j\|^2$, where $\| \ \|$ is the Euclidean

norm on X and β is a positive constant. It is also possible to use an ellipsoidal distance function for A_{ij}, which we do later in Chapters 7 and 8. The term λ_j is called *valence* and was introduced earlier. The k-vector θ_j represents the effect of the k different sociodemographic parameters (class, domicile, education, income, etc.) on voting for the party j while η_i is a k-vector denoting the ith individual's relevant sociodemographic characteristics. We use θ_j^{T} to denote the transpose of θ_j so $\theta_j^{\mathrm{T}}\eta_i$ is a scalar. The abbreviation SD is used throughout to refer to models involving sociodemographic characteristics. The vector ε_j is a stochastic error term, associated with the jth party. Early models of this kind assume that the elements of the random vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_p)$ are independently distributed so the covariance matrix Ω of the error vector is diagonal. In the case the errors are also identically distributed, with variance σ^2, then the covariance matrix of ε is $I\sigma^2$, where I is the identity matrix.

In their study of U.S. presidential elections, Poole and Rosenthal (1984) assumed $\{e_j\}$ to be multivariate normal and pair-wise independent. More recent empirical analyses have been based on Markov Chain Monte Carlo (MCMC) methods, allowing for estimation when the errors are covariant (Chib and Greenberg, 1996). Assuming that the errors are independent and identically distributed via the Type I extreme value (or log-Weibull distribution) gives a *multinomial logit* (MNL) model, whereas assuming that the errors are distributed multivariate normal, and thus covariant, gives the *multinomial probit* (MNP) model. MNP models are generally preferable because they do not require the restrictive assumption of "independence of irrelevant alternatives" (Alvarez and Nagler, 1998). However, a comparison of MNP and MNL models suggests that the results are broadly comparable (Quinn, Martin, and Whitford, 1999). We use an MNL model in this chapter because comparison of MNL and MNP models suggest that the simpler MNL model gives an adequate account of voter choice. It is also much easier to use the MNL empirical model to simulate parties' vote-maximizing strategies (Quinn and Martin, 2002).

A variety of methods have been used to measure the distance or "policy" component $A_{ij}(x_i, z_j)$. Alvarez, Nagler, and Bowler (2000) used a National Election Survey for Britain to locate each voter (in a sample N, of size n) with regard to preferred positions on a large number of policy issues. Each voter was asked to locate the parties and the average across the survey population was used to estimate the position, on this large number of issues, of each party. This method has the virtue that data were not lost, but had the disadvantage that no representation of policy issues was possible.

In their study of U.S. presidential elections, Poole and Rosenthal (1984) used factor analysis to estimate the distribution of voter bliss points in a two-dimensional policy space, X, and also located presidential candidate positions in the same space. In their analysis, the second noneconomic dimension "capture[ed] the traditional identification of southern conservatives with the Democratic party" (Poole and Rosenthal, 1984: 287). For the election of 1968, they estimated identical λ-valence terms and β-coefficients for Humphrey and Nixon and found a much higher λ-valence term and β-coefficient for Wallace. They also noted that there was no evidence that candidates tended to converge to the electoral mean (cf. Hinich, 1977), but gave no explanation for this phenomenon.

There are many possible explanations for nonconvergence of candidate positions. For example, primaries may lead to the choice of more radical candidates for each party. In this chapter we make use of the formal model presented in Chapter 3.

Figures 4.1 and 4.2 are reproduced from Chapter 2, and show the "smoothed" distributions of voter ideal points for 1996 and 1992, while Figure 4.3 gives the distribution for 1988. (The outer contour line in each figure contains 95 percent of the voter ideal points).

All three figures were obtained by factor analysis of the surveys conducted by Arian and Shamir (1990, 1995, and 1999) for these three elections. Party positions were estimated by expert analysis of party manifestos, using the same survey questionnaires. (Earlier work on these estimations can be found in Schofield, Sened, and Nixon, 1998; and Ofek, Quinn, and Sened, 1998).

Each respondent for the survey is characterized by a point in the resulting two-dimensional policy space, X. Thus, the smoothed electoral distribution can be taken as an estimation of the underlying probability density function for the voter ideal points.

Table A4.1 (in the appendix to this chapter) presents the factor loadings for the 1996 analysis of the survey questions. "Security" refers to attitudes toward peace initiatives. "Religion" refers to the significance of religious considerations in government policy. The axes of the figures are oriented so that "left" on the security axis can be interpreted as supportive of negotiations with the PLO, while "north" on the vertical or religious axis is indicative of support for the importance of the Jewish faith in Israel. Comparing Figure 4.3 for 1988 with Figure 4.1 for 1996 suggests that the covariance between the two factors has declined over time.

Since the competition between the two major parties, Labour and Likud, is pronounced, it is surprising that these parties do not move to the

Elections in Israel, 1988–1996

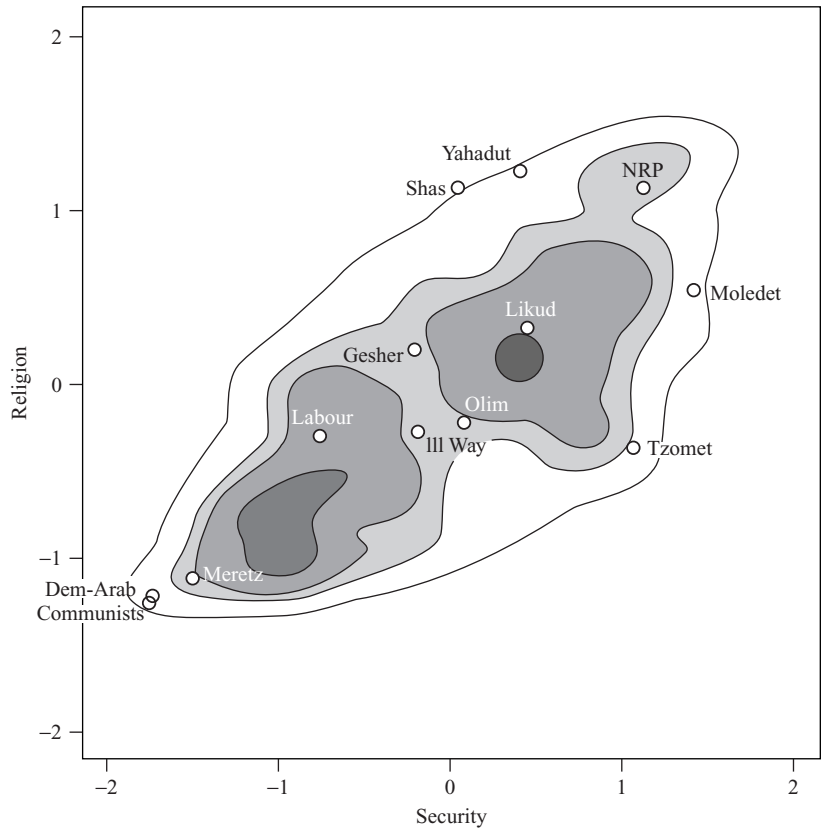


Figure 4.1. Party positions and electoral distribution (at the 95%, 75%, 50%, and 10% levels) in the Knesset at the election of 1996.

electoral mean (as suggested by the formal vote model) in order to increase vote and seat shares. The data on seats in the Knesset given in Chapter 2 (Table 2.1) suggests the vote share of the small Sephardic orthodox party, Shas, increased significantly between 1992 and 1996. As Figures 4.1 and 4.2 illustrate, however, there was no significant move by Shas to the electoral center. Our inference is that the shifts of electoral support are the result of changes in party valence. To be more explicit, we contend that prior to an election each voter i forms a judgment about the relative capability of each party leader. Let λ_{ij} denote the weight given by voter i to party j in the voter's utility calculation. The voter utility is then given by the expression:

$$u_{ij}(x_i, z_j) = \lambda_{ij} - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i. \tag{4.3}$$

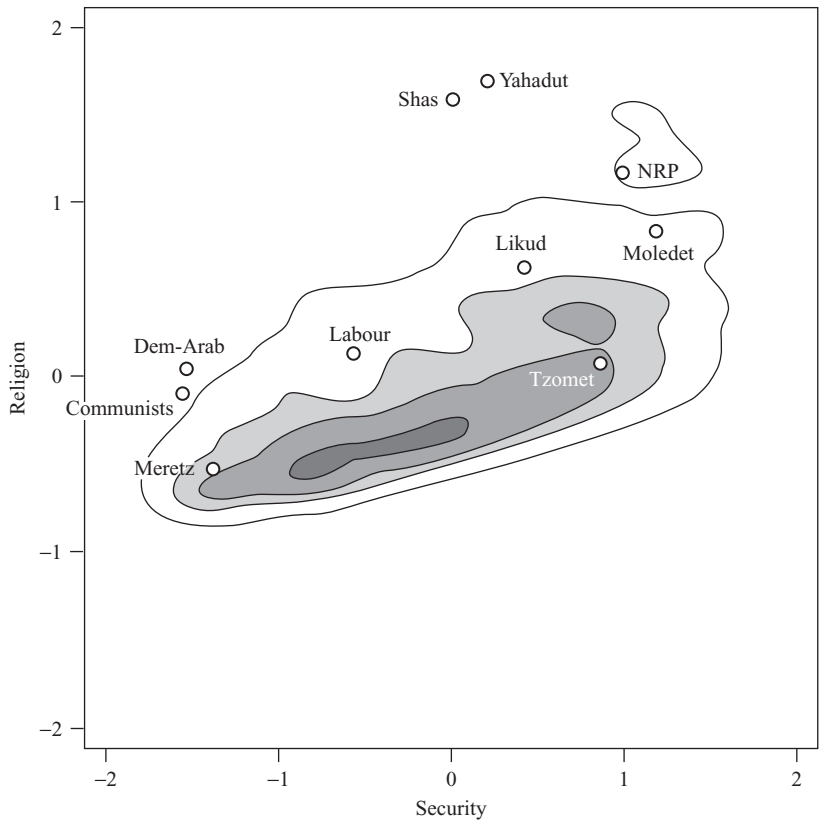


Figure 4.2. Party positions and electoral distribution (at the 95%, 75%, 50%, and 10% levels) in the Knesset at the election of 1992.

However, these weights are subjective, and may well be influenced by idiosyncratic characteristics of voters and parties. For empirical analysis, we shall assume $\lambda_{ij} = \lambda_j + \Delta_{ij}$, where Δ_{ij} is drawn at random from a Type I extreme value distribution. The expected value $\mathcal{E}xp(\lambda_{ij})$, of λ_{ij} is λ_j, and so we write $\lambda_{ij} = \lambda_j + \varepsilon_j$, giving (4.2). Since in this chapter we are mainly concerned with the voter's choice, we shall assume here that λ_j is exogenously determined. We relax this assumption in Chapter 6, where we focus on party behavior. Full details of the estimations of (4.3) for the parameters β and $\{\lambda_j$ for $j = 1, \ldots, p\}$, and for the k by p matrix θ for the three elections are given in the appendix to this chapter.

Estimating the voter model given by (4.2) requires information about sample voter behavior. It is assumed that data is available about voter intentions; this information is encoded, for each sample voter i by the

Elections in Israel, 1988–1996

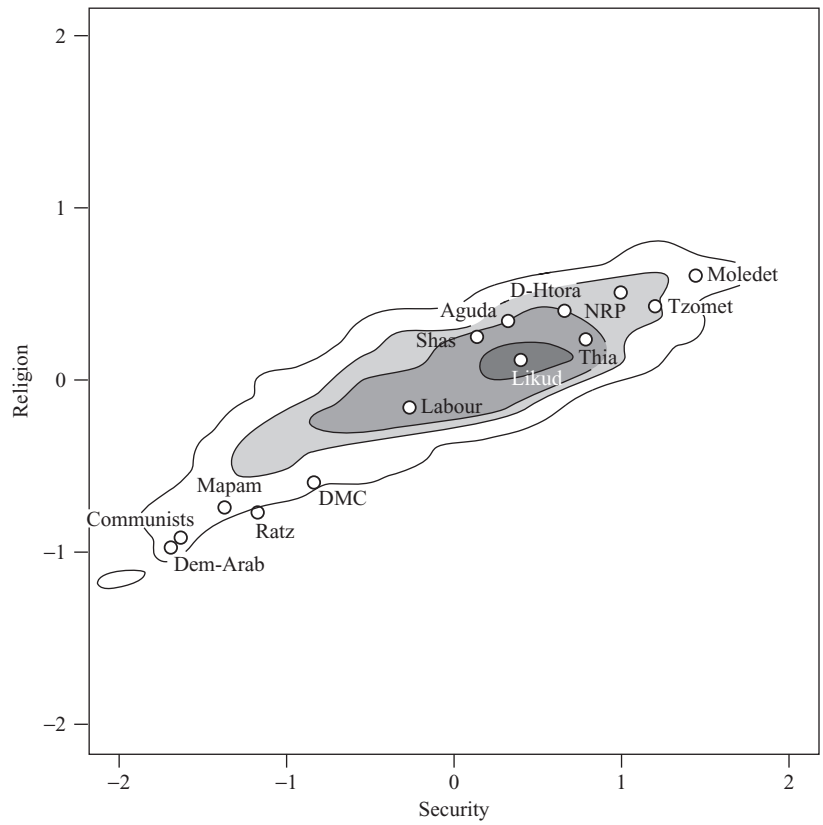


Figure 4.3. Party positions and electoral distribution (at the 95%, 75%, 50%, and 10% levels) in the Knesset at the election of 1988.

vector $c_i = (c_{i1}, \ldots, c_{ip})$, where $c_{ij} = 1$ if and only if j intends to vote (or did indeed vote) for agent j. Given the data set $\{x_i, \eta_i, c_i\}_N$ for the sample N (of size n) and $\{z_j\}_P$, for the political agents, a set $\{\rho_i^*\}_N$ of stochastic variables is estimated. The first moment of ρ_i^* is the probability vector $\rho_i = (\rho_{i1}, \ldots, \rho_{ip})$. Here ρ_{ij} is the probability that voter i chooses agent j.

There are standard procedures for estimating the model given by (4.2). The technique is to choose estimators for the coefficients so that the estimated probability takes the form:

$$\bar{\rho}_{ij}(\mathbf{z}) = \Pr[\,\bar{u}_{ij}(x_i, z_j) > \bar{u}_{il}(x_i, z_l) \text{ for all } l \in P\backslash\{j\}]. \qquad (4.4)$$

Here, $\bar{u}_{ij}$ is the jth component of the estimated latent utility function for i. The estimator for the choice is $\bar{c}_{ij} = 1$ if and only if $\bar{\rho}_{ij} > \bar{\rho}_{il}$ for all

$l \in P \backslash \{j\}$. The procedure minimizes the errors between the n by p matrix $[c]$ and the n by p estimated matrix $[\bar{c}]$. The vote share, $V_j^*(\mathbf{z})$, of agent i, given the vector z of strategies, is defined to be:

$$V_j^*(\mathbf{z}) = \frac{1}{n} \sum_i \rho_{ij}^*(\mathbf{z}). \tag{4.5}$$

Note that, since $V_j^*(\mathbf{z})$ is a stochastic variable, it is characterized by its first moment (its expectation), as well as higher moments (its standard variance, etc.). We follow the theory presented in Chapter 3 and focus on the expectation $\mathcal{E}xp(V_j^*(\mathbf{z}))$. As in the formal analysis, the estimate of this expectation, denoted $\mathcal{E}_j(\mathbf{z})$, is given by:

$$\mathcal{E}_j(\mathbf{z}) = \frac{1}{n} \sum_i \bar{\rho}_{ij}(\mathbf{z}). \tag{4.6}$$

A virtue of using the general voting model (4.3) is that the Bayes' factor, B_{st} (differences in log likelihoods associated with the comparison of model s against model t) can be used to determine which of various possible models is statistically superior (Kass and Raftery, 1995; Quinn, Martin, and Whitford, 1999).

We compared a variety of different MNL models against a pure MNP model for each election. The models were:

(i) MNP: a pure spatial multinomial probit model with $\beta \neq 0$ but $\theta \equiv 0$ and $\lambda = 0$
(ii) MNLSD: a pure logit sociodemographic (SD) model, with $\beta = 0$, involving the component θ, based on respondent age, education, religious observance, and origin (whether Sephardic, etc.)
(iii) MNL1: a pure multinomial logit spatial model with $\beta \neq 0$, but $\theta \equiv 0$ and $\lambda = 0$
(iv) MNL2: a multinomial logit model with $\beta \neq 0$, $\theta \neq 0$, and $\lambda = 0$
(v) Joint MNL: a multinomial logit model with $\beta \neq 0$, $\theta \neq 0$, and $\lambda \neq 0$

The pure sociodemographic model MNLSD gave poor results and this model was not considered further.

Full details of the joint MNL models are given in Tables A4.2, A4.3, and A4.4 in the appendix to this chapter.

For comparison of the models, Table 4.1 gives standard interpretations of the Bayes' factors of model comparisons, while Tables 4.2 to 4.4 give the comparisons for MNP, MNL1, MNL2, and Joint MNL for the three elections. Note that the MNP model had no valence terms. Observe from Table 4.4 that, for the 1996 election, the Bayes' factor for the comparison

Table 4.1. *Interpretation of Evidence Provided by the Bayes Factor B_{st}.*

$\ln(B_{st})$	B_{st}	Evidence in favor of M_s against M_t
0 to 1	1 to 3	Insignificant
1 to 3	3 to 20	Positive
3 to 5	20 to 150	Strong
>5	>150	Very strong

Table 4.2. *Bayes Factor B_{st} for M_s against M_t for the 1988 Election.*

	M_t	Spatial MNP	Spatial MNL1	Spatial MNL2	Joint MNL
	Spatial MNP	n.a.	278***	−38	−73
M_s	Spatial MNL1	−278	n.a.	−316	−352
	Spatial MNL2	38***	316***	n.a.	−35
	Joint MNL	73***	352***	35***	n.a.

*** = very strong support for M_s.

Table 4.3. *Bayes Factor B_{st} for M_s against M_t for the 1992 Election.*

	M_t	Spatial MNP	Spatial MNL1	Spatial MNL2	Joint MNL
	Spatial MNP	n.a.	50***	−24	−71
M_s	Spatial MNL1	−50	n.a.	−74	−121
	Spatial MNL2	24***	74***	n.a.	−47
	Joint MNL	71***	121***	47***	n.a.

*** = very strong support for M_s.

Table 4.4. *Bayes Factor B_{st} for M_s against M_t for the 1996 Election.*

	M_t	Spatial MNP	Spatial MNL1	Spatial MNL2	Joint MNL
	Spatial MNP	n.a.	239***	−17	−49
M_s	Spatial MNL1	−239	n.a.	−255	−288
	Spatial MNL2	17***	255***	n.a.	−33
	Joint MNL	49***	288***	33***	n.a.

*** = very strong support for M_s.

Table 4.5. *National and Sample Vote-Shares and Valence Coefficients for Israel, 1988–1996.*

	1988		1992		1996		1988	1992	1996
	National	Sample	National	Sample	National	Sample	λ^*	λ^*	λ^*
Labour	30.7	28.5	36.0	35.8	27.5	44.0	0.30	0.91	4.15
Meretz, Ratz	4.4	8.3	10.0	11.9	7.6	6.0	0.00	0.00	0.00
Dem, Arab	1.0	–	1.6	–	3.0	–	–	–	–
Communist	2.8	–	2.5	–	4.3	–	–	–	–
Olim	–	–	–	–	5.8	–	–	–	–
Third Way	–	–	–	–	3.2	1.8	–	–	−2.34
Likud	31.8	49.7	26.2	30.2	25.8	43.0	2.84	2.73	3.14
Tzomet	2.0	2.5	6.7	9.6	–	–	−0.75	−0.37	–
Shas	4.8	3.6	5.2	3.6	8.7	2.0	−5.78	−4.67	−2.96
Yahadut	–	–	3.3	–	3.3	–	–		–
NRP	4.0	2.9	5.2	4.6	8.0	5.1	−3.00	−0.44	−4.52
Moledet	1.5	–	2.5	4.4	2.4	1.8	–	0.38	−0.89
Techiya, Thia	3.2	4.4	–	–	–	–	0.39	–	–
Others	8.0	–	0.8	–	0.4	–	–	–	–
β coefficients							1.32	1.25	1.12
Log marginal likelihoods							−597	−834	−465
n							505	781	794
Vote % correctly predicted							52%	46%	64%

* Normalized with respect to a zero valence for Meretz.

of the Joint MNL model with MNL1 was of order 288, so clearly sociodemographic variables add to predictive power. However, the valence constants add further to the power of the model. The spatial distance, as expected, exerts a very strong negative effect on the propensity of a voter to choose a given party. To illustrate, Table 4.5 shows that, in 1996, the β coefficient was estimated to be approximately 1.12. In short, Israeli voters cast ballots, to a very large extent, on the basis of the issue positions of the parties. This is true even after taking the demographic and religious factors into account. The coefficients on "religious observation" for Shas and the NRP (both religious parties) were estimated to be 3.022 and 2.161, respectively. Consequently, a voter who is observant has a high probability of voting for one of these parties, but this probability appears

to fall off rapidly the further the voter's ideal position is from the party position.

In each election, factors such as age, education, and religious observance play a role in determining voter choice. Obviously, this suggests that some parties are more successful among some groups in the electorate than would be implied by a simple estimation based only on policy positions.

Tables 4.2 through 4.4 indicate that, in all three elections, the best model is the joint MNL that includes valence and the sociodemographic factors along with the spatial coefficient β. In particular, there is strong support, in all three elections, for the inclusion of valence. This model provides the best estimates of the vote shares of parties and predicts the vote choices of the individual voters remarkably well. Therefore, this is clearly the model of choice to use as our best estimator for what we refer to as the *stochastic electoral response function*. Adding valence to the MNL model makes it superior to both MNL and MNP models without valence. Adding the sociological factors increases the statistical validity of the model. Table 4.5 provides a summary of the estimation results for the three elections. Note that the 1996 estimation correctly predicts 64 percent of the vote choice and 72 percent and 71 percent of survey participants who voted Labour and Likud, respectively. This success rate is particularly impressive in light of the number of parties that participated in this electoral campaign.

It is possible that an MNP valence model of these elections would have been statistically superior. However, such a model with seven parties would have been difficult to estimate. Moreover, comparison of MNP and MNL models for the Netherlands reported by Quinn, Martin, and Whitford (1999) and discussed in Chapter 6, suggests that the two classes of models are broadly comparable. Dow and Endersby (2004: 111) also suggest that "researchers are justified in using MNL specifications." Since our purpose in constructing the empirical model was to examine the mean voter theorem, as given by Theorem 3.1, it was appropriate to adopt the MNL assumption of independent errors with a Type I extreme value distribution.

Throughout our analyses, we assume that because the sociodemographic components of the model are independent of party strategies, we are able to use the estimated parameters of the model to simulate party movement in order to increase the expected vote share of each party. Hill-climbing algorithms were used for this purpose. Such algorithms involve small changes in party position, and are therefore only capable of obtaining local optima for each party. Consequently, a vector

$\mathbf{z}^* = (z_1^*, \ldots, z_p^*)$ of party positions that results from such a search is what we have called a *local Nash equilibrium* or LNE. We now repeat the definition of an LNE as given in Chapter 3 for the context of the empirical vote-maximizing game defined by $\mathcal{E} : X^p \to \mathbb{R}^p$.

Definition 4.1 ***Equilibrium Concepts for the Empirical Model***

(i) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile function $\mathcal{E} : X^p \to \mathbb{R}^p$ iff for each agent $j \in P$, there exists a neighborhood X_j of z_j^* in X such that

$$\mathcal{E}_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) > \mathcal{E}_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$
$$\text{for all } z_j \in X_j - \{z_j^*\}.$$

(ii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *local weak Nash equilibrium* (LNE) for $\mathcal{E}$ iff for each agent j there exists a neighborhood X_j of z_j^* in X such that

$$\mathcal{E}_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \geq \mathcal{E}_j(z_1^*, \ldots, z_j, \ldots, z_p^*)$$
$$\text{for all } z_j \in X_j.$$

(iii) A strategy vector $\mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *strict*, respectively, *weak, pure strategy Nash equilibrium* (PSNE, respectively, PNE) for $\mathcal{E}$ iff X_j can be replaced by X in (i), (ii), respectively.

(iv) The strategy z_j^* is termed a *local strict best response*, a *local weak best response*, a *global weak best response*, a *global strict best response*, respectively, to $\mathbf{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_{j+1}^*, \ldots, z_p^*)$.

As noted previously, in these definitions *weak* refers to the condition that z_j^* is no worse than any other strategy. While LSNE are easier to use in the formal analysis, we focus on LNE for empirical anlysis. Clearly, a PNE must be an LNE, but not conversely. As we have emphasized, above, a condition that is sufficient to guarantee that an LNE is a PNE for the electoral game is *concavity* of the vote functions.

Definition 4.2 *The profile $\mathcal{E} : X^p \to \mathbb{R}^p$ is concave iff for each j, and any real α and $x, y \in X$, then*

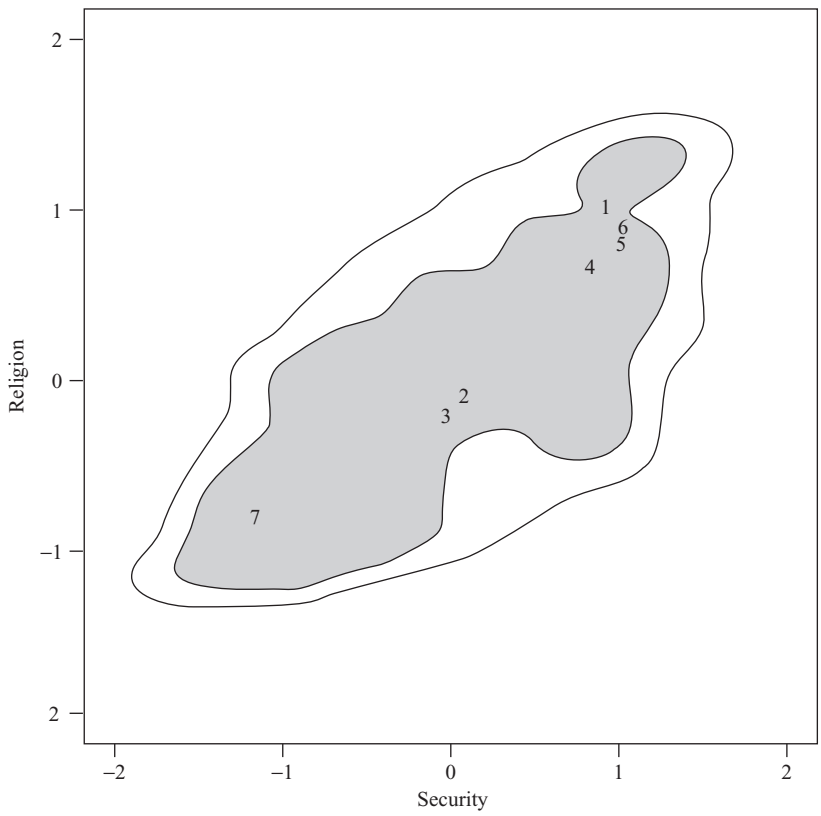
$$\mathcal{E}_j(\alpha x + (1-\alpha)y) \geq \alpha \mathcal{E}_j(x) + (1-\alpha)\mathcal{E}_j(y).$$

Concavity of the payoff functions $\{\mathcal{E}_j\}$ in the jth strategy z_j, together with continuity in z_j and compactness and convexity of X is sufficient for existence of a PNE (Banks and Duggan, 2005).

Elections in Israel, 1988–1996

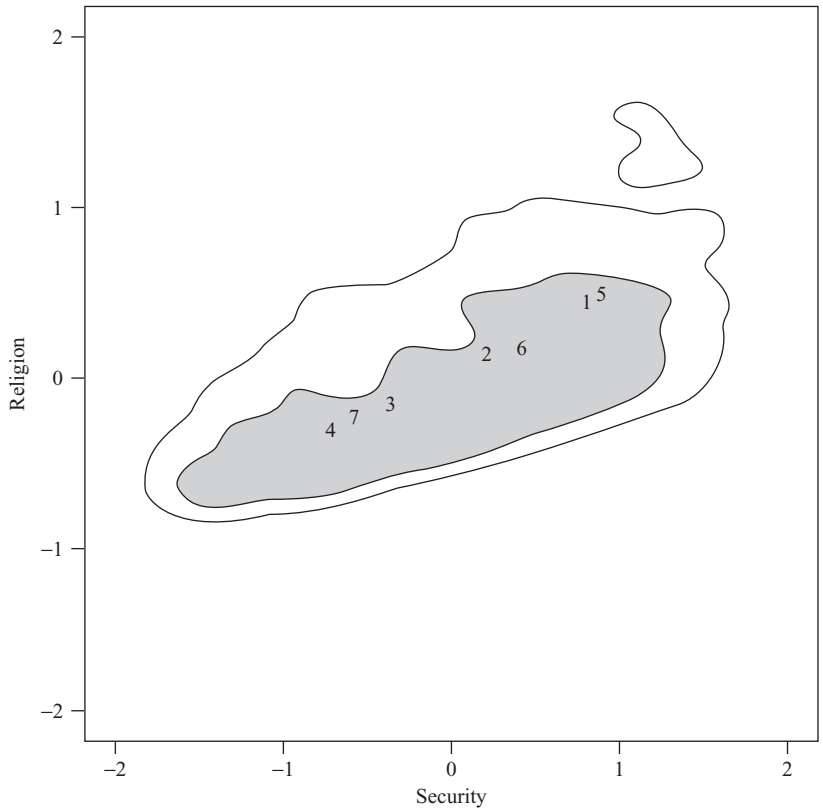
In the following section we discuss the relevance of mean voter theorem for empirical analysis. As previously mentioned, this theorem asserts that the vector $\mathbf{z}^* = (x^*, \ldots, x^*)$ (where x^* is the mean of the distribution of voter ideal points) is a PNE for the vote-maximizing electoral game (Hinich, 1977; Enelow and Hinich, 1984; Lin et al., 1999). As in the formal discussion, we call $(x^*, \ldots, x^*)$ the *joint electoral mean*. Since the electoral distribution can be readily normalized, so $x^* = 0$, we also use the term *joint electoral origin*. We used a hill-climbing algorithm to determine the LNE of the empirical vote models for the three elections.

Our simulation of the empirical models found five distinct LNE for the 1996 election in Israel. A representative LNE is given in Figure 4.4.



Key: 1= Shas, 2 = Likud, 3 = Labour, 4 = NRP, 5 = Moledet, 6 = Third Way, 7 = Meretz

Figure 4.4. A representative local Nash equilibrium of the vote-maximizing game in the Knesset for the 1996 election.



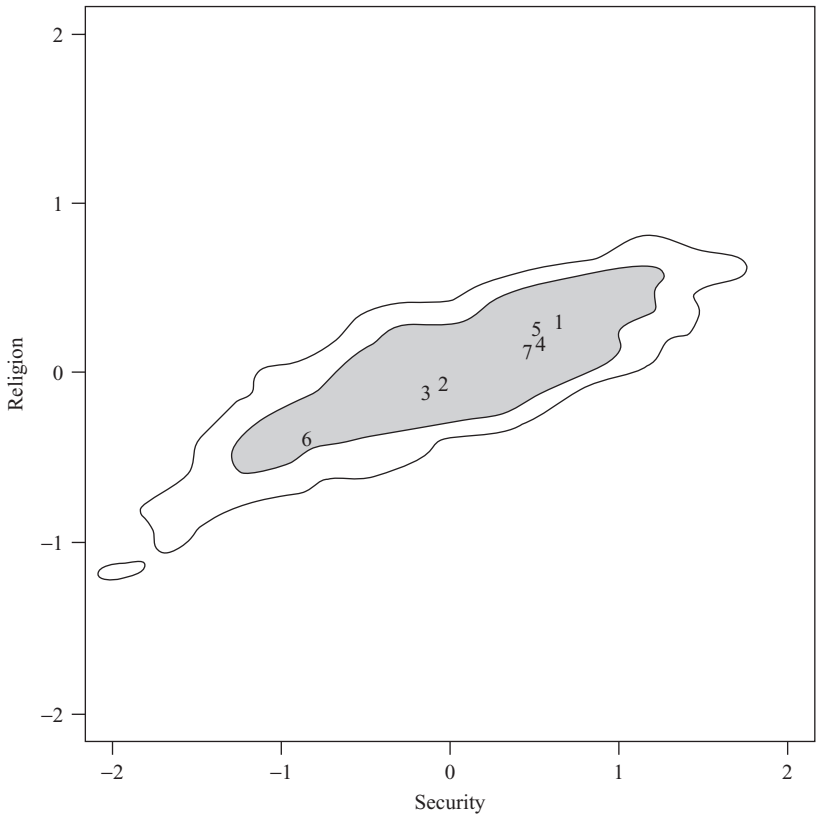
Key: 1 = Shas, 2 = Likud, 3 = Labour, 4 = Meretz, 5 = NRP, 6 = Moledet, 7 = Tzomet

Figure 4.5. A representative local Nash equilibrium of the vote-maximizing game in the Knesset for the 1992 election.

Notice that the locations of the two high-valence parties, Labour and Likud, in Figure 4.1 closely match their simulated positions in Figure 4.4. Obviously, none of the estimated equilibrium vectors in Figure 4.4 correspond to the convergent situation at the electoral mean. Figures 4.5 and 4.6 give representative LNE for 1992 and 1988. The close match with the Labour and Likud positions was observed in all simulated LNE.

It has been noted many times before that parties do not converge to an electoral mean. Various theoretical models have been offered to account for this phenomenon. Our analysis in this chapter is meant as a further contribution to this literature. Before we begin our discussion of

Elections in Israel, 1988–1996



Key: 1 = Shas, 2 = Likud, 3 = Labour, 4 = Tzomet, 5 = NRP, 6 = Ratz, 7 = Thia

Figure 4.6. A representative local Nash equilibrium of the vote-maximizing game in the Knesset for the 1988 election.

the results just presented, several preliminary conclusions appear to be of interest.

1) First, the empirical MNL model and the formal model based on the extreme value distribution Ψ (as discussed in Chapter 3) are mutually compatible.
2) Second, the set of LNE obtained by simulation of the empirical model must contain any PNE for this model (if any exist). Since no LNE was found at the joint mean position, it follows that the mean voter theorem is invalid, given the estimated parameter values of the empirical model. This conclusion is not susceptible to any counter-argument that the parties may have utilized evaluation functions other than

expected vote shares, because only vote-share maximization was allowed to count in the hill-climbing algorithm used to generate the LSNE.

3) A comparison of Figures 4.1, 4.2, and 4.3 with the simulation Figures 4.4, 4.5, and 4.6 makes it clear that there are marked similarities between estimated and simulated positions. This is most obvious for the high-valence parties, Labour and Likud, but also for the low-valence party Meretz. This suggests that the set of expected vote-share functions $\{\mathcal{E}_j\}$ is a close proxy to the actual, but unknown, utility functions $\{U_j\}$, deployed by the party leaders.
4) Although the equilibrium notion of LNE that we deploy is not utilized in the game theoretic literature, it has a number of virtues. In particular, Theorem 3.3 shows that an LSNE, and thus an LNE, will exist, for "almost all" party utility profiles $\{\mathcal{E}_j\}$, as long as these profiles are differentiable in the strategy variables and satisfy the "boundary condition" on the set X_o^p of feasible strategy profiles. Clearly, X_o^p can be chosen sufficiently extensive so that all gradients point toward its interior. Moreover, the definition of $\{\mathcal{E}_j\}$ makes it obvious that it is differentiable. On the other hand, existence for PNE is problematic when concavity fails.
5) Although the local equilibrium concept is indeed "local," there is no formal reason why each of the various LNE that we obtain should be, in fact, "close" to one another. It is noticeable in Figures 4.4, 4.5, and 4.6 that the LNE for each election are approximate permutations of one another, with low-valence parties strung along what we call the *electoral principal axis*.

In the following section, we examine the formal vote model in order to determine why the mean voter theorem appears to be invalid for the estimated model of Israel. The formal result will explain why low-valence parties in the simulations are far from the electoral mean, and why all parties lie on a single electoral axis.

4.2 COMPARING THE FORMAL AND EMPIRICAL MODELS

The point of this section is to use the Israeli example to present a case in which the necessary condition of Theorem 3.1 is not satisfied. This failure has significant consequences for the behavior of political parties in this electoral competition. As we demonstrate here, in such an electoral environment, some parties have a clear incentive to formulate

divergent policy positions rather than converge at an LNE at the origin of the distribution of the voters' ideal points. We first note that the expected vote-share functions $\{\mathcal{E}_j\}$ of the empirical model just discussed are not exactly the same as the formal vote functions presented in Chapter 3. The principal difference is that the empirical model incorporates sociodemographic characteristics. In the simulation, these characteristics were held fixed, because by definition they are unaffected by party policy choices. We should expect that when the values of the empirical parameters are utilized in the formal model, then the equilibrium characteristics of the model should mirror the results of simulation. In fact, we find an exact parallel between the model and simulation.

In 1996, the lowest-valence party is the NRP with valence -4.52. The spatial coefficient is $\beta = 1.12$. As Table A4.1 shows, the electoral variances are 1.0 and 0.732 with covariance 0.591. We can use Theorem 3.1 to compute

$$\rho_{NRP} \simeq \frac{1}{1 + e^{4.15+4.52} + e^{3.14+4.52}} \simeq 0.$$

$$\text{Thus } A_{NRP} = \beta = 1.12.$$

$$\text{Hence } C_{NRP} = 2(1.12)\begin{pmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{pmatrix} - I = \begin{pmatrix} 1.24 & 1.32 \\ 1.32 & 0.64 \end{pmatrix}$$

$$\text{and } c(\Psi) = 3.88.$$

Then the eigenvalues for the NRP are 2.28 and -0.40, giving a saddlepoint, and a value for the convergence coefficient of 3.88. The major eigenvector for the NRP is (1.0, 0.8), and along this axis the NRP vote-share function increases as the party moves away from the origin. The minor, perpendicular axis is given by the vector $(1, -1.25)$ and on this axis the NRP vote-share decreases. Figure 4.4 gives one of the local equilibria in 1996, obtained by simulation of the model. The figure makes it clear that the vote-maximizing positions lie on the principal electoral axis through the origin and the point (1.0, 0.8). As we noted, five different LNE were located; in all cases, the two high-valence parties, Labour and Likud, were located at almost precisely the same positions. The only difference between the various equilibria were that the positions of the low-valence parties were perturbations of one another. Compare this analysis with Figure 4.4.

We next analyze the situation for 1992, by computing the eigenvalues using Theorem 3.1 again. From the empirical model we obtain $\lambda_{shas} = -4.67$, $\lambda_{likud} = 2.73$, $\lambda_{labour} = 0.91$, $\beta = 1.25$. When all parties are at the

origin, then the probability that a voter chooses Shas is

$$\rho_{shas} \simeq \frac{1}{1 + e^{2.73+4.67} + e^{0.91+4.67}} \simeq 0.$$

Thus $A_{shas} = \beta = 1.25.$

$$C_{shas} = 2(1.25)\begin{pmatrix} 1.0 & 0.453 \\ 0.453 & 0.435 \end{pmatrix} - I = \begin{pmatrix} 1.5 & 1.13 \\ 1.13 & 0.08 \end{pmatrix}$$

$$c(\Psi) = 3.6.$$

Then the two eigenvalues for Shas can be calculated to be +2.12 and −0.52 with a convergence coefficient for the model of 3.6. Thus we find that the origin is a saddlepoint for the Shas Hessian. The eigenvector for the large, positive eigenvalue is the vector (1.0, 0.55). Again, this vector coincides with the principal electoral axis. The eigenvector for the negative eigenvalue is perpendicular to the principal axis. To maximize vote share, Shas should adjust its position—but only on the principal axis. This is exactly what the simulation found. Notice that the probability of voting for Labour is $[1 + e^{1.82}]^{-1} = 0.14$, and $A_{labour} = 0.9$, so even Labour will have a positive eigenvalue at the origin. Figure 4.5 gives one of the two different LNE obtained from simulation of the empirical model. Again, the prediction obtained from the formal model and the simulation are consistent.

Calculation for the model for 1988 gives eigenvalues for Shas of +2.0 and −0.83 with a convergence coefficient of 3.16, and a principal axis through (1.0, 0.5). Again, vote-maximizing behavior by Shas should oblige it to stay strictly to the principal electoral axis. The three simulated vote-maximizing local equilibrium positions indicated that there was no deviation by parties off the principal axis or eigenspace associated with the positive eigenvalue. Again, compare the prediction with the representative LNE given in Figure 4.6.

Thus the simulations for all three elections were compatible with the predictions of the formal model based on the extreme value distribution. All parties were able to increase vote shares by moving away from the origin, along the principal axis, as determined by the large, positive principal eigenvalue. In particular, the simulation confirms the logic of the above analysis. Low-valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center. Their optimal positions will lie either in the "northeast" quadrant or the "southwest" quadrant. The vote-maximizing model, without any additional information, cannot determine which way the low-valence parties should move.

As noted above, the simulations of the empirical models found multiple LNE essentially differing only in permutations of the low-valence party positions.

In contrast, since the valence difference between Labour and Likud was relatively low in all three elections, their optimal positions would be relatively close to, but not identical to, the electoral mean. The simulation figures for all three elections are also compatible with this theoretical inference. It is clear that once the low-valence parties vacate the origin, then high-valence parties, like Likud and Labour, will position themselves almost symmetrically about the origin, and along the major axis. It should be noted that the positions of Labour and Likud, particularly, closely match their positions in the simulated vote-maximizing equilibria.

The correlation between the two electoral axes was much higher in 1988 ($r^2 = 0.70$) than in 1992 or 1996 (when $r^2 \simeq 0.47$). It is worth observing that as r^2 falls from 1988 to 1996, a counter-clockwise rotation of the principal axis can be observed. This can be seen in the change from the eigenvalue (1.0, 0.5) in 1988 to (1.0, 0.55) in 1992 and then to (1.0, 0.8) in 1996. Notice also that the total electoral variance increased from 1988 to 1992 and again to 1996. Indeed, Figure 4.1 indicates that there is evidence of bifurcation in the electoral distribution in 1996.

In comparing Figure 4.1, of the estimated party positions, and Figure 4.4, of simulated equilibrium positions, there is a notable disparity particularly in the position of Shas. In 1996, Shas was pivotal between Labour and Likud, in the sense that to form a winning coalition government, either of the two larger parties required the support of Shas. It is obvious that the location of Shas in Figure 4.1 suggests that it was able to bargain effectively over policy, and presumably perquisites. Indeed, it is plausible that the leader of Shas was aware of this situation, and incorporated this awareness in the utility function of the party.

The relationship between the empirical work and the formal model, together with the possibility of strategic reasoning of this kind, suggests the following conclusion.

Conjecture 4.1 The close correspondence between the simulated LNE based on the empirical analysis and the estimated actual political configuration suggests that the true utility function for each party j has the form $U_j(\mathbf{z}) = \mathcal{E}_j(\mathbf{z}) + \delta_j(\mathbf{z})$, where $\delta_j(\mathbf{z})$ may depend on the beliefs of party leaders about the post-election coalition possibilities, as well as the effect of activist support for the party.

Developing a formal model based on this conjecture could be used to show that the LNE for $\{U_j\}$ would be close to the LNE for $\{\mathcal{E}_j\}$.

If this were true as a general conjecture, it would be possible to use a combination of multinomial logit electoral models, simulations of these models, and the formal electoral model based on exogenous valence to study general equilibrium characteristics of multiparty democracies. In the next section we offer one way of constructing this more complex formal model.

4.3 COALITION BARGAINING

In this section we discuss the formation of coalition government in order to provide a tentative account for the discrepancy we have noted between vote-maximizing positions, as obtained from simulation and predicted by the formal model, and estimated party positions.

Six coalition governments formed during the period covered in Table 2.1. Following the 1988 election, Likud and Labour formed a national unity coalition. Figure 4.3 shows that Likud and Labour were the closest and therefore the most likely coalition partners. The coalition that formed in 1988, however, was clearly oversized. It included Labour, Likud, Shas, NRP, Aguda, and Degel HaTora for a total of 92 seats, which is more than three quarters of the 120 seats in the Knesset.

Three points are noteworthy. First, at this point in time, the riots in the occupied territories, the so-called First Intifada, reached new peaks of violence. Riker (1962) gave one reason for oversized coalitions: national crisis in terms of external threat. Second, the national unity government formed after both major parties failed to form minimal winning coalitions on their own. (As before, we use the standard term *minimal winning*, MW, for a coalition that is winning but may lose no member and still win). The left block had 55 seats, including 2 independent Arab Nationalists (Progress and Democratic Arab) and 4 Communist delegates. The right had 65, including 2 from Tzomet, 3 from Techiya, and 2 from Moledet. These were all regarded as too extreme right-wing parties to be admitted into the coalition at that time. Finally, a common interpretation of the situation suggests that while neither Labour nor Likud could form coalitions on their own, they both wanted to include the religious parties in order to keep future options open.

However, this coalition did not last. Eighteen months after it was sworn in it collapsed and Likud formed the second, slightly oversized, coalition including Likud, Shas, NRP, Yahadut, and the three extremist parties

of Moledet, Tzomet, and Thia. This coalition formally controlled 65 of the 120 seats, but Moledet and Tzomet constantly complained about the "soft" policy of the government toward the Arabs in the occupied territories and the willingness of Likud to endorse the conference for peace that was held in Madrid in 1991. When the conference started, both Tzomet and Moledet left the government, leaving behind a strictly minimum winning coalition. As Figure 4.3 shows, this was a natural coalition in terms of ideological proximity. The coalition lasted until the election of 1992.

The first coalition to form after the 1992 election was a minimal winning coalition of Shas, Labour, and Meretz, controlling 62 seats. Observers soon realized two basic facts about the newly elected Knesset and the new government. First, Labour was at the structurally stable core position, $\mathbb{SC}_1^0(\mathbf{z})$, given the post-election decisive structure. Chapter 2 and the example in Chapter 3 both discuss this characteristic of the configuration of party positions. Second, Meretz and Shas were unlikely partners in the same coalition (Sened, 1996). Seventeen months after its conception, Shas left the coalition, leaving Rabin at the head of a minority coalition of 56 seats. This minority government proved to be not only remarkably stable—it lasted 31 months and longer than any coalition in the last two decades—but remarkably effective in pursuing an audacious policy toward a peace agreement with the PLO and Jordan and introducing major reforms in the public sector.

Sened (1996) gives a lengthy account of how this coalition came to be and how effective it was in legislation and in pursuing its peace initiative in spite of its minority status. One important aspect of this account is what led Shas to abandon the 1992 coalition. As the coalition agreement was signed, Prime Minister Itzhak Rabin promised Shas that he would delay the passage of several basic laws in the Knesset. In Israel, basic laws serve as substitutes for the constitution. They have special status, as they require special majorities in order to be amended or discontinued. In 1992, Shas was particularly concerned about two such basic laws:

(1) Freedom and Human Dignity, and
(2) Freedom of Occupation.

Both laws were appropriately interpreted by the spiritual leadership of the ultra orthodox Shas party as serious constraints on the ability of the religious establishment in Israel to intervene in the private choices of Israeli citizens. Rabin was unable to keep his promise, the laws passed, and Shas resigned (Sened, 1996: 366). The lesson of this important political event is three fold.

First, the laws coincided with the core policy position of the Labour party. While a Prime Minister gave his word to a coalition partner to delay the passage of the law he could not keep his promise because it was the Knesset that passed the laws. As we argue throughout this book, it is Parliament and not any particular coalition that passes legislation. Moreover, it is the structure of Parliament, and not the composition of any particular coalition, that determines the final legislative outcome.

Second, while Rabin promised repeatedly to enlarge his coalition, he never bothered to do so. This coalition remained unbeaten until the 1996 election, surviving the controversy over its policies that eventually brought about the assassination of Prime Minister Rabin in November of 1995.

Finally, this coalition was the "cheapest" coalition to occur in Israeli politics, in the sense that Labour kept almost all the important portfolios to itself (Nachmias and Sened, 1999).

The first coalition to form after the 1996 elections was again slightly oversized. It included all the parties of the upper right quadrant of Figure 4.1 (except Moledet) as well as Gesher and III Way. Together the 8 parties in this coalition controlled 66 of 120 Knesset seats. Figure 4.1 illustrates remarkable spread of the ideological positions of the coalition members and the inflated number of coalition members. The bargaining model that we introduce below would predict that partners in this coalition should be able to extract significant government perquisites out of the formateur (Likud). Nachmias and Sened (1999) have tested this hypothesis. They show that the first Netanyahu government ranked 4th among 34 coalition governments in terms of government perquisite allocated per seat held by a coalition partner other than Likud. On average, each such seat earned the Knesset member approximately 3.5 times more government perquisites than a seat held by a Likud member. (We measure perquisites in terms of the percentage of the annual government budget controlled by the coalition member divided by the number of seats this party has in the Knesset). A seat held by a coalition partner other than Likud was worth 2.3 percent of the annual budget, while a seat held by a Likud member was worth 0.65 percent. This difference was statistically significant, and substantially higher than the average percentage calculated across the 33 previous coalitions. Netanyahu, the leader of Likud, eventually refused to allocate additional resources to Gesher, and this led Gesher to leave the coalition. Netanyahu remained at the head of a strictly minimal winning coalition government that stayed in power until the 1999 election.

The most important lesson to draw from these results is that parties may position themselves away from simple vote-maximizing positions if

in doing so they become more attractive coalition partners. There are at least three reasons why a party may move away from its vote-maximizing position.

First, a central party may try to capture the core of the polity in order to obtain more of the government perquisites through its position as a dominant party. We conjecture that this was the strategy of Labour in 1992. The estimated position of Labour in Figure 4.2 is somewhat "north" and "west" of the simulated vote-maximizing position given in Figure 4.5.

A second incentive suggests itself on the basis of the bargaining model outlined in Section 3.3. If the party believes that there will be no core party after the election, and it is able to guess at the location of the heart, then it may be able to adjust its position to take advantage of this estimate.

A third incentive, particularly relevant to a pivotal party like Shas, is to be closer to both potential coalition formateurs.

Schofield, Sened, and Nixon (1998) suggest that a combination of these two last incentives explains the position of Shas in Figure 4.1. Obviously, the Shas position is at the center of the security dimension and very far "north" on the religious dimension. This position is far from a simple vote-maximizing position on the basis of the electoral model based on fixed, or exogenous, valences.

It is interesting to note in this respect how Shas seems to have behaved in an increasingly sophisticated fashion. We suggest that at the time of the 1992 election, Shas may have calculated that the coalition structure $\mathbb{D}_0$ was most likely. As the example in Chapter 3 indicated, this would lead Shas to adopt a fairly radical position in order to extract perquisites from government. Labour ended up capturing the structurally stable core in the Knesset and Shas ended up too far away to be an attractive coalition member. In 1996, the loss of votes for Labour meant that the $\mathbb{D}_0$ coalition structure did occur. Shas adjusted its position by moving "south" on the religious axis and was able to bargain its way into lucrative membership in both of Netanyahu's coalitions (Nachmias and Sened, 1999). Since then, Shas has remained pivotal between Labour coalitions, led by Barak, and Likud coalitions led by Sharon. We discuss in Chapter 9 the recent changes in the political configuration of the Knesset.

4.4 CONCLUSION: ELECTIONS AND LEGISLATIVE BARGAINING

In a very simple sense, legislative bargaining models often assume that the composition of the coalition government determines the nature of

legislation and policy implementation. In contrast, the previous section suggests that it is necessary to tie the pre-election party positioning to the expected final coalitional outcome. As we have discussed in Chapters 2 and 3, under the post-election coalition structures given by $\mathbb{D}_1$, the structurally stable core $\mathbb{SC}_1^0(\mathbf{z})$ at the vector $\mathbf{z}$ is nonempty, and the heart $\mathcal{H}_1^0(\mathbf{z})$ collapses to $\mathbb{SC}_1^0(\mathbf{z})$. The discussion of the 1992 election suggests that the policy position of Labour meant that it was not only the *strongest* party, in terms of seat shares, but the configuration of party positions meant that it was also *dominant,* in the sense that its position could be expected to be implemented with certainty. We can then expect a minority government, as did occur under Rabin's leadership. In contrast, under a coalition structure belonging to $\mathbb{D}_0$, the core is empty, and the vector of party positions $\mathbf{z}$, together with the distribution of seat shares, defines the heart $\mathcal{H}_0^0(\mathbf{z})$ of the legislature. In such a situation, one expects one of a number of possible coalition governments. Indeed, all such governments must command the support of at least a majority of the seats in the Parliament. If they do not, then a majority counter-coalition will be able to engineer a vote of no confidence. Although this argument is clearest when nonpolicy perquisites are irrelevant, we argue that a similar argument holds when perquisites are incorporated. This observation about the fundamental difference between the core situation $\mathcal{H}_1$ and the noncore situation $\mathcal{H}_0$ is crucial, we believe, to an understanding of the sharp qualitative shift that can occur in legislative bargaining.

As the Israel examples in Chapter 2 and 3 illustrate, the potentially dominant party, Labour, should attempt to maximize the probability π_1 that the election outcome $\mathbb{D}_1$ occurs. In contrast, since Likud had available no feasible position, prior to 2003, that would allow it to be dominant, then it should attempt to maximize the probability π_0 that $\mathbb{D}_0$ occurs. As a first approximation, we may assume that $U_j(\mathbf{z}) = \mathcal{E}_j(\mathbf{z})$ for $j =$ Labour or Likud. This provides an explanation of why the positions of Labour and Likud are close to their estimated vote-maximizing positions in the elections of 1988, 1992, and 1996.

The parties with low valence may have more complex incentives depending on their beliefs concerning the game form $\tilde{g}^\alpha = \{\tilde{g}_t^\alpha, \pi_t\}$. The vote-maximizing model suggests that they will adopt positions on the periphery of the voter distribution, but their precise location may be off the principal electoral axis if they believe that such a position can be advantageous in coalition bargaining.

We return to this analysis in Chapter 9.

It should be possible to test this model against other hypotheses that point to the composition of the coalition as the main determinant of final policy outcomes in multiparty parliaments (see, for example, Laver and Shepsle, 1990, 1994, 1996).

4.5 APPENDIX

Table A4.1. *Factor Analysis Results for Israel for the Election of 1996.*

	Factor weights*	
Issue Question	Security	Religion
Chance for peace	0.494 (0.024)	–
Land for peace	0.867 (0.013)	–
Religious law vs. democracy	0.287 (0.038)	0.613 (0.054)
Must stop peace process	−0.656 (0.020)	–
Agreement with Oslo accord	−0.843 (0.012)	–
Oslo accord contributed to safety	−0.798 (0.016)	–
Personal safety after Oslo	−0.761 (0.020)	–
Israel should talk with PLO	−0.853 (0.016)	–
Opinion of settlers	0.885 (0.015)	–
Agree that Palestinians want peace	−0.745 (0.016)	–
Peace agreement will end Arab-Israeli conflict	−0.748 (0.018)	–
Agreement with Palestinian state	−0.789 (0.016)	–
Should encourage Arabs to emigrate	0.618 (0.022)	–
What must Israel do to prevent war?	−0.843 (0.019)	–
Settlements 1	0.712 (0.014)	–
Settlements 2	0.856 (0.014)	–
National security	0.552 (0.023)	–
Equal rights for Arabs and Jews in Israel	−0.766 (0.018)	–
More government spending toward religious institutions	–	0.890 (0.035)
More government spending toward security	0.528 (0.049)	0.214 (0.065)
More government spending on immigrant absorption	0.342 (0.052)	0.470 (0.065)
More government spending on settlements	0.597 (0.040)	0.234 (0.055)
More government spending in Arab sector	−0.680 (0.019)	–
Public life should be in accordance with Jewish tradition	–	1.000 (constant)
Views toward an Arab minister	−0.747 (0.019)	–
Var (Security)	1.000 (constant)	
Var (Religion)	0.732	
Covar (Security, Religion)	0.591	

* Standard errors in parenthesis.

Table A4.2. *Multinomial Logit Analysis of the 1996 Election in Israel (normalized with respect to Meretz).*

	Posterior		95% Confidence Interval	
	Party	Mean	Lower Bound	Upper Bound
Spatial Distance	β	1.117	0.974	1.278
Constant λ coefficients	Shas	−2.960	−7.736	1.018
	Likud	3.140	0.709	5.800
	Labour	4.153	1.972	6.640
	NRP	−4.519	−8.132	−1.062
	Moledet	−0.893	−4.284	2.706
	Third Way	−2.340	−4.998	0.411
Ashkenazi	Shas	0.066	−1.831	1.678
	Likud	−0.625	−1.510	0.271
	Labour	−0.219	−0.938	0.492
	NRP	1.055	−0.206	2.242
	Moledet	0.819	−0.560	2.185
	Third Way	−0.283	−1.594	1.134
Age	Shas	0.014	−0.058	0.086
	Likud	−0.024	−0.063	0.008
	Labour	−0.040	−0.077	−0.012
	NRP	−0.064	−0.111	−0.020
	Moledet	−0.026	−0.088	0.026
	Third Way	0.014	−0.034	0.063
Education	Shas	−0.377	−0.693	−0.063
	Likud	−0.032	−0.180	0.115
	Labour	0.011	−0.099	0.120
	NRP	0.386	0.180	0.599
	Moledet	0.049	−0.219	0.305
	Third Way	−0.067	−0.298	0.150
Religious Observation	Shas	3.022	1.737	4.308
	Likud	0.930	0.270	1.629
	Labour	0.645	0.077	1.272
	NRP	2.161	1.299	3.103
	Moledet	0.897	−0.051	1.827
	Third Way	0.954	0.031	1.869
Correctly Predicted	Shas	0.309	0.210	0.414
	Likud	0.707	0.672	0.740
	Labour	0.717	0.681	0.752
	NRP	0.408	0.324	0.493
	Moledet	0.078	0.046	0.115
	Third Way	0.029	0.017	0.043
	Meretz	0.286	0.226	0.349
	Entire Model	0.638	0.623	0.654
$n = 794$		Log marginal likelihood = −465.		

Table A4.3. *Multinomial Logit Analysis of the 1992 Election in Israel (normalized with respect to Meretz).*

	Posterior		95% Confidence Interval	
	Party	Mean	Lower Bound	Upper Bound
Spatial Distance	β	1.253	1.098	1.408
Constant λ coefficients	Shas	−4.671	−7.993	−1.539
	Likud	2.725	1.100	4.304
	Labour	0.914	−0.627	2.560
	NRP	−0.442	−2.291	1.247
	Tzomet	−0.369	−1.653	0.905
	Moledet	0.380	−1.348	2.108
Ashkenazi	Shas	0.453	−0.907	2.106
	Likud	−0.881	−1.463	−0.283
	Labour	−0.727	−1.360	−0.125
	NRP	−0.083	−1.020	0.815
	Tzomet	−0.559	−1.385	0.200
	Moledet	−0.918	−1.772	−0.854
Age	Shas	0.011	−0.028	0.053
	Likud	0.019	−0.006	0.047
	Labour	0.042	0.013	0.069
	NRP	0.018	−0.014	0.054
	Tzomet	0.010	−0.025	0.044
	Moledet	−0.020	−0.052	0.012
Education	Shas	0.077	−0.087	0.228
	Likud	−0.193	−0.305	−0.089
	Labour	−0.069	−0.088	0.028
	NRP	−0.138	−0.267	−0.015
	Tzomet	−0.027	−0.136	0.079
	Moledet	0.032	−0.078	0.142
Religious Observation	Shas	2.024	1.394	2.791
	Likud	0.658	0.320	0.977
	Labour	0.141	−0.255	0.536
	NRP	1.595	1.081	2.203
	Tzomet	0.526	0.022	0.983
	Moledet	0.446	−0.166	01058
Correctly Predicted	Shas	0.405	0.315	0.504
	Likud	0.503	0.463	0.545
	Labour	0.595	0.552	0.633
	NRP	0.234	0.176	0.299
	Tzomet	0.253	0.207	0.303
	Meretz	0.470	0.415	0.530
	Moledet	0.114	0.080	0.149
	Entire Model	0.475	0.460	0.490
$n = 781$		Log marginal likelihood = −834.		

Table A4.4. *Multinomial Logit Analysis of the 1988 Election in Israel (normalized with respect to Meretz).*

	Posterior		95% Confidence Interval	
	Party	Mean	Lower Bound	Upper Bound
Spatial Distance	β	1.315	1.083	1.569
Constant λ coefficients	Shas	−5.777	−10.092	−2.324
	Likud	2.839	0.325	5.047
	Labour	0.297	−2.547	3.011
	NRP	−2.989	−6.874	0.333
	Tzomet	−0.745	−4.480	2.570
	Thia	0.388	−3.247	4.023
Ashkenazi	Shas	−1.108	−2.326	0.075
	Likud	−2.384	−3.171	−1.731
	Labour	−1.461	−2.320	−0.594
	NRP	−1.485	−2.300	−0.674
	Tzomet	−1.760	−2.797	−0.696
	Thia	−1.418	−2.549	−0.287
Age	Shas	0.031	−0.024	0.091
	Likud	0.017	−0.024	0.066
	Labour	0.034	−0.466	0.988
	NRP	−0.059	−0.144	0.013
	Tzomet	−0.017	−0.078	0.045
	Thia	−0.042	−0.094	0.010
Education	Shas	−0.169	−0.443	0.094
	Likud	−0.183	−0.369	0.006
	Labour	−0.069	−0.272	0.126
	NRP	−0.063	−0.360	0.179
	Tzomet	−0.045	−0.274	0.215
	Thia	−0.042	−0.305	0.221
Religious Observation	Shas	3.163	2.133	4.141
	Likud	0.650	0.094	1.261
	Labour	0.235	−0.375	0.924
	NRP	2.856	1.936	3.997
	Tzomet	0.716	−0.113	1.683
	Thia	0.807	0.009	1.605
Correctly Predicted	Shas	0.261	0.157	0.371
	Likud	0.648	0.605	0.688
	Labour	0.504	0.454	0.552
	NRP	0.219	0.136	0.331
	Tzomet	0.046	0.023	0.076
	Meretz	0.449	0.373	0.515
	Thia	0.112	0.074	0.163
	Entire Model	0.525	0.506	0.544
$n = 505$		Log marginal likelihood = −597		

5

Elections in Italy, 1992–1996

5.1 INTRODUCTION

Understanding Italian politics in terms of coalition theory has proved very difficult. From the office-seeking perspective, the common occurrence of both minority and surplus coalitions during the 1970s and the 1980s seemed puzzling (Axelrod, 1980; Laver and Schofield, 1990; Strom, 1990). Other writers have been intrigued by the apparent instability of Italian coalition governments during this same period (Sartori, 1976; Pridham, 1987). The theoretical challenge has become even harder after the institutional upheaval of the early 1990s. So much has changed in terms of electoral rule, party alignment, and party composition that it has been hard to follow, let alone explain.

Recently, Mershon (1996a,b, 2002) has made a significant contribution to the study of Italian politics by combining a theoretical approach with careful data analysis. Our own theoretical model of multiparty politics is offered as an extension of Mershon's earlier work.

Different sources of data are used in this chapter. For party policy positions before 1996 we rely on the most updated version of the Comparative Manifesto Project (CMP) (Budge et al., 2001). The methodological status of the CMP data set, obtained via content analysis of party platforms, has been challenged on various grounds. First, the CMP research strategy is meant to ascertain salience of issues rather than party positions on those issues (Laver, 2001). Second, party positions derived from the content analysis of party platforms do not necessarily coincide with voter perceptions of these positions. We use the CMP analyses only to give an approximate indication of party positions prior to 1996. For the 1996 election, we use original data obtained by Giannetti and Sened (2004). These include mass and expert surveys. We believe that this methodological strategy is

better suited to determine parties' policy positions since it is based on expert judgments and voter perceptions, both of which can be represented by locations in the same policy space. As in Chapter 4, we use a visual approach to the data to make the complexities of Italian politics more readily explicable. This facilitates examination of the Italian political system with simple policy diagrams.

In Section 5.2, we give a systematic account of Italian electoral and coalition politics before 1992. In Section 5.3, we discuss the institutional revolution of the 1990s. Sections 5.4 and 5.5 interpret election and coalition formation following the 1994 and 1996 campaigns, respectively.

One preliminary remark immediately illustrates the advantage of our theoretical approach and will prove very useful for the discussion that follows. As in the case of Israel, our distinction between the two generic coalition structures is very helpful in modelling the transition from the "old" Italian politics that persisted until the early 1990s to the recent "new" Italian politics. The latter is characterized by a $\mathbb{D}_0$ coalition structure, where the core is empty, whereas the former was characterized by a $\mathbb{D}_1$ structure with a structurally stable core at the position of the dominant Christian Democrat (DC) Party. As we demonstrate in the sections that follow, this observation allows us to make sense of this transformation in Italian politics. We use this framework to illustrate the usefulness of the model in understanding such political transformations.

5.2 ITALIAN POLITICS BEFORE 1992

> Governments in Italy both change and remain the same. The Christian Democratic Party (DC) always held governing power. But almost no government stayed in office more than a few years, and many governments collapsed after only a few months. How can instability coexist with stability in this way? (Mershon, 1996a: 534)

> [T]he core Christian Democrat Party leads a dance with three or four partners often forming new governments after less than a year. The 1992 election and the appearance of the Lombardy/Northern League [NL] may have resulted in a major transformation in Italy, with the destruction of the core. (Schofield, 1993: 9)

The first question posed by Mershon (1996a) provides a central motivation for her work on politics in Italy for the period 1947 to 1987 (Mershon, 1996b, 2002). While the Christian Democrats (DC) headed every cabinet between 1946 and 1981 and was always in government until the election of 1992, government coalitions were typically unstable.

The average duration of minimum winning and surplus coalitions was 17 months and 9 months for minority coalitions, for the period from 1945 to 1987 (Laver and Schofield, 1990).

The model presented in Chapter 3 provides a straightforward solution to this puzzle. Laver and Schofield (1990) were the first to suggest that the DC simply occupied the core position from 1945 to 1987. They proposed a one-dimensional model, in which the core always exists and coincides with the party that controls the median legislator. Schofield (1995) then extended the model to a two-dimensional one where the structurally stable core coincides with the position of the largest party located at a central position. He called such a party *dominant*. The second quotation from Schofield (1993) reflects his observation that the changes in party strengths, and particularly the emergence of the Northern League (NL) in 1992, destroyed the dominance of the DC.

The following hypothesis is derived by Schofield (1995) and Sened (1996) based on an earlier version of the general coalition model presented in Chapter 3 above, and developed by Schofield and Sened (2002) and Giannetti and Sened (2004).

Hypothesis 5.1: *If the structurally stable core of the political game is nonempty and coincides with the position of the largest party, then this dominant party will always be a member of the government coalition.*

Figure 5.1 represents the estimates of party positions in 1987, based on the CMP data and using the technique given in Laver (2001). The two dimensions are an economic left–right dimension and a (vertical) liberal–conservative social dimension (partially based on religious attitudes).

In Figure 5.1, the "median" lines are given by the arcs such as

$$\{DC - PCI,\ DC - PSDI,\ DC - PRI,\ DC - PLI\}.$$

As mentioned before, a median line bisects the policy space, so that coalition majorities lie on either side of the line. These medians all intersect at the policy position of the DC. This property is a sufficient condition for DC to be located at the core position. Another way to see this is to consider the convex compromise sets associated with winning coalitions. The DC position in Figure 5.1 belongs to the convex compromise set associated with the winning coalition

$$\{PCI, PSI, PSDI, PRI, PLI\}$$

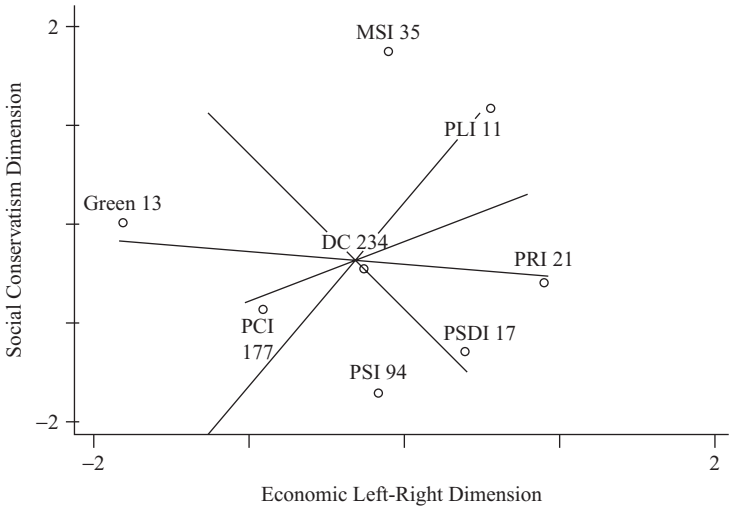


Figure 5.1. Party policy positions (based on the manifesto data set) and party seat strength in Italy in 1987.

If the DC position lay outside this set, then this large, though somewhat unlikely coalition, could theoretically agree to a policy position different from that of the DC. Assuming the DC position did indeed belong to the larger coalition compromise set, then it follows that bargaining between the parties will result in the DC obtaining the policy position that it had chosen (Schofield, 1995; Sened, 1996; Banks and Duggan, 2000). Moreover, this conclusion is not affected by small perturbations of party positions. Thus DC can be seen to be a core party, located at the structurally stable core position (Giannetti and Sened, 2004).

If the results obtained for 1987 could be generalized, it is plausible to argue that a fundamental underlying $\mathbb{D}_1$ coalition structure characterized Italian politics until 1992. It is our understanding that the $\mathbb{D}_1$ structure, illustrated in Figure 5.1, was typical of Italian politics during the entire period between 1946 and 1992. This explains the otherwise puzzling apparent coalition instability combined with outcome stability noted by Mershon (1996a,b, 2002).

The model does not explain the phenomenon of short-lived coalition governments in Italy. To date, no comprehensive model of government termination has been elaborated in the formal literature (Laver, 2003). In her study of coalition politics in Italy, Mershon (2002) offers the low costs of "making and breaking governments" by Italian political parties as a plausible explanation for constant government turnover. We suggest that

because the DC was positioned at the core, it was able to implement its policy, even through minority government when it so chose. On occasion it would form minimal winning or surplus coalitions in order to placate other parties in the Chamber of Deputies with nonpolicy perquisites. The dominance of the DC disappeared in the election of 1992.

5.3 THE NEW INSTITUTIONAL DIMENSION: 1991–1996

In the early 1990s, Italian politics experienced a dramatic change. Corruption scandals shook the Italian political elites. A political crisis resulted and a major institutional revolution followed, changing the entire electoral system after almost 40 years of PR. This marked the beginning of what has been called the "Second Italian Republic." This prompted an extensive literature on the "Italian transition." (See for instance, Bartolini and D'Alimonte, 1995; D'Alimonte and Bartolini, 1997).

The first and most notable change affected the identity and the set of relevant actors. Old parties either disappeared or went through major transformations in ideologies and electoral strategies. New parties emerged or split off from old parties. The main changes in parties' identities between 1991 and 1996 are discussed below.

The Communist Party (PCI) transformed into the Democratic Party of the Left (PDS), splitting off from the "far left" RC.

On January 18, 1994, the last National Assembly of the DC was held. The party renamed itself Partito Popolare Italiano (PPI). A right-wing faction, Centro Cristiano-Democratico (CCD), split off. Between 1994 and 1996, the Socialist Party (PSI) and other center parties

$$\{PSDI, PRI, PLI\}$$

(that had systematically formed the pentapartito coalition governments with DC in the 1980s) dissolved. The PSI dropped from a vote share of 13.6 percent in 1992 to a vote share of 2.2 percent in 1994.

In January 1994, Forza Italia (FI) led by the media magnate Silvio Berlusconi formed, just a few months before the elections. In January 1995, the fascist party, the Movimento Social Italian (or MSI) transformed into Alleanza Nazionale (MSI-AN), generating a splinter party, MSFT, on its right. Figure 5.2 provides a simplified, graphic presentation of this major party realignment that is but one aspect of this major transformation of the Italian political landscape in the late 1980s and early 1990s.

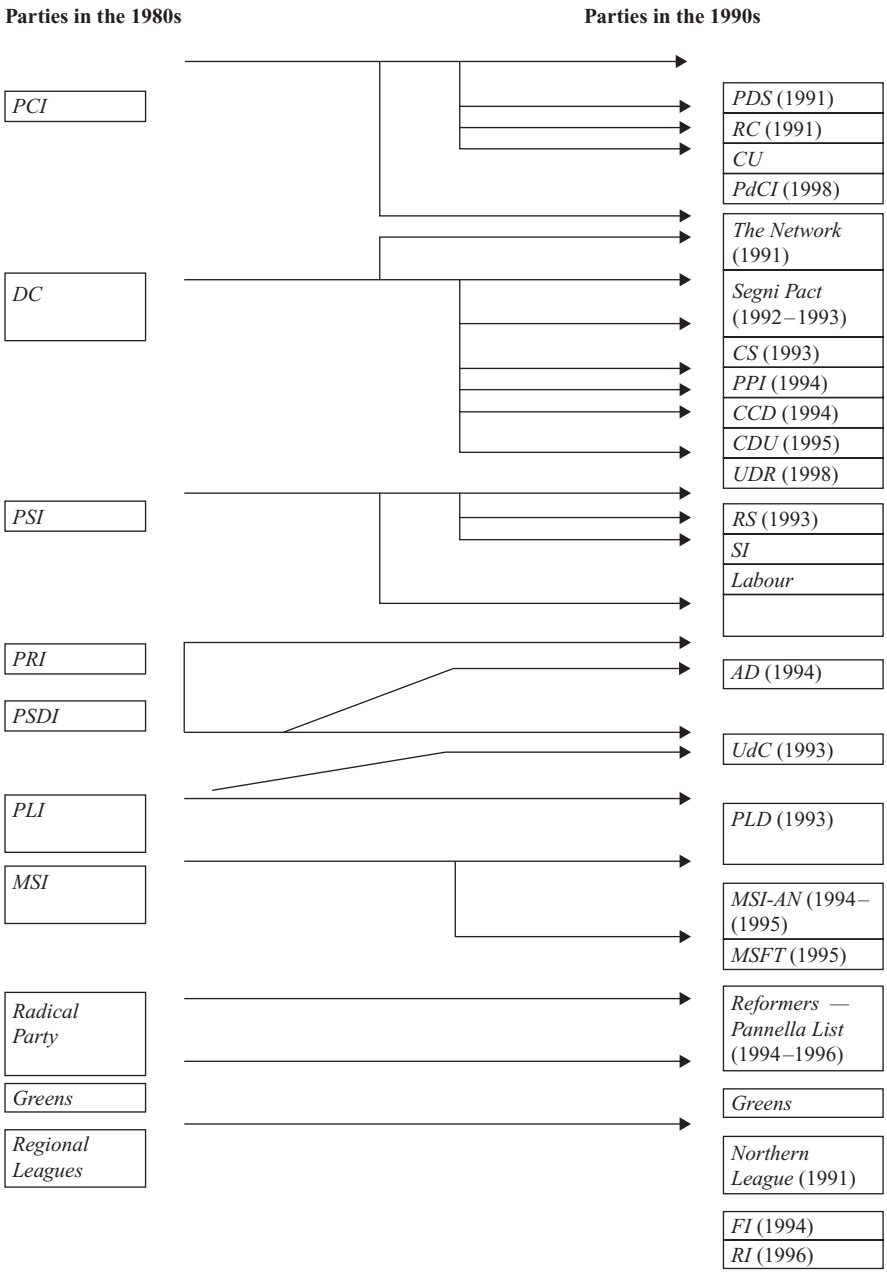


Figure 5.2. Changes in the political party landscape between the 1980s and 1990s.

Elections in Italy, 1992–1996

Table 5.1. *Italian Elections: Votes/Seats in the Chamber of Deputies, 1987–1996.**

	1987		1992		1994		1996	
Parties	Vote share %	Seats	Vote share %	Seats	Vote share %	Seats	Vote share %	Seats
DP	1.7	8	–	–	–	–	–	–
RC	–	–	5.6	35	6.0	39	8.6	35
Greens	2.5	13	2.8	16	2.7	11	2.5	14
PCI-PDS	26.6	177	16.1	107	20.4	125	21.1	175
Network	–	–	1.9	12	1.9	6	–	–
PSI	14.3	94	13.6	92	2.2	14	–	–
PSDI	2.9	17	2.7	16	–	–	–	–
PRI	3.7	21	4.4	27	–	–	–	–
AD	–	–	–	–	1.2	18	–	–
DC-PPI	34.3	234	29.7	206	11.1	33	6.8	70
Segni Pact	–	–	–	–	4.7	13	–	–
PLI	2.1	11	2.9	17	–	–	–	–
RI	–	–	–	–	–	–	4.3	26
PR-Pannella List	2.6	13	1.2	7	3.5	–	1.9	0
Northern League (NL)	0.5	1	8.7	55	8.4	117	10.1	59
CCD	–	–	–	–	–	29	5.8	19
CDU	–	–	–	–	–	–		11
FI	–	–	–	–	21.0	111	20.6	123
MSI-AN	5.9	35	5.4	34	13.5	109	15.7	93
Others	2.9	6	5.0	6	3.4	5	2.7	5
Total	100	630	100	630	100	630	100	630

* *Note*. In order to compare electoral results before and after electoral reform in 1993, the results given for 1994 and 1996 refer to PR contests only.

Table 5.1 shows the vote shares of the main party lists and their respective seat weights in the chamber between 1987 and 1996. The 1992 election gave the first indication of the coming transformation. The popular vote for the DC fell below 30 percent and the main beneficiary of shifting voter choice was the Northern League (NL), a federation of regionalist groups that won 8.7 percent of the national vote and 55 seats. NL became the second most popular party in Northern Italy (with 20.5 percent compared with 25.5 percent for the DC).

We can illustrate the effect of this election with Figure 5.3, which develops the idea about the destruction of the core proposed by Schofield

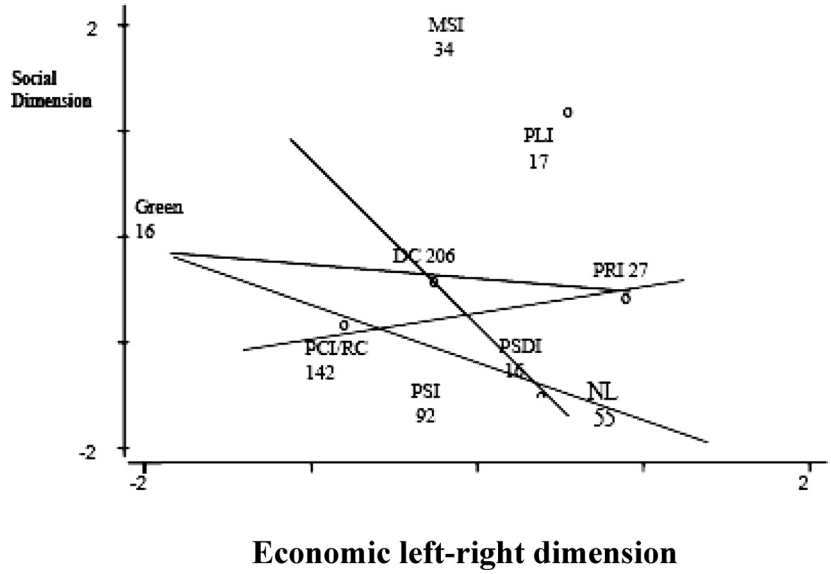


Figure 5.3. Hypothetical party policy positions and seats in Italy in 1992.

(1993). Assuming the traditional parties were positioned as they were in 1987, and the NL was positioned in the southwest of the figure, then the coalition {PCI/RC,PSI,PSDI,PRI,NL} obtained a majority of 332 seats. (In the figure, the position marked PCI is taken to represent both PCI, with 107 seats, and the RC with 35 seats.) More importantly, the compromise set of this coalition no longer contained the DC position. Another way of showing this is that there is a new median line from the PRI to the PCI that does not intersect the DC position. Thus, the DC was no longer at a core position, and was therefore no longer a dominant party. This suggestion is of course somewhat hypothetical, but it accords with the changes that were to come.

These changes were accompanied by a transformation in the perceptions of the defining features of Italian politics. The emergence of a North–South dimension, partially overlapping with the issue of corruption, is central. This *institutional dimension,* as we refer to it here, is really a compound one, composed of demands for federal reforms led by the Northern League and the reactive proposals by the establishment parties for electoral reforms. These competing calls for reform evolved in an environment pervaded by judicial investigations of political corruption.

In a "heresthetic move" (Riker, 1986), Umberto Bossi, leader of the NL, put the North–South issue on the political agenda in the late 1980s.

A socioeconomic North–South divide had preceded the formation of the unitary state (Putnam, 1993). The strategy of the Northern League reversed the traditional *questione meridionale* ("the Southern issue") into a Northern issue, putting the demand for federal reform at the center of the political agenda. This strategy is central in four of the Northern League's electoral campaign issues in the early 1990s. First, there was the fight against disproportionate party power (*partitocrazia*), which was regarded as the source of patronage, clientelism, and corruption. Second, the League's anti-southern stand was tied to the common perception of the inefficiency of public services in Southern Italy. Third, the League's anti-immigrant stance was related to the influx of third-world illegal immigrants from the South. Finally, the *partitocrazia* was portrayed by the League as ineffective in dealing with the mafia, following accusations that the party establishment relied on the mafia to govern the South (Leonardi and Kovacks, 1993).

The Northern League's recreation of the North–South dimension can be seen as an example of the transformation of policy dimensions. In the same way, the issue of race and civil rights in the United States has the capacity to alter "the political environment within which [it] originated and evolved ... replacing one dominant alignment with another and transforming the character of the parties themselves" (Carmines and Stimson, 1989: 11; Miller and Schofield, 2003).

As a reaction to the reemergence of the North–South tensions, leaders of the winning majority attempted to bring about more accountable democratic institutions. The Christian Democrat leader Mario Segni championed a referendum on reducing the number of preferential votes in parliamentary elections, allegedly associated with corrupt vote trading in the South. (The electoral law allowed voters to express up to four preferential votes for candidates in the party lists.)

On June 9, 1991, the multiple-preference vote procedure was discontinued by an overwhelming majority of 95.6 percent. After the success of the 1991 referendum, a new referendum committee was set up to abolish clauses of the existing electoral law for the Senate. On April 18 1993, 82.7 percent of voters cast their ballot for change. On August 1993, a Parliament still dominated by the old political elite, approved a new electoral law at the national level. Italy switched from an almost pure PR system to a mixed system that allocated 75 percent of the seats by plurality and only the remaining 25 percent by PR.

Thus, the North–South tension reintroduced by Bossi and the Northern League was transformed into a new dimension of institutional change that

reshaped political competition and brought about new party alignments. The general issue of reform was central in that a strong demand for change determined a transformation of the rules of political competition, which then contributed to the reshaping of the entire party system.

On these grounds, our a priori assumption that the institutional dimension is most relevant for understanding Italian politics from the early to mid-1990s seems justified.

In the next two sections we return to a close examination of the theory in the context of the two electoral campaigns that followed. A central theme in this elaboration is Schofield's (1993) notion of the "evaporation of the core" of Italian politics. We contend that this transformation has similarities to the changes in Israel described in the Chapter 4. The transition was from a $\mathbb{D}_1$ coalition structure, with the dominant or core DC party at its center, so characteristic of Italian politics from 1945 to 1987, to a $\mathbb{D}_0$ structure, with an empty core. This has had a profound effect on the nature and dynamics of Italian politics in the 1990s. Our analysis of the 1994 and 1996 elections illustrates this observation.

5.4 THE 1994 ELECTION

The introduction of a new dimension to the issue space of Italian politics, coupled with the demise of old parties and the emergence of new ones, led to a significant transformation of Italian politics to a parliamentary system characterized by a $\mathbb{D}_0$ structure, where the core is empty. Our theory suggests that the expected set of outcomes is typically characterized by the policy heart of the Parliament. This means less stability in the outcome space and a very different type of political game. We no longer expect "policy stability" through the exercise of power by the dominant DC party. Instead we expect policy instability as each governing coalition is replaced with one of a very different composition. Indeed, we might expect a degree of political chaos, reminiscent of the formal results on voting.

5.4.1 *The Pre-Election Stage*

In March 1994 Italy had its first election under the new electoral system. (A discussion of this election, and the one following in 1996 can be found in Corbetta and Parisi, 1997). The plurality part of the new electoral law sets up a coalition-formation phase before, rather than after, the election. Parties form pre-electoral coalitions, declare common policy packages to be implemented once in government, and bargain over the allocation of

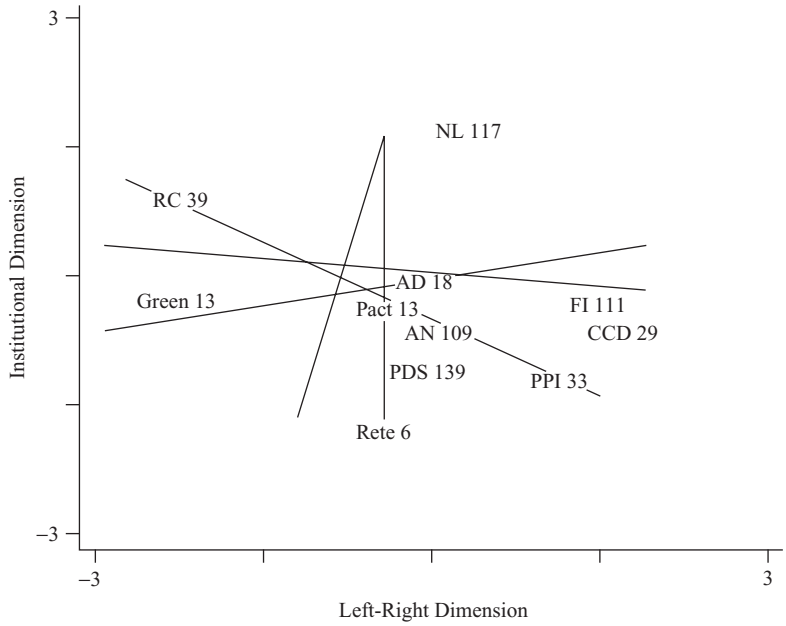


Figure 5.4. Party policy positions and seats in Italy after the 1994 election.

seats. But the PR tier still gives parties a strong incentive to maintain separate policy positions.

The parties' positions in Figure 5.4 for 1994 were estimated from CMP data. A left–right scale was constructed from parties' scores on economic and social issues. (Party positions may appear at variance with common perceptions as far as the AN is concerned. The "low" score of this party on the left–right dimension may be partially explained by the fact that the MSI has always been more of a populist than a "Thatcherite" right-wing party. While expert and mass surveys data commonly agree on placing the party at the extreme right of the scale, estimates obtained from content analysis of party manifestos between 1946 and 1996 suggest that our estimate of the party location may be quite accurate.)

We operationalized the "institutional dimension" as party scores on issues of decentralization. The NL scored the highest on this dimension. Notice that the vertical dimensions is Figures 5.1 and 5.4 are quite different.

In 1994, four pre-electoral coalitions, Progressisti on the left, Patto per l'Italia at the center, and Polo delle Libertà and Polo del Buon Governo on the right, contested the plurality component. They are best seen as

mere electoral alliances. Parties agreed on the presentation of common candidates in the districts but did not campaign on a common policy platform.

Progressisti was composed of Partito Democratico della Sinistra (PDS), Regio Calabria (RC), Greens, La Rete (The Network), various factions of the PSI, minor left parties, and the new movement of moderate left, Democratic Alliance (AD). The members of the Progressisti alliance issued a brief joint document. The campaign revealed clear differences between their policy positions.

DC was divided in three: the Popolari per la Riforma, founded by Segni, the Partito Popolare Italiano (PPI), and the right-wing faction, Centro Cristiano Democratico (CCD). The Northern League explored the possibility of reaching an agreement with Segni. The failure of this agreement on January 24, 1994, marks the end of the attempts to unite the center political forces. Eventually, PPI and Segni formed the electoral alliance Patto per l'Italia.

On January 24, Berlusconi launched the new political movement, Forza Italia (FI) on a liberal right program, advocating lower taxes, fiscal federalism, and direct election of the head of the state. Berlusconi formed two electoral alliances: one with the Northern League in the North (Polo delle Libertà), and the other with the MSI-AN in the South (Polo del Buon Governo). In the North, AN contested the elections on its own. The Northern League (NL) did not run in the South. The NL managed to stress its policy differences with FI. Bossi was confident that NL would defeat FI on the PR ballot and could dictate institutional reforms to the new government. In Southern Italy, FI allied with AN, while MSI-AN downgraded its policy differences with FI. Despite the project of a radical renovation launched by secretary Fini in January 1994, the AN was still very conservative on the institutional dimension, positioning itself as a radical party, supporting national unity, and against federalism, though still stressing its antiestablishment stance.

5.4.2 *The Electoral Stage*

The elections resulted in a major transformation of the political scene. Most striking was the success of FI, a party that did not exist just months before the election. FI became the leading national party with 21 percent of the vote, which translated into 15.7 percent of the seats. The NL kept its vote share close to its 1992 share. Thanks to the pre-electoral agreement that gave 63.4 percent of single-member districts in Northern and Central

Italy to the NL candidates, the NL became the largest parliamentary party, with 18.6 percent of the seats in the Chamber but only 8.4 percent of the vote. AN more than doubled the electoral strength of the former MSI (from 5.4 percent to 13.5 percent). The splinter factions of the former CD ended up with roughly half of the vote (15.8 percent) that they had in 1992. The translation of votes into seats further penalized the centrist alliance, which ended up with only 7.3 percent of the seats despite having a PR vote share of 15.8 percent. Table 5.2 displays the result of the 1994 elections. For the sake of the discussion we divided parties into three blocks: Progressisti (left), Patto per l'Italia (center) and Polo (right). We also give seat subtotals for the PDS and FI groups.

We do not have a good data set to model voter choice for this election. We present the results of the election in Table 5.2 for the sake of completeness and without further interpretation.

5.4.3 *The Coalition Bargaining Game*

Following the 1994 election, FI, AN, NL, and CCD formed a winning coalition controlling 366 seats: 111 for FI, 117 for NL, 109 for AN, and 29 for CCD. Again, a coalition is *minimal winning* (MW) if it is winning but cannot lose a coalition member and remain winning. Thus this coalition is MW if the CCD, which contested the election under the FI label, is counted as part of FI. CCD formed a parliamentary group after the election. If we count CCD as a distinct party the coalition is not MW.

In the Senate, the coalition was short of a majority, controlling 156 seats out of 315. It passed the investiture vote due to the defection of four PPI deputies who voted in its favor.

Figure 5.4 shows the fundamental change that took place in the structure of the Italian Parliament: The core is now empty. The intrinsic instability of this structure sheds some light on the puzzling question of why Bossi decided to withdraw his support from the Berlusconi government after only 8 months, although NL was overrepresented in Parliament and controlled five ministers, including Budget and Constitutional Reform. From a pure office-seeking perspective, it is possible to argue that the legislative weights' distribution, which made the NL a pivotal party, and the actual allocation of ministerial positions, gave the party a strong incentive to defect (Giannetti and Laver, 2001).

An alternative explanation of the NL strategy relies on future electoral concerns. The European elections, held under the PR electoral system on June 12, 1994, can be regarded as an important event that provided

Table 5.2. *The 1994 Election Results in Italy: Chamber and Senate.**

	Chamber					Senate		
Party Blocks	Parties	PR Vote Share %	Plurality Vote Share %	Seats	Seats %	Vote Share %	Seats	Seats %
	RC	6.0	–	39	6.2	–	18	5.7
	PDS	20.4	–	109	17.3	–	60	19.0
	PSI	2.2	–	14	2.2	–	12	3.8
	Ind Left	–	–	10	1.6	–	4	1.3
	CS	–	–	5	0.8	–	4	1.3
	RS	–	–	1	0.2	–	1	0.3
	Total PDS	–	–	139	22.1	–	81	25.7
	AD	1.2	–	18	2.9	–	10	3.2
	Greens	2.7	–	11	1.7	–	7	2.2
	Network	1.9	–	6	1.0	–	6	1.9
Progressi (Left)	Total	34.4	32.8	213	33.8	32.9	122	38.7
	PPI	11.1	–	33	5.2	–	30	9.5
	Segni/Pact	4.7	–	13	2.1	–	1	0.3
Patto (Center)	Total	15.8	15.6	46	7.3	16.7	31	9.8
	FI	21.0	–	99	15.7	–	32	10.2
	UdC	–	–	4	0.6	–	3	1.0
	PLD	–	–	2	0.3	–	–	–
	Reformers	–	–	6	1.0	–	1	0.3
	Total FI	–	–	111	17.6	–	36	11.5
	Northern League	8.4	–	117	18.6	–	60	19.0
	AN	13.5	–	109	17.3	–	47	14.9
	CCD	–	–	29	4.6	–	12	3.8
	Pannella	3.5	–	–	–	–	1	0.3
Polo (Right)	Total	46.4	46.2	366	58.1	42.7	156	49.6
	SVP	0.6	–	3	0.5	0.7	3	1.0
	PvdA		–	1	0.2	0.1	1	0.3
	LdAM	0.2	–	1	0.2	–	–	–
	Others	2.6	4.3	–	–	6.9	2	0.6
Total All		100.0	98.9	630	100.0	100.0	315	100.0

* *Source*: Data derived from Ministry of Interior and political parties.
Note: These results differ slightly from those of Bartolini and D'Alimonte (1995) due to different aggregation procedures being used.

parties critical information about shifting voter choice. The NL's support fell to 6.6 percent of the national vote compared with FI's 30.6 percent. The NL faced the serious prospect of being absorbed by FI, which created a strong incentive for the NL to ask for earlier national elections.

From our theoretical perspective, the plausible explanation to Bossi's move is that, following his defeat in the European elections, he realized that the policy implemented by the FI-led government was too far from the declared position of NL. The office-related perquisites were no longer enough to compensate for the deviation from the NL ideal point. This also explains why NL adopted a more radical stance inside the government, and eventually, on December 17, advanced a motion of no confidence against the government; this motion was also signed by the PPI. Berlusconi's attempts at keeping a parliamentary majority failed. On December 22, 1994, Berlusconi resigned. The head of the state entrusted Dini, former Treasury Minister in Berlusconi's cabinet, with the formation of a new government. Dini's cabinet was non-partisan. All ministers were professionals with no parliamentary affiliation, including the Prime Minister himself. But the government was supported by a parliamentary majority that included center-left parties plus the NL. On January 25, 1995 the Dini cabinet carried the vote of confidence: 302 voted in favour (PDS, PPI, NL), 39 opposed (RC), and 270 abstained (FI, AN, CCD, plus 5 deputies of the NL). Then on February 1, Dini carried the confidence vote in the Senate: 191 voted in favour (PDS, PPI, NL), 17 opposed (RC), and 2 abstained (1 NL and 1 AN). The senators of the Polo (FI, AN, CCD) did not take part in the vote as a sign of protest.

The Dini cabinet lasted about a year. Facing 13 no-confidence votes and resorting quite often to restrictive procedures such as emergency decrees, Dini eventually resigned in January 1996.

According to the theory offered in Chapter 3, the transformation to a $\mathbb{D}_0$ coalition structure with empty core results in a set of policy outcomes – the heart of the Parliament. Since possible outcomes are associated with lotteries over this set, one can expect coalition instability. Indeed, two coalitions lasted less than a year each. This was not uncommon in Italian politics, even prior to 1992. What is new, and what we can attribute to the shift to a $\mathbb{D}_0$ structure, is that the consecutive coalitions were different in composition and in policy goals.

Just as the $\mathbb{D}_1$ structure typified Italian politics up until 1987, so does it appear that the more unstable $\mathbb{D}_0$ structure will characterize politics in the future. Certainly, it appears unlikely that the PDS or FI will receive sufficient electoral support to become dominant parties.

Our analysis of the 1996 election in the next section shows that no party became a dominant, core party. Indeed, the analysis indicates that, in this election, the centrifugal forces associated with factionalized vote maximizing became even more important.

5.5 THE 1996 ELECTION

For the 1996 election we obtained survey data from attitudinal questions. Just as in Chapter 3, the data were analyzed using exploratory and then confirmatory factor analysis. The analysis yielded two underlying factors. One factor is related to questions on the future institutional design of Italy. The other is the common left–right dimension (but with the commonly observed new twist, in Europe, of issues related to foreign workers and postmodernist moral values). Just as in the analysis of Israel, the questions that related to these two factors were given to experts on Italian politics, who were asked to answer the questions as the party leaders would. The responses allowed us to locate the parties in the same policy space used to represent voters' opinions. Figure 5.5 displays the distribution of the Italian electorate and the positions of the parties.

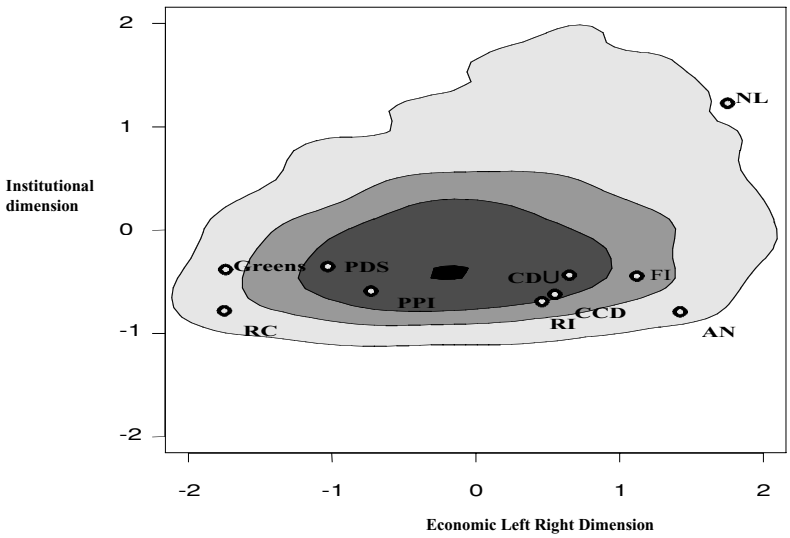


Figure 5.5. Distribution of Italian voter ideal points and party positions in 1996. The contours give the 95%, 75%, 50%, and 10% highest density regions of the distribution.

5.5.1 The Pre-Election Stage

The 1996 election saw significant changes in the formation of pre-electoral coalitions. In line with Duverger's (1954) famous prediction, only two pre-electoral coalitions formed: center left and center right. More importantly, parties that formed electoral coalitions did not issue their own electoral platforms but subscribed to joint platforms. But parties were still the most important actors in the pre-electoral and post-electoral legislative game.

The center-left coalition, Ulivo, consisted of PDS, PPI, Greens, center, socialist, and local parties. RC was no longer a member of the left alliance but made electoral agreements to avoid contesting some plurality seats. RC supported candidates of the Ulivo except in two districts; the Ulivo supported candidates of RC in 27 single-member districts for the election of the Chamber and 17 single-member districts for the election of the Senate. RC ran the elections with its own electoral platform and declared that it would not take part in the future government in the event of a victory of the left. On the other hand, the Ulivo claimed that the electoral agreement with RC would make it easier to gain a "self-sufficient" parliamentary majority.

Before the election, a new party, RI, led by Dini, joined the Ulivo coalition. The political debate about the meaning of the Ulivo coalition highlights political actors' electoral strategies, given the incentives set up by the new electoral law.

Trying to position the PDS at the center of the policy space, the new secretary D'Alema made clear that the PDS could aspire to rule Italy only if it detached itself from the neocommunists and joined forces with the PPI. On the other hand, according to prospective Prime Minister, Prodi, and other prominent political leaders, the Ulivo was to be seen as the first step in the process of federating center-left political groups, leading eventually to a unified party. Once in government, Prodi declared:

> The government that today is going to ask the investiture vote is aware that this Parliament is profoundly different from the previous ones. For the first time, the electoral competition has not been dominated by distinct parties or mere electoral alliances but by two coalitions, that campaigned on their own distinct platforms in order to rule the country. . . . This government will be bound to the program that was submitted to the electorate. . . . It is not incidental that the head of the state wanted to point out the political novelty of the electoral competition receiving not parties' delegations but the two coalitions' delegations. (*Atti Parlamentari*: May, 22, 1996).

We may interpret this as an attempt to recreate a dominant party.

Following a similar strategic plan, Dini, the leader of RI, attempted to position himself at the median position on the relevant dimensions. Eventually Dini allied with the left. Dini's party ended up pivotal to the coalition of the left. As Table 5.3 shows, the left coalition, if combined with RI, attained a majority. If RI joined the right, the coalition of the right would still have remained a minority. It is plausible that Dini joined the left for this reason. As he himself declared: "Without us the Ulivo will not win. Prodi may capture those voters who sympathize with the PDS already. It is RI that will capture the center electorate. We are the surplus value of the coalition" (quoted in Giannetti and Sened, 2004).

On the right, FI and AN consolidated the 1994 alliance forming Polo della Libertà, which for the first time ran candidates nationwide. MSI-AN renamed itself AN in 1995 and for the first time declared its commitment to decentralization and privatization. The fact that AN moved toward the center can be inferred also from the birth of a splinter on its right, MSFT. The AN position on both dimensions was closer to FI than in 1994. This must have helped consolidate the Polo coalition. The other two members of the Polo coalition were CCD and CDU, both splinters of the PPI.

The NL refused any alliance and contested the elections separately. According to Diamanti (1997), "The 1996 election is a turning point in the Northern League political strategy." The key word was no longer "federalism" but "secession." The leader, Bossi, presented the 1996 election as a referendum on the "independence of Northern Italy," claiming that the NL was the only force capable of fighting against the resurgent *partitocrazia* and of defending the interests of the North. The creation of the "Parliament of the North" and the organization of mass demonstrations in favor of the "independence of Padania" highlight this strategic change. As Figure 5.5 illustrates, the NL positioned itself at an extreme pro-federalist position on the institutional dimension.

We speculate that it may have positioned itself hoping to be pivotal between a center-left and a center-right coalition. Given the complexities of the electoral system, a tie between the two coalitions was probable. If this is a correct interpretation of the NL position, then it parallels our inference about the strategic maneuvering of Shas in the case of the 1992 and 1996 elections in Israel. As Table A5.1, shows, the NL had the lowest valence of all the parties. With Ulivo and Polo positioned near the electoral center, and both coalitions led by high-valence parties, the NL would be at a vote-minimizing position anywhere near these parties. We suggest that its strategy was to attempt to achieve two goals. First, by adopting a position to the "north" in Figure 5.5, it affected the location

Elections in Italy, 1992–1996

Table 5.3. *The 1996 Election Results in Italy: Chamber and Senate.**

		Chamber				Senate		
Party Blocks	Parties	PR Vote Share %	Plurality Vote Share %	Seats	Seats %	Vote Share %	Seats	Seats %
RC		8.6	2.6	35	5.6	2.9	11	3.5
Ulivo	PDS	–	–	139	–	–	–	–
	Ind. Left	–	–	12	–	–	–	–
	CU	–	–	8	–	–	–	–
	Labour	–	–	6	–	–	–	–
	CS	–	–	5	–	–	–	–
	Network	–	–	5	–	–	–	–
	Total PDS	21.1	–	175	27.8	–	98	31.1
	PPI	–	–	61	–	–	–	–
	UD	–	–	6	–	–	–	–
	PRI	–	–	2	–	–	–	–
	UL	–	–	1	–	–	–	–
	Total PPI	6.8	–	70	11.1	–	32	10.2
	RI-Dini List	–	–	10	–	–	–	–
	Segni Pact	–	–	9	–	–	–	–
	SI	–	–	7	–	–	–	–
	Total RI	4.3	–	26	4.1	–	11	3.5
	Greens	2.5	–	14	2.2	–	14	4.4
	PsdA	0.1	–	–	–	–	1	0.3
Ulivo	Total	34.8	42.2	285	45.2	41.2	156	49.5
Local	SVP	–	0.4	3	0.5	0.5	2	0.6
	PvdA	–	0.1	1	0.2	0.1	1	0.3
	Total Local Parties		0.5	4	0.7	0.6	3	0.9
Left: Ulivo+ RC	Total	43.4	45.3	324	51.4	41.8	170	53.9
	CCD	5.8	–	19	3.0	–	16	5.1
	CDU	–	–	11	1.7	–	9	2.9
	FI	20.6	–	123	19.8	–	47	14.9
	AN	15.7	–	93	14.6	–	44	14.0
Polo	Total	42.1	40.3	246	39.0	37.3	116	36.8
Right	LdAM	0.2	0.2	1	0.2	0.2	–	–
	MSFT	0.9	1.7	–	–	2.3	1	0.3
	Pannella List	1.9	0.2	–	–	1.6	1	0.3
Polo+ Right	Total	45.1	42.4	247	39.2	41.4	118	37.4
	NL	10.1	10.8	59	9.4	10.4	27	8.6
	Other Lists	1.4	1.5	–	–	3.5	–	–
Total		100.0	100.0	630	100.0	100.0	315	100.0

* *Source*: Data derived from Ministry of Interior and political parties.

Note: Note that these electoral results differ slightly from thse of Bartolini and D'Alimonte (1997) due to different aggregation procedures being used.

of the heart of the Italian polity, moving it further north in the literal sense of the words. Second, it may have chosen this extreme position in order to affect its expected reward from coalition government. In the illustration of our theoretical model in Chapter 3, we attributed similar motives designed to extract more office-related perquisites by the orthodox religious party Shas in the 1992 and 1996 elections in Israel. We believe that this model provides a general explanation for the puzzling, but recurrent, phenomenon of extremist parties in coalitional polities adopting positions that are more radical than those their voters actually support.

5.5.2 *The Electoral Stage*

Italian politics remains very factionalized, and the new-found institutional structures will take time to mature. The electoral centers of the two coalitions, Ulivo and Polo, are not sufficiently powerful to create strong centripetal forces in the system. Our interpretation of the 1996 electoral results is that the high-valence pre-electoral bi-coalitional struggle at the center provided the motivation for low-valence parties to head to the periphery of the electoral distribution. This phenomenon, which we can call the *centrifugal tendency* is clearly illustrated in Figures 5.5 and 5.6. It is also apparent in the electoral results themselves.

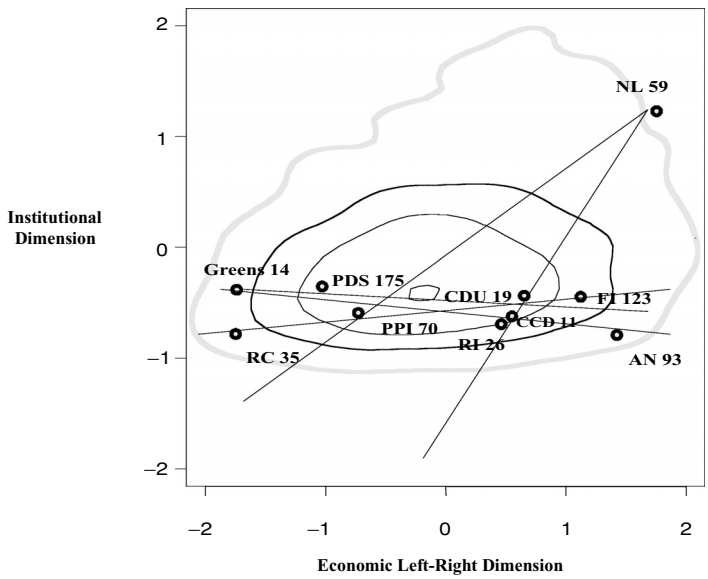


Figure 5.6. Party policy positions and the empty core following the 1996 election in Italy.

Elections in Italy, 1992–1996

Table 5.3 reports the electoral results for the 1996 election in Italy, both for the Chamber and the Senate. In the Chamber, Ulivo took 42.2 percent of the vote on the plurality ballot and 34.8 percent on the proportional ballot. This vote share translated to 285 seats (45.2 percent). RC got 8.6 percent of the vote on the proportional ballot and 35 seats (5.6 percent). With several minor local parties, the center-left coalition controlled a total of 324 seats (51.4 percent).

The Polo coalition obtained 40.3 percent of the vote on the plurality ballot and 42.1 percent on the proportional ballot. This vote share translated into a total of 246 seats (39 percent). The NL actually raised its vote share to 10.8 percent of the national vote on the plurality part and 10.1 percent on the PR component (from the 8.4 percent PR it had in 1994). This electoral success translated into a total of only 59 seats (9.4 percent). Thus, in spite of its electoral success, the NL was unable to play a pivotal role between left and right in the coalition bargaining game that followed. Similar to the mistake made by Shas in the elections of 1992, the NL may have gone too far with its strategy of secession, allowing the center-left coalition to obtain enough seats to form a coalition without it. By refusing to form pre-electoral coalitions with any of the two major pre-electoral coalitions, it paid a heavy price in getting very little out of the by-now dominant share of the seats obtained by plurality.

The results of an MNL estimation for the election are shown in Table A5.1 in the appendix. As in the analysis for Israel, the empirical model includes sociodemographic (SD) parameters. The effects for age and education that have so greatly preoccupied previous studies of vote choices in Italy (e.g., Ricolfi, 1993; Corbetta and Parisi, 1997) appear insignificant. (Significance is based on the 95 percent confidence intervals reported in the two columns on the right of the table. Because zero belongs to this confidence interval for the age and education coefficients, for all parties, we cannot reject, at the 95 percent level, the hypothesis that these parameters are indeed zero.) This does not imply that these variables do not have a causal effect. As in our analysis of the Netherlands in Chapter 6, we infer that the voter sociodemographic variables partially influence beliefs, but the beliefs (or voter ideal points) are predominant in characterizing voter choice.

Three important aspects of the voter choice in Italy come out very clearly from Table A5.1. First, as in our other tests of the model, party policy positions were the most important factor in explaining vote choice in Italy in the 1996 election. This can be seen from the confidence interval on the spatial coefficient, β. Second, the party constants, interpreted

throughout the book as measures of party valence, are all significantly different from zero. The fact that they all have negative signs is easy to interpret. These constants are all relative to the valence score of the RC, which is normalized to be zero. In terms of the formal model, the important comparison is between the lowest valence (namely that of NL) and the valence of RC. This difference is clearly statistically significant. It is also relevant that the estimated valence of NL is not contained in the confidence intervals for the valences of the PDS and FI. This lends support to our theoretical argument that low-valance parties will position themselves at the electoral extreme, in any vote-maximizing equilibrium. In other words, a party such as the NL should rationally avoid competition with the high-valance parties. Here, as in Israel, these parties eventually counter the centripetal forces of the electoral system by leading the more centrist parties to move away from the center to better compete with parties at the periphery. In light of the political discussion in Italy prior to the 1996 election over the importance of capturing the center and creating a dominant party, it is interesting that low-valance parties like the Greens, the NL, and the AN exert strong centrifugal pressure on the entire political system, forcing even the parties regarded as centrist to move away from the center. In this respect it is worthwhile to compare the party policy positions maps of 1994 and 1996. These two maps are not directly comparable because of the different methods of estimation. But general trends can be observed. The AN appears to have moved out to the right while the declared intentions of the PDS and FI to move to the center were checked by the AN on the right, the Greens and RC on the left, and the NL to the north.

The pull of the NL toward the north seems so much more powerful once one observes the remarkable relative advantage of the NL in Italy's northeast, northwest and central geographic regions. These are demonstrated by the very large positive estimates for these SD parameters for the NL (see Table A5.1 for these regions). While the 95 percent confidence intervals include zero, the parameters are significant at the 90 percent level.

The fact that the model does not seem to predict the vote choice of individual voters is not particularly significant. To expect a statistical model to predict the vote choice of the Italian voter among nine different parties is a little too much to ask. The relative success of the model in predicting the vote choice for the PDS, FI, and NL suggest that the problem stems from the complexity of the computation and estimation effort required rather than any misspecification of the model itself.

Elections in Italy, 1992–1996

Before considering the coalition game we first note that vote maximizing should lead to convergence in Italy for the election in 1996, at least on the basis of exogenous valence as developed in Theorem 3.1. The high valence difference between the lowest valence party, NL, and RC was 20.1 for the election. With all parties at the electoral mean, the probability of voting for NL is zero. Since the spatial coefficient $\beta = 0.21$ and the total electoral variance is 1.50, with negligible electoral covariance, we obtain a value for the convergence coefficient of $c = 0.6$, well below the sufficient bound of 1.0. On the other hand, the very high SD coefficients for the NL in the nothern regions of Italy suggest that the party should adopt a vote maximizing position that was radical on the institutional axis. By doing so, it will increase its vote share from zero. Although this does not follow directly from Theorem 3.1, it would be consistent with a formal model that estimated differing party valences in different subsets of the population. Thus, we can infer that NL should move away from the origin on both axes. Obviously this inference is mirrored in the position of NL in Figure 5.5.

Once NL moves from the origin, then so will the other parties. However, since the electoral variance on the institutional axis is much smaller than on the economic axis, the eigenvalue on the institutional axis will generally be negative. In other words, it appears that the origin will be a saddlepoint for the other parties. We therefore have an explanation of why all parties other than the NL are positioned on this axis. As in the case of Israel, we may refer to the economic axis as the principal electoral axis. Notice also that no party has a valence very much higher than the other, although the RC has the highest valence ($\lambda_{RC} = 0$). From the formal theory we would expect no party to be located near the electoral origin. This prediction is clearly substantiated. Theory thus indicates that the positions of the parties in Figure 5.5 are close to a local equilibrium of the vote-maximizing game. As we found in Israel, there are indications that the NL position was chosen not simply to maximize votes, but to affect coalition bargaining. It should also be mentioned that the significant role of the regional SD parameters in the NL vote share indicate that activists are important in influencing the NL policy position. We take up this possibility in the next chapter in the discussion of politics in the Netherlands.

5.5.3 The Coalition Bargaining Game

Figure 5.6 clearly shows that the core of the 1996 Chamber is empty, since the median lines of NL–RI, NL–RC, PDS–FI, PDS–AN, and FI–PPI

do not intersect. The relevant coalition structure of the Italian Parliament remained $\mathbb{D}_0$ after 1996. Following the elections, Prodi formed a center-left minority coalition comprising the Ulivo (PDS, PPI, RI, Greens) and small local parties (the SVP with three seats and the PvdA with one). The coalition controlled 285 seats and relied on the external support of RC (35 seats) to pass the majority threshold (of 316) in the Chamber. In the Senate, Ulivo controlled 155 seats (98 of PDS, 32 of PPI, 11 of RI, 14 of Greens), together with the support of RC (11 seats) and the 4 seats of local parties (1 PSdA, 2 SVP, 1 PVdA) giving it 170.

The Prodi government just managed to survive for two years. Eventually, on October 9, 1998, it fell after the leader of RC refused to support the annual budget bill. The coalition government was defeated on a vote of no confidence by one vote (312 yes, 313 no).

After the 1996 election the strategy of the NL changed substantially. Prodi succeeded in bringing Italy into the first round of the European monetary union (or EMU) in May 1998. This deprived the NL of a powerful weapon to use against the government. NL suffered substantial losses in the local elections of June 1998. Bossi perhaps realized that he had gone too far with his policy declaration preceding the 1996 election. In August 1998, Bossi declared that the NL had given up its goal of secession. The "Parliament of the North" was dissolved as well. Bossi, the principal of the NL, seems to have made the same mistake that Shas had made in 1992. In 1992, the leader of Labour, Rabin, in Israel, preferred to form a minority government rather than acquiesce to the demands of Shas over policy and government perquisites (Sened, 1996). In the same way, Prodi in Italy preferred to lead a coalition with a shaky minuscule majority rather than coalesce with Bossi (Giannetti and Sened, 2004). This miscalculation cost Bossi and his party dearly.

5.6 CONCLUSION

The analysis conducted so far clearly illustrates the importance of the post-election coalition structure in Parliament together with the trade-off between vote-maximizing positions and party positioning focused on coalition risk. A $\mathbb{D}_1$ structure, with a nonempty core, guarantees some stability. Though this need not enhance government duration, it does appear to affect policy coherence. An empty core or $\mathbb{D}_0$ structure tends to lead to constantly shifting government coalitions. As for the two pressures that decide the positioning of the party, a particular position may be appropriate in terms of a party's vote share but detrimental to its bargaining

position in the coalition bargaining stage of the game. Taking a risk in positioning with the coalition bargaining game in mind may lead to loss of electoral support, or to being outmaneuvered by a clever party leader. For both the Shas in Israel and the NL in Italy, this electoral effect may take time to make itself felt. This explains why parties may be willing to bet on such a risky strategy. The hope, presumably, is that the party's inclusion in the government coalition will enable them to repay voters for its deviation from the voters' perceived interests. It is also possible that the party can be hijacked by activists.

The stochastic nature of the electoral response function adds yet another level of uncertainty to the party positioning strategy prior to each election. Not just the risk involved, but the need to constantly balance vote-maximizing strategies with the resource availability, when resources depend so much on activists who may push agendas that are not necessarily vote maximizing, makes the calculus of party positioning difficult both for party principals and modelers. To maintain a high valence so as to be able to compete at the center of the voter distribution, a party needs activist resources. The next two chapters will discuss the tension between obtaining activist support and adopting an electorally advantageous position.

One purpose of this chapter was to show how the formal model applied to multiparty competition under a roughly proportional electoral rule captured some intriguing aspects of political change in Italy in the last three decades. A stable coalition structure characterized the system until 1987. The emergence of a new dimension, together with the electoral success of the NL in 1992, brought about the destruction of the prevailing decisive structure and opened up a new era in coalition politics. Governments that formed after the two elections held under the new electoral system found themselves struggling to survive. This kind of coalitional instability is different from the situation prior to 1992. Under the $\mathbb{D}_1$ structure, governments appeared to change regularly but the DC remained dominant. After 1992 and the emergence of the new, $\mathbb{D}_0$, empty core structure, consecutive coalitions are more likely to be different both in composition and in policy goals. The confusing Italian election of April, 2006, suggests that Italian coalition politics will remain unstable for some time to come.

We also hope to have shown the usefulness of the spatial model in establishing the empirical relevance of formal theory in the study of politics. Logit models of elections are commonly used to estimate voter response, but the theory of how party principals respond to the electorate is less

developed. The formal vote model developed in Chapter 3 can be applied to this substantive question. The difference between the theoretically predicted positions and those determined by the empirical model then allows us to extend the theory to include other party motivations. In this chapter, and the previous one on Israel, we hope to have shown that some of the discrepancy can be accommodated by developing the cooperative theory of the core and the heart. In the next three chapters we turn our attention to more complex electoral models.

5.7 APPENDIX

Table A5.1 *Logit Analysis for the 1996 Election in Italy (normalized with respect to RC).*

Parameter	Party	Posterior Mean	95% Confidence Lower	95% Confidence Upper
Spatial Distance β		0.206	0.024	0.388
Constant λ	PDS	−1.353	−2.501	−0.205
	Greens	−2.533	−4.649	−0.417
	PPI	−2.374	−4.538	−0.210
	RI	−7.490	−14.714	−0.266
	NL	−20.110	−39.350	−0.870
	CCD	−5.866	−11.331	−0.401
	FI	−1.845	−3.532	−0.158
	AN	−1.133	−2.152	−0.114
Age	PDS	0.039	−0.009	0.086
	Greens	0.005	−0.012	0.022
	PPI	0.051	−0.010	0.113
	RI	0.074	−0.013	0.160
	NL	0.009	−0.011	0.028
	CCD	0.054	−0.012	0.119
	FI	0.035	−0.011	0.082
	AN	0.028	−0.010	0.065
Education	PDS	0.196	−0.081	0.474
	Greens	0.481	−0.178	1.139
	PPI	0.035	−0.098	0.168
	RI	0.805	−0.252	1.861
	NL	0.092	−0.093	0.276
	CCD	0.705	−0.148	1.558
	FI	0.357	−0.084	0.798
	AN	0.343	−0.100	0.785

Elections in Italy, 1992–1996

Parameter	Party	Posterior Mean	95% Confidence Lower	95% Confidence Upper
Northwest region	PDS	−0.393	−0.672	−0.115
	Greens	−0.416	−0.641	−0.191
	PPI	−0.123	−0.179	−0.067
	RI	−0.456	−0.800	−0.111
	NL	20.410	−0.630	41.450
	CCD	−0.616	−0.914	−0.318
	FI	−0.313	−0.409	−0.216
	AN	−1.244	−2.385	−0.103
Northeast region	PDS	0.430	−0.101	0.960
	Greens	0.382	−0.151	0.915
	PPI	0.635	−0.105	1.374
	RI	0.521	−0.217	1.258
	NL	20.880	−0.640	42.400
	CCD	0.110	−0.127	0.346
	FI	0.020	−0.254	0.294
	AN	−0.363	−0.569	−0.157
Center region	PDS	−0.236	−0.340	−0.131
	Greens	−0.178	−0.189	−0.168
	PPI	−0.380	−0.527	−0.234
	RI	0.115	−0.180	0.409
	NL	17.760	−0.660	36.180
	CCD	−0.026	−0.198	0.147
	FI	−0.710	−1.016	−0.404
	AN	−0.838	−1.548	−0.128
Correctly Predicted	PDS	0.287		
	Greens	0.071		
	PPI	0.135		
	RI	0.054		
	NL	0.302		
	CCD	0.044		
	FI	0.214		
	AN	0.189		
	RC	0.130		
	Model	0.208		
$n = 1367$				

6

Elections in the Netherlands, 1979–1981

6.1 THE SPATIAL MODEL WITH ACTIVISTS

As our discussion of Israel in Chapter 4 illustrated, government in multiparty polities, based on proportional electoral methods, requires the cooperation of several parties. The model of coalition bargaining indicates that a large, centrally located party, at a core position, will be dominant. Such a core party can, if it chooses, form a minority government by itself and control policy outcomes.* If party leaders are aware of the fact that they can control policy from the core, then this centripetal tendency should lead parties to position themselves at the center.

Yet, contrary to this intuition there is ample empirical evidence that party leaders or political contenders do not necessarily adopt centrist positions. For example, Budge et al. (1987) and Laver and Budge (1992), in their study of European party manifestos, found no evidence of a strong centripetal tendency. The electoral models for Israel and Italy presented in the previous two chapters estimated party positions in various ways, and concluded that there is no indication of policy convergence by parties. Theorem 3.1 indicates why convergence does not occur in these two polities. In this chapter, we re-examine the earlier empirical analyses for the Netherlands (Schofield et al., 1998; Quinn, Martin, and Whitford, 1999; Quinn and Martin, 2002) to determine if the nonconvergence noted previously can be accounted for by the electoral theorem.

Contrary to the results of Chapter 4, we show that the valence terms, while relevant, are insufficiently different in the Netherlands for the elections of 1979 and 1981 so that convergence to the electoral center is

* See Schofield, Grofman, and Feld (1989), Laver and Schofield (1990), Schofield (1993, 1995), Sened (1995, 1996), Banks and Duggan (2000), Schofield and Sened (2002).

indeed predicted for the vote-maximizing electoral model. The conflict between theory and evidence suggests that the models be modified to provide a better explanation of party policy choice (Riker, 1965). This can be done either by changing the model of voter choice (e.g., Adams, 1999a,b, 2001; Merrill III and Grofman, 1999) or by considering more complex versions of the rational calculations of politicians.

In this chapter, we use a variety of empirical analyses to estimate the degree of centripetal tendency in the Netherlands. As far as electoral models are concerned, we develop the idea of valence, introduced in the previous chapters. We examine party positioning strategies in the Netherlands to show why these terms are required. We use Theorems 3.1 and 3.2 from Chapter 3 to examine whether local Nash equilibrium can occur at the electoral origin. We conduct additional empirical analysis to determine whether convergence should be expected on theoretical grounds at various electoral competitions.

While using the same theoretical model as in the previous chapter, our preoccupation in this chapter is with party's strategic behavior and not with voters' choice. Therefore, it is of great interest to us that our estimations for the election in the Netherlands suggest that the valence terms of the leaders of the major parties were quite similar. Under the assumption that these valence terms were exogenously determined, the mean voter theorem should have been valid and convergence to the mean should have occurred. Since there is no evidence of convergence by the major parties, we consider, instead, a more general valence model based on activist support for the parties (Aldrich and McGinnis, 1989). This activist valence model (Schofield, 2005a) presupposes that party activists donate time and other resources to their party. Such resources allow parties to present themselves more effectively to the electorate, increasing their valence. Thus, choosing an optimal position for the party becomes a difficult choice between the more radical preferences of activists and electoral considerations.

In the model of voting that we introduced in Chapter 3 and applied in Chapters 4 and 5, we have shown that many local equilibria exist, all of which can be found by simulation. Since this set of LNE contains all PNE, it is possible, in principle, to examine these LNE to see if any one of them would qualify as a PNE. The usual sufficient condition for existence of PNE is concavity of the party utility functions. Theorem 3.1 shows that the local version of this property, namely *local concavity* at the origin typically fails in these electoral games. This immediately implies that concavity fails. The failure of a sufficient condition for existence

of equilibrium does not, of course, imply nonexistence. Nonetheless, it suggests that PNE are unlikely to exist in the vote-maximization game. In the absence of a PNE and in the presence of multiple LNE, party leaders may be unable to coordinate on which particular local equilibrium to adopt. Thus, every local equilibrium of the model is a potential outcome of the political situation.

In the previous empirical analyses, valence terms, associated with each party, were crucial for the validity of the electoral model. Such valence terms were assumed be an exogenous feature of the election, characterizing each party by an average electoral evaluation of the competence of the party leader. We now consider the possibility that these terms are determined by party position.

By representing a coalition of activists, the party obtains resources. These contributions allow it to advertise its effectiveness, and thus gain electoral support (Aldrich and McGinnis, 1989). Since activist coalitions tend to be more radical than the average voter, parties are faced with a dilemma. By accommodating the political demands of activists, a party gains resources that it can use to enhance its valence; but by adopting radical policies to accommodate the demands of activists, it may lose electoral support due to the policy effect on voters. In this more general framework the party must balance the electoral effect, determined by its position, against the activist valence effect. One crucial difference emerges when valence is interpreted in this more general fashion. In the model where valence is fixed, our results indicate that concavity fails, casting doubt on the existence of PNE. However, when valence is affected by activist support, then it will naturally exhibit "decreasing returns to scale" (i.e., concavity). Consequently, when concavity of activists' valence is sufficiently, pronounced, then a PNE will exist but it will most assuredly not coincide with the electoral mean. In some polities, activists' valence is pronounced and so only one PNE exists. To determine whether such a PNE exists is extremely difficult, since the model requires data not just on voter-preferred positions but also a detailed examination of activist motivations. Nonetheless, the general model that we propose appears to be compatible with the rich diversity of party systems that we survey.

In this chapter, we study the elections in the Netherlands in 1977 and 1981 to illustrate the interaction among activists, the valance effect, policy preferences of voters at large, and the vote-maximizing motivations of party leaders. We use party delegate positions to construct an electoral model based on the implicit assumption that activists control party position. It turns out that the parameters of the multinomial logit (MNL) and

multinomial probit (MNP) models, with and without sociodemographic components, suggest that parties should have converged to the electoral center. Thus, in contrast to the empirical analysis of Israel, there is indirect evidence that activists did influence the policy positions of the parties.

6.2 MODELS OF ELECTIONS WITH ACTIVISTS IN THE NETHERLANDS

In Chapter 3, we introduced a formal model where each voter i, when presented with a choice between p different parties whose policy positions are described by the vector $\mathbf{z} = (z_1, \ldots, z_p)$, then chooses party $j \in P$ with some probability ρ_{ij}.

Recall that in this model, each party j is identified with a policy point, z_j, in a policy space X of dimension w. Each voter i is similarly identified with an ideal policy point x_i, together with individual characteristics, η_i. Let $\mathbf{x}$ denote the $(n \times w)$ matrix representing the voter ideal points. The variate $c_i = (\ldots c_{ij} \ldots)$ describes i's choice. If voter i actually chooses party j, then $c_{ij} = 1$; otherwise, $c_{ij} = 0$. As before, we concentrate on the probability ρ_{ij} that $c_{ij} = 1$, noting that $\Sigma_{i \in P} \rho_{ij} = 1$. Since c_{ij} is a binary variable, the expectation $\mathcal{E}xp(c_{ij})$ is ρ_{ij}. The expectation $\mathcal{E}_j(\mathbf{z})$ at the vector $\mathbf{z}$ of the stochastic vote share V_j^* of party j, can be estimated by taking the average of the estimations $\{\bar{\rho}_{ij}(\mathbf{z})\}$ across the sample. Thus,

$$\mathcal{E}_j(\mathbf{z}) = \frac{1}{n} \sum_i \bar{\rho}_{ij}(\mathbf{z}). \tag{6.1}$$

In general, the empirical variance of V_i^* will be significant. This is illustrated by Figure A6.1 in the appendix to this chapter, which shows the estimated stochastic vote share functions for the electoral model of the Netherlands. (This figure is taken from Schofield et al., 1998)

We now modify the earlier notation and write

$$\rho(\mathbf{x} : \mathbf{z}) = \rho(\mathbf{x} : z_1, \ldots, z_p) = (\rho_{ij}) \tag{6.2}$$

to denote that this is an n by p matrix which depends on both $\mathbf{x}$ and $\mathbf{z}$. The formal stochastic model introduced in Chapter 3 assumes that this matrix is derived from the $(n \times p)$ matrix of distances $(\delta_{ij}) = (||x_j - z_i||)$ where, as before, $|| \; ||$ is the Euclidean norm on X. Again, we assume the error vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_p)$ has a cumulative distribution function Ψ. The probability function ρ_{ij} depends on the assumption made on Ψ, and

is given by

$$\rho_{ij}(\mathbf{z}) = \Pr\left[\varepsilon_j - \beta\delta_{ij}^2 + \lambda_j + \theta_j^{\mathrm{T}}\eta_i > \varepsilon_k - \beta\delta_{ik}^2 + \lambda_k + \theta_k^{\mathrm{T}}\eta_i : \ k \neq j\right]. \tag{6.3}$$

As before, β is the positive spatial coefficient, λ_j is the valence of party j, $\theta_j^{\mathrm{T}}\eta_i$ gives the effect of sociodemographic influence on i's vote, and Pr stands for the probability operator derived from the cumulative distribution.

Computation of this probability obviously depends on the distribution assumption made on the errors. Most formal voting models with stochastic voters assume that voter choice is pairwise statistically independent. The analogous empirical MNL model already discussed in Chapters 3 and 4 assumes "Independence of Irrelevant Alternatives" (IIA). That is, for any two parties, j, k, the ratio

$$\frac{\rho_{ij}(\mathbf{z})}{\rho_{ik}(\mathbf{z})} = \frac{\exp[-\beta\delta_{ij}^2 + \lambda_j + \theta_j^{\mathrm{T}}\eta_i]}{\exp[-\beta\delta_{ik}^2 + \lambda_k + \theta_k^{\mathrm{T}}\eta_i]} \tag{6.4}$$

is independent of $\rho_{il}(\mathbf{z})$ for a third party l. It has generally been inferred that assuming the Type I extreme value distribution (or log Weibull) and thus IIA would result in existence of equilibrium at the electoral mean (Adams, 2001). The simulation of the MNL model for Israel given in Chapter 4 has already shown this to be incorrect. The IIA assumption is not satisfied by the more general stochastic MNP model. Such a model does not require the assumption of independent errors. A Markov Chain Monte Carlo (MCMC) technique due to Chib and Greenberg (1996) was used by Schofield et al. (1998) and Quinn, Martin, and Whitford (1999) to model elections in the Netherlands, Germany, and Britain. Here we re-examine these earlier analyses for the Netherlands for 1977 to 1981 in the light of the new formal results reported in Chapter 3.

In the MNP model, with constant valence terms $\{\ldots, \lambda_j, \ldots\}$, the probability matrix (ρ_{ij}) is determined by the $(p-1)$ dimensional vector of error differences $e_j = (\varepsilon_p - \varepsilon_j, \ldots, \varepsilon_{j-1} - \varepsilon_j, \ldots, \varepsilon_1 - \varepsilon_j)$. If the covariance matrix of ε is known to be Ω, then, the covariance of e_j is given by the matrix

$$\sum_j(\Omega) = \begin{pmatrix} \mathcal{E}xp(\epsilon_p - \epsilon_j, \epsilon_p - \epsilon_j) & . & . \\ \mathcal{E}xp(\epsilon_p - \epsilon_j, \epsilon_{p-1} - \epsilon_j) & & . \\ \mathcal{E}xp(\epsilon_p - \epsilon_j, \epsilon_1 - \epsilon_j). & . & \mathcal{E}xp(\epsilon_1 - \epsilon_j, \epsilon_1 - \epsilon_j) \end{pmatrix}.$$

Once this is estimated then we obtain the multivariate probability density function, φ of the $(p-1)$ variate. In parallel to the proof of Theorem 3.2 we use

$$g_{ij}(\mathbf{z}) = \left(\ldots, \beta\delta_{ik}^2 - \beta\delta_{ij}^2 - \lambda_k + \lambda_j - \theta_k^{\mathrm{T}}\eta_i + \theta_j^{\mathrm{T}}\eta_i, \ldots\right) \qquad (6.5)$$

to denote the $(p-1)$ comparison vector, by which we model the calculation made by voter i of the choice between party j with the other parties $k \in \{1, \ldots, j-1, j+1, \ldots, p\}$.

By definition, $\rho_{ij}(\mathbf{z})$ is given by $\int \varphi(e_j)de_j$, with bounds from $-\infty$ to $g_{ij}(\mathbf{z})$.

Theorem 3.1 assumed that the distribution function Ψ of the errors was the Type I extreme value distribution. Here we examine empirical estimation carried out under the more general assumption that the errors are multivariate normal, with nondiagonal covariance matrix Ω and error difference covariance matrices.

To estimate voter ideal points in the two elections in the Netherlands, Schofield et al. (1998) and Quinn, Martin, and Whitford (1999) used survey data for 1979, collected for a number of European countries by Rabier and Inglehart (1981). We use these data and the previous exploratory factor analysis based on the voter response profile to estimate the nature of the underlying policy space X. In the Netherlands, two dimensions were significant: the usual left–right dimension and a second concerned with scope of government. (Table A6.1 in the appendix to this chapter reports the weights associated with the two policy dimensions.) The response of voter i to the survey gave the location of the individual's ideal point in the policy space. For each party j, the data set (ISEIUM, 1983) was used to estimate the ideal points of the elite members (or delegates) of that party, namely $\{x_l^j : l \in N_j\}$, where N_j, represents the elite of party j. Since the estimated policy space was two-dimensional, the position $z_j \in X$ of party j was obtained by taking the two-dimensional median of the delegate positions. This position was taken to represent the "sincere" ideal point of party j. The representative delegate of party j whose ideal point is z_j we call the *principal* of party j.

Figure 6.1 gives the resulting estimation of the distribution of voter ideal points, together with the estimated positions of the party principal positions of the four parties. Labor (PvdA), Christian Democratic Appeal (CDA), Liberals (VVD), and Democrats 66 (D'66). Table 6.1 gives the election results for 1977 and 1981.

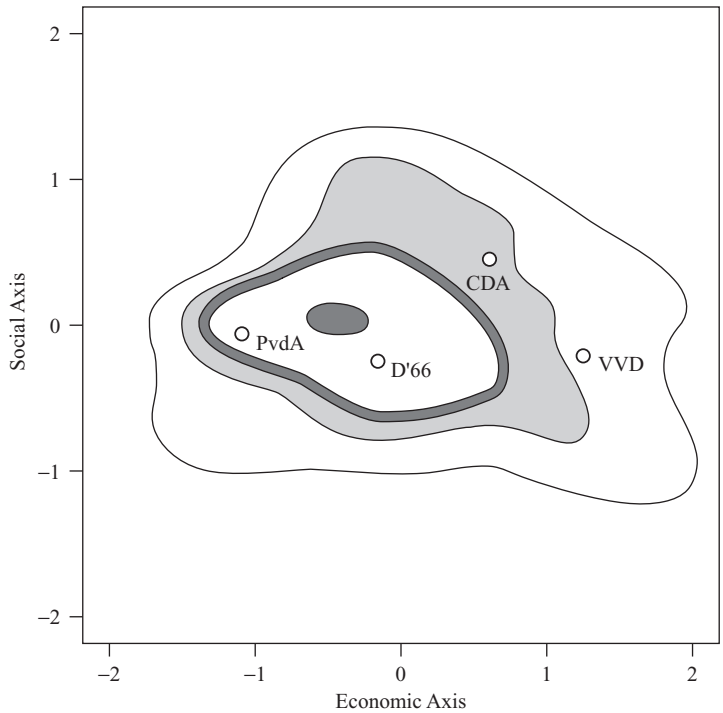


Figure 6.1. Distribution of voter ideal points and party positions in the Netherlands.

For the electoral estimations we adopt the following hypothesis.

Hypothesis 6.1: *The positions of the party principals can be used as proxies for the electorally perceived positions of the parties.*

On the basis of this hypothesis, a number of separate estimations using these data were carried out. The results are given in Table 6.2.

The first MNP model is discussed in Schofield et al. (1998). In this model, all valence terms were set to zero. It included a comparison of the "pure" spatial model, based on $\rho(\mathbf{x}, \mathbf{z})$; a sociodemographic model (SD), based on $\rho(\eta)$, where η represents the vector of such individual characteristics; and a joint model $\rho(\eta, \mathbf{x}, \mathbf{z})$, using the spatial component as well as η. As expected, sociodemographic characteristics were significant in predicting voter choice. For example, status as a manual worker would be expected to increase the probability of voting for the PvdA. Table 6.2 gives the national vote shares in the two elections of 1977 and 1981, as well as the sample vote shares, calculated for these four parties. The

Table 6.1. *Election Results in the Netherlands, 1977–1981.*

Party (acronym)	1977 (Seats)	1981 (Seats)
Labour (PvdA)	53	44
Democrats '66 (D'66)	8	17
Liberals (VVD)	28	26
Christian Dem Appeal (CDA)	49	48
	(138)	(135)
Communists (CPN)	2	3
Dem '70 (D70)	1	0
Radicals (PPR)	3	3
Pacific Socialists (PSP)	–	3
Reform Federation (RPF)	–	2
Reform Pol Ass (GDV)	1	1
Farmers Party (BP)	1	0
State Reform Party (SGP)	3	3
	(11)	(15)
Total	149	150

survey sample vote shares in Table 6.2 can be compared with the party seat distributions given in Table 6.1. Note that the national vote share of the Labour Party (PvdA) declined from 38 percent in 1977 to 32.4 percent in 1981. Its sample share was 36.9 percent in 1979 and the estimated

Table 6.2. *Vote-Shares, Valences, and Spatial Coefficients for Empirical Models in the Elections in the Netherlands, 1977–1981.*

Party	National Vote, 1977 %	National Vote, 1981 %	Sample Vote, %	Esatimated Share[a] %	Confidence Interval of Vote Share[b] %	Valences in MNL[c]	Valences in MNP[d]
D'66	6.1	12.6	10.4	10.6	(6.1, 16.1)	0	0
PvdA	38.0	32.4	36.9	35.3	(30.6, 38.9)	1.596	0.622
CDA	35.9	35.2	33.8	29.9	(30.1, 40.1)	1.403	0.655
VVD	20.0	19.8	18.9	24.2	(14.9, 24.6)	1.015	0.334
β				0.456	0.499	0.737	0.420

[a] Estimated by an MNP spatial model without valence or SD.
[b] Estimated by an MNP spatial model without valence but with SD.
[c] Estimated by an MNL model with valence without SD.
[d] Estimated by an MNP model with valence without SD.

expectation from the MNP model, without the SD terms, was 35.3 percent with a 95 percent confidence interval of (30.9, 39.7).

We have emphasized that the vote-share functions are stochastic variables, with significant variance. This can be illustrated by Figure A6.1 in the appendix to this chapter. The estimated shares based on the MNP model without SD or valence are fairly close to the sample shares, though the VVD estimation was poor. The log marginal likelihood (LML) was calculated to be −626. Adding sociodemographic characteristics to the MNP model improved the prediction, as the 95 percent confidence intervals in Table 6.2 indicate. The LML changed to −596, so the Bayes' factor (Kass and Raftery, 1995) or the difference between log likelihoods of the MNP spatial model with SD and without was 30 (= 626 − 596), suggesting that the joint SD model was statistically superior to the pure spatial model.

Simulation of these two models found that each of the parties could have increased vote share by moving away from their locations in Figure 6.1 toward the electoral mean. We shall show below that this inference is consistent with Theorem 3.2 when applied to empirical models including valence. Schofield et al. (1998) raised the question: If the positions given in Figure 6.1 are indeed the party positions, then why do the parties not approach the electoral center to increase vote share?

To study this question further, an MNL model based on Hypothesis 6.1 was estimated to include valence (λ), but without SD. The estimated valences are also reported in Table 6.2. Notice that in the model with $\lambda \neq 0$, the valences are normalized by setting the valence of D'66 to zero. Comparing this MNL valence model with the MNL model without valence gave a very significant Bayes' factor of 75, corresponding to a chi-square of 149. Even comparing it to the above MNP model with SD but without valence, gave a Bayes' factor of 64 (= 596 − 532). Clearly, the valence terms increase the statistical likelihood of the voter model.

It should be pointed out that the coefficients, β and λ, are not directly comparable between the MNL and MNP models. The MNL models are based on the (iid) extreme value distribution with error variance $\sigma^2 = \frac{1}{6}\pi^2 = 1.6449$, while the MNP models are based on some appropriate normalization for the error difference variances.

Although probit models have theoretical advantages, it would appear from the above that the MNL and MNP models give comparable results in terms of predictions about party vote shares. To more fully examine the effect of valence, Tables A6.2 and A6.3 in the appendix present the result of MNP and MNL estimation for models involving both SD and

Table 6.3. *Log Likelihoods and Eigenvalues in the Dutch Electoral Model.*

Spatial Models	Convergence Coefficient	Eigenvalue First	Second	LML[a]
MNL no valence or SD[b]	na	na	na	−606
MNL no valence, with SD[b]	na	na	na	−565
MNL with valence, no SD[c]	1.19	−0.18	−0.64	−532
MNL with valence and SD[c]	1.38	−0.04	−0.58	−465
MNP without valence or SD[d]	na	na	na	−626
MNP without valence, with SD[d]	na	na	na	−596
MNP with valence, no SD[c]	0.75	−0.48	−0.77	−545
MNP with valence and SD[c]	1.55	+0.05	−0.50	−427
Non-Spatial, MNP pure SD[c]				−596

[a] LML = log marginal likelihood.
[b] Based on Schofield and Sened (2005b), using extreme value distribution.
[c] Based on Quinn, Martin, and Whitford (1999).
[d] Based on Schofield, Martin, Quinn, and Whitford (1998).

valence. Quinn et al. (1999) previously computed the Bayes' factors for the various models and found the joint spatial MNP and MNL models, $\rho(\eta, \mathbf{x}, \mathbf{z})$, with valence superior to the pure MNP and MNL sociodemographic models $\rho(\eta)$ without a spatial component. This suggests that the appropriate causal model is one in which SD characteristics (η_i) influence beliefs (x_i) which in turn affect the probability vector of voter choice (ρ_i). Table 6.3 reports the log marginal likelihoods of the eight different models.

An important inference for our argument here is that, as in the case of Israel, the explanatory power of each empirical model is much increased by adding in the valence terms (Stokes, 1963, 1992). Indeed, pairwise comparison of a model with valence, but without SD, against one without valence, but with SD, suggests that the valence terms, to some degree, substitute for using the individual characteristics of voters. We draw three conclusions from the log likelihoods presented in Table 6.3.

Conclusion 6.1

(i) There is strong justification for Hypothesis 6.1. The log marginal likelihoods of all spatial models, when compared with the pure SD models, indicate that these estimated party positions provide a useful basis for modelling electoral choice. Indeed, the 95 percent confidence intervals of the β coefficients in Tables A6.2 and A6.3 allow us to reject the hypothesis that the spatial coefficients are zero.

(ii) The valence terms are all significant. More importantly, the confidence interval on the high-valence party, the PvdA, excludes 0, so we can infer that there is a significant valence difference.
(iii) Although the sociodemographic terms are important, their effect can to some extent be captured by valence.
(iv) The valence differences are reduced when SD terms are included. As a consequence, when examining the models to determine whether convergence is to be expected, it is important to include SD.

Given that there is evidence for the statistical significance of the estimation, we can examine the question of convergence.

It is obvious that if the valence of party j is increased, then the probability that a voter chooses the party also increases. As we have observed, it is not the absolute values of the valences that are relevant but the pairwise differences in the valences. For estimation purposes we set the lowest valence of one party to zero. For example, in the MNL model presented in Table A6.2, the valence of D'66 was normalized to be zero. In the MNP model with SD however it turns out that the religious sociodemographic variable affects the vote choice. The result is that the CDA is estimated to have the lowest valence for this model. We now utilize the results of the formal model given in Chapter 3 on the basis of the following hypothesis.

Hypothesis 6.2: *The results of the formal model given in Chapter 3 are applicable to the analysis of empirical models.*

These empirical models are not directly comparable to the formal electoral model presented in Chapter 3. In particular, the sociodemographic components are not included in the formal model. In computing the coefficients and eigenvalues for the MNL models we used the results given in Theorem 3.1 for the extreme value distribution, Ψ. For the MNP models it is necessary to modify the definition of the Hessians to account for error covariance. First, we note that the electoral variance on the first axis is 0.658, whereas on the second it is 0.289. The reason these are both different from 1.0, is that the normalization was done with respect to the variance of the delegate points on the first axis.

Table 6.3 also presents the results of the computation of the eigenvalues of the Hessians at the origin for the lowest-valence party. These computations are presented in a technical appendix to this chapter. Tables A6.1

and A6.2 in the appendix give the estimation results, including the valences for the various parties as well as the sociodemographic coefficients for the MNL and MNP models. According to the results of Chapter 3, if the convergence coefficient is bounded above by 1.0, then we may argue that the origin will, for sure, be a local equilibrium. It is evident that the convergence coefficients of three of the four baseline formal models satisfy this condition. We regard this as strong evidence that the earlier inference made by Schofield et al. (1998) about convergence to the electoral origin is generally unaffected by the addition of valence to the models. An additional simulation by Quinn and Martin (2002) provides further support for the convergence result.

As we have noted, adding sociodemographic terms tends to reduce the valence coefficients, because these explain less of the voter choice. This has the effect of reducing valence difference between high- and low-valence parties, thus changing the estimated convergence coefficients. However, as Table 6.3 indicates, the effect on the MNL models is trivial. The only model that gives a noncentrist equilibrium is the MNP model with valence and SD. Because the correlation between the two electoral axes is negligible, we can treat the two axes separately. Table 6.3 shows that for this model, the eigenvalue of the CDA Hessian on the second axis is negative. This implies that, in local equilibrium, all parties should be at the zero position on the second axis. Because the eigenvalue for the CDA on the economic axis is positive (albeit small), then it is possible that its vote-maximizing position will be away from the origin. We cannot predict whether it should move to the right or the left. We can infer, however, that all parties, in equilibrium in this model, should be strung along the economic axis. It is also the case that the vote-share functions of the parties were "close" to concave. This can be seen from examining the vote probability functions presented in Figures A6.2 and A6.3 (in the appendix to this chapter), based on the positions of the parties given in Figure 6.1. The inference is that the parties should adopt positions on the economic axis, but very close to the electoral origin. Note also that, for three of the four models, because the eigenvalues are typically negative, and "large" in magnitude with respect to the parameters of the various models, then the origin is not only likely to be a local equilibrium with respect to vote-maximizing, but also the unique Nash equilibrium. Comparing Figure 6.1 with the predictions of the formal model we therefore infer that it is very unlikely that the CDA position is a component of a vote-maximizing equilibrium. Although the positions of the PvdA, VVD,

and D'66 are not in obvious contradiction to the formal interpretation of the MNP/SD empirical model, there is evidence that these parties could have increased vote share by moving from their presumed positions in Figure 6.1 toward the electoral center. On the basis of Hypothesis 6.2 we are led to the following conclusion.

Conclusion 6.2 *It is unlikely that the estimated positions given in Figure 6.1 can belong to a local equilibrium on the basis of an electoral model with fixed exogenous valences.*

It is possible that the CDA position is one chosen in response to coalition risk, as discussed in Section 3.1 in Chapter 3, as well as in the empirical illustrations from Israel and Italy in Chapters 4 and 5. There are two distinct coalition structures relevant to politics in the Netherlands:

$$\mathbb{D}_0 = \{\text{PvdA,CDA}\}, \{\text{PvdA,VVD}\}, \{\text{CDA,VVD}\}, \text{and}$$

$$\mathbb{D}_{PvdA} = \{\text{PvdA,CDA}\}, \{\text{PvdA,VVD,D'66}\}, \{\text{CDA,VVD,D'66}\}.$$

After the May 1977 election, structure $\mathbb{D}_0$ can be taken to represent the electoral outcome since the {CDA,VVD} coalition had 77 seats out of 149, and was therefore winning. This coalition did indeed form a government, but only after 6 months of negotiation. After the 1981 election, this coalition controlled only 74 seats (out of 150) so we can represent the outcome by $\mathbb{D}_{PvdA}$. A {PvdA,D'66,CDA} coalition government with 109 seats first formed and then collapsed to a minority {D'66,CDA} government. A new election had to be called in September 1982. Although the post-1981 election situation is designated a $\mathbb{D}_{PvdA}$ coalition structure, the PvdA could only be at a core position if it adopted a position inside the convex hull of the {CDA,VVD,D'66} positions. In fact, the heart, given the positions in Figure 6.1, together with the seat strengths in 1981, is the convex hull of the three positions {PvdA,D'66, CDA}. Thus, the minority coalition government that did indeed form is compatible with the notion of the heart. Moreover, as Section 3.4 illustrated, the CDA may gain advantage in coalition bargaining if it adopts a radical strategic position on the second axis.

Notice that the model suggests that there is strong centripetal pressure on the PvdA, in terms of adopting a centrist position to both gain seats and possibly control the core policy position. The coefficient for the PVdA for manual labor given in Appendix Table A6.3, is high and significant, suggesting that activists had a centrifugal influence on the policy preferences of the party. This influence appears to have overcome the

centripetal tendency generated by the formal model with fixed exogenous valences.

It is also noticeable from the tables that the sociodemographic coefficient associated with religion was highly significant for the CDA, in both MNL and MNP models. This also suggests that activists concerned about policy on this axis were influential in determining the CDA position. We are therefore led to the conclusion that activists for both these parties generated centrifugal forces within each party and that these countered the centripetal effect our analysis has shown to be associated with the model of vote maximizing.

Instead of supposing that valence is exogenously determined at the time of the election, we now consider the more general hypothesis that valence is determined by the effect of activists on party support and that these valence *functions* affect the local Nash equilibrium positions that parties adopt. By contributing support, the party elite enhances the popularity of the party. We conjecture that the activist valence terms will not, in fact, be constant, but will be maximized at the center of the distribution of the positions of the elite or delegates of the parties. This follows because at this position the contributions of the party activists will be maximized. Consequently, it is plausible that the valence functions will be concave in the positions adopted by the parties. We conjecture that noncentrist LNE may exist, and that they may indeed be PNE of this more complex electoral game.

Our analysis of these elections in the Netherlands suggests the following conclusion concerning the interplay of electoral and coalition risk in the strategic calculations of policy-motivated party activists.

Conclusion 6.3 *Because the coalition structure $\mathbb{D}_{PvdA}$ is advantageous to the PvdA, this party should attempt to maximize the probability π_{Pvda} that this is the election outcome, and a proxy for this is to maximize the expected vote-share function $\mathcal{E}_{PvdA}$. On the other hand, while the CDA should attempt to maximize the probability π_0 that the coalition structure $\mathbb{D}_0$ occurs, it can be rational for the party to consider the consequences of coalition risk, and choose a position that allows it to bargain effectively with its probable coalition partners.*

As mentioned above, the estimates for party locations in Figure 6.1 were derived from the ISEIUM delegate surveys. It is a reasonable assumption that each delegate of a party has a preferred position to offer to the electorate. Obviously, there is a calculus involved as delegates optimize

between their own preferences and the desire to gain votes. The empirical analysis of the Netherlands is based on the assumption that the principal's position for each party is the one that is offered to the electorate by the party. In fact, the positions given in Figure 6.1 closely correspond to the positions estimated by De Vries (1999) using an entirely different methodology based on policy choices of the parties. These chosen positions then generate activist support and the estimated valences. Conclusion 6.3 is compatible with the more complex model, articulated in Chapter 3, in which the party principal chooses a party leader with a different position because of the realization that the chosen position not only affects vote share, but independently influences the probability that the party will join in coalition government. These observations suggest the following general hypothesis on the nature of the centipetal and centrifugal tendencies.

Hypothesis 6.3: *The centripetal tendency associated with simple vote maximization in the model with exogenous valence is balanced by:*

(i) the motivation of concerned party principals to affect the final coalition government policy, and
(ii) the requirement to gain support from activists, thus indirectly increasing overall electoral support for the party.

We have suggested in this chapter that there is some evidence that both influences can affect party position. It is difficult to determine which of these two effects is more important. However, one way to examine the influence of activists is to consider a polity where the coalition effect can be disregarded. The next two chapters will examine the activist hypothesis in the context of empirical models of elections in Britain and the United States.

6.3 TECHNICAL APPENDIX: COMPUTATION OF EIGENVALUES

Here we show how the coefficients and eigenvalues given in Table 6.3 can be computed. As Figure 6.1 indicates, the electoral variance on the first economic axis is $v_1^2 = 0.658$, whereas on the second it is $v_2^2 = 0.289$. The covariance is negligible.

We can calculate the various coefficients and eigenvalues for the four models with valence.

(i) As an illustration of Theorem 3.1, for the extreme value formal model $M(\Psi)$ without SD we find $\lambda_{\text{d'}66} = 0$, $\lambda_{Pvda} = 1.596$, $\lambda_{cda} = 1.1403$, $\lambda_{vvd} = 1.015$, and $\beta = 0.737$. At the joint origin the probability of voting for D'66 is

$$\rho_{\text{d'}66} = \frac{1}{1 + e^{1.596} + e^{1.403} + e^{1.015}} = 0.074.$$

Thus, $A_{\text{d'}66} = 0.737(0.852) = 0.627$.

$$C_{\text{d'}66} = (1.25)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I = \begin{pmatrix} -0.18 & 0 \\ 0 & -0.64 \end{pmatrix}$$
$$c(\Psi) = 2A_{d66}(0.658 + 0.289) = 1.187$$

Since the eigenvalues are -0.18 and -0.64, both negative, it is obvious that the model based on the extreme value distribution gives an LSNE at the joint origin.

(ii) When sociodemographic variables are added to the MNL model (Quinn, Martin, and Whitford, 1999) the valence differences are changed and we find that the CDA is the lowest-valence party, $\lambda_{cda} = -0.784$. The other valences are now $\lambda_{vvd} = 0.313$, $\lambda_{Pvda} = 2.112$ and $\lambda_{\text{d'}66} = 0$. Using the model Ψ we find $\beta = 0.665$. Thus,

$$\rho_{cda} = \frac{1}{1 + e^{2.896} + e^{1.097} + e^{0.784}} = 0.04.$$

Hence, $A_{cda} = 0.0665(0.99) = 0.664$.

$$C_{cda} = (1.33)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I = \begin{pmatrix} -0.12 & 0 \\ 0 & -0.61 \end{pmatrix}$$
$$c(\Psi, SD) = 1.25.$$

Again, both eigenvalues are negative and the necessary condition is satisfied. Note the large negative eigenvalue on the second axis in contrast to the small eigenvalue on the first axis.

(iii) With the probit model (without SD), we find $\lambda_{\text{d'}66} = 0$, $\lambda_{cda} = 0.655$, $\lambda_{Pvda} = 0.622$, $\lambda_{vvd} = 0.334$, and $\beta = 0.420$. Thus, the average valence excluding D'66 is $\lambda_{av(\text{d'}66)} = 0.537$.

Because the errors have nondiagonal covariance matrix Ω, we now compute the covariance matrix of the difference vector,

$e_{\text{d'66}} = (\epsilon_{cda} - \epsilon_{\text{d'66}}, \epsilon_{Pvda} - \epsilon_{\text{d'66}}, \cdots, \epsilon_{vvd} - \epsilon_{\text{d'66}})$. This will be the symmetric matrix

$$\Sigma_{\text{d'66}}(\Omega)$$
$$= \begin{pmatrix} \mathcal{E}xp(\epsilon_{cda} - \epsilon_{\text{d'66}}, \epsilon_{cda} - \epsilon_{\text{d'66}}) & . & . \\ \mathcal{E}xp(\epsilon_{cda} - \epsilon_{\text{d'66}}, \epsilon_{Pvda} - \epsilon_{\text{d'66}}) & & . \\ \mathcal{E}xp(\epsilon_{cda} - \epsilon_{\text{d'66}}, \epsilon_{vvd} - \epsilon_{\text{d'66}}). & . & \mathcal{E}xp(\epsilon_{vvd} - \epsilon_{\text{d'66}}, \epsilon_{vvd} - \epsilon_{\text{d'66}}) \end{pmatrix}.$$

Here $\mathcal{E}xp$ denotes expectation. Normalization gives the following matrix:

$$\Sigma_{\text{d'66}}(\Omega) = \begin{pmatrix} 1.0 & -0.06 & 1.258 \\ -0.06 & 0.186 & 0.558 \\ 1.258 & 0.558 & 0.454 \end{pmatrix}$$

The sum of the terms in this matrix is $var(\Sigma_{\text{d'66}}(\Omega)) = 5.15$.

From Theorem 3.2, we find that the Hessian matrix $C_{\text{d'66}}$ associated with the D'66 is given by

$$C_{\text{d'66}}(\Omega) = \left[2A_{\text{d'66}}\left(\frac{\nabla}{n}\right) - I\right],$$

where $\frac{\nabla}{n}$ is the electoral covariance matrix and

$$A_{\text{d'66}} = \left[\frac{(p-1)^2\beta}{var(\Sigma_{\text{d'66}})}\right]\left[\lambda_{av(\text{d'66})} - \lambda_{\text{d'66}}\right] = 0.39.$$

Since the number of parties $p = 4$, we find

$$C_{\text{d'66}}(\Omega) = (0.78)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I$$

$$= \begin{pmatrix} -0.48 & 0 \\ 0 & -0.77 \end{pmatrix}$$

so again the eigenvalues are negative. The convergence coefficient is given by

$$c(\Omega) = 2A_{\text{d'66}}(0.658 + 0.289) = 0.75.$$

(iv) Finally, for the MNP model with SD we find $\lambda_{cda} = -0.408$, $\lambda_{av(cda)} = 0.443$, and $\beta = 0.455$. The error difference covariance matrix is

$$\Sigma_{cda} = \begin{pmatrix} 1.0 & -0.141 & 0.170 \\ -0.141 & 1.383 & 0.489 \\ 0.170 & 0.489 & 0.936 \end{pmatrix}$$

so $var(\Sigma_{cda}) = 4.355$. Thus,

$$A_{cda}(\Omega) = 0.8 \text{ and } C_{cda} = \begin{pmatrix} 0.05 & 0 \\ 0 & -0.5 \end{pmatrix}.$$

The coefficient $c(\Omega, SD) = 1.55$. Obviously the sufficient condition fails, and one of the eigenvalues is positive. Although the necessary condition does not fail, it is clear that the origin is now a saddlepoint for the CDA for this model. Thus, under a pure vote-maximizing model, incorporating sociodemographic characteristics of the voters, the CDA may well move away from the origin, along the first, high-variance economic axis so as to gain votes. However, because this eigenvalue on the economic axis is small in modulus (at least in comparison to the eigenvalue on the second axis) in equilibrium we expect the PvdA, D'66 and VVD to be close to the origin on the second axis. That is, in the equilibrium for the MNP model with SD, all parties should be located on the economic axis.

Naturally there is uncertainty about the correct model. However, the analyses indicate that it is unlikely that the positions in Figure 6.1 can constitute a local equilibrium under the assumption of exogeneous valence.

6.4 EMPIRICAL APPENDIX

Appendix Table A6.1. *Factor Weights for the Policy Space in the Netherlands.*

Issue	Dimension 1	Dimension 2
Income distribution	+0.510 (10.86)	−0.148 (1.92)
Terrorism	−0.232 (4.28)	−0.253 (2.51)
Nuclear energy	−0.297 (6.74)	–
Enterprises	+0.526 (12.0)	–
Environment	+0.306 (7.46)	–
MNC	+0.612 (12.6)	−0.229 (2.42)
Abortion	+0.327 (5.56)	+0.390 (2.45)

Chi-square over d.o.f. = 1.76; Sample size (n) = 529.

Appendix Table A6.2. *Probit Analysis of the 1979 Dutch Survey Data (normalized with respect to D'66).*

Model Variable	Party	Mean	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
β (MNP)		0.455	0.341	0.571
Valence	PvdA	1.298	0.165	2.433
	VVD	0.031	−1.116	1.175
	CDA	−0.408	−3.530	2.640
Manual Labour	PvdA	0.865	0.377	1.408
	VVD	−0.522	−1.350	0.268
	CDA	0.537	0.023	1.151
Religion	PvdA	−0.082	−0.295	0.124
	VVD	−0.012	−0.271	0.242
	CDA	0.736	0.427	1.108
Income	PvdA	−0.043	−0.089	0.002
	VVD	0.059	0.005	0.116
	CDA	0.021	−0.029	0.068
Town Size	PvdA	0.162	−0.073	0.407
	VVD	0.017	−0.283	0.321
	CDA	−0.236	−0.015	0.517
Education	PvdA	−0.113	−0.170	−0.054
	VVD	−0.002	−0.074	0.073
	CDA	−0.062	−0.128	0.000
Correctly Predicted %	PvdA	61.2		
	VVD	49.5		
	CDA	62.3		
	D66	20.4		
	MNP Model	56.4		
Sample Size = 529				

Source: Quinn, Martin, and Whitford (1999) and Schofield, Martin, Quinn, and Whitford (1998).

Appendix Table A6.3. *Multinomial Logit Analysis of the 1979 Dutch Survey Data (normalized with respect to D'66).*

Model Variable	Party	Mean	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
β (MNL)		0.665	0.557	0.785
Valence	PvdA	2.112	1.276	2.927
	VVD	0.313	−0.920	1.273
	CDA	−0.784	−1.206	−0.265
Manual Labour	PvdA	1.406	0.444	2.508
	VVD	−0.547	−1.996	0.937
	CDA	1.064	0.012	2.330
Religion	PvdA	0.080	−0.234	0.408
	VVD	0.080	−0.236	0.434
	CDA	1.382	1.070	1.757
Income	PvdA	−0.056	−0.141	0.038
	VVD	0.095	0.004	0.201
	CDA	0.019	−0.065	0.115
Town Size	PvdA	0.373	−0.055	0.878
	VVD	0.221	−0.271	0.774
	CDA	0.517	0.058	1.044
Education	PvdA	−0.203	−0.311	−0.086
	VVD	−0.056	−0.173	0.078
	CDA	−0.148	−0.266	−0.033
Correctly Predicted %	PvdA	62.5		
	VVD	49.9		
	CDA	62.2		
	D66	19.3		
	MNL Model	55.5		
Sample Size = 529				

Source: Quinn, Martin, and Whitford (1999) and Schofield, Martin, Quinn, and Whitford (1998).

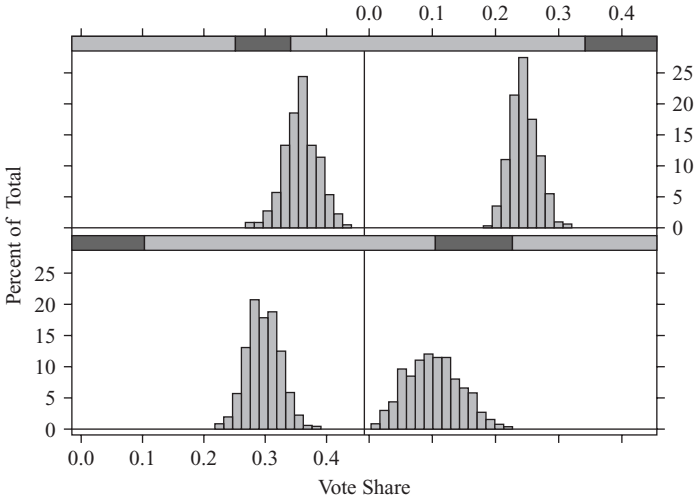


Figure A6.1. Estimated stochastic vote-share functions for the Pvda, VVD, CDA, and D'66 (based on 1979 data and the party positions given in Figure 6.1). (Source: Schofield, Martin, Quinn, and Whitford, 1998.)

Elections in the Netherlands, 1979–1981

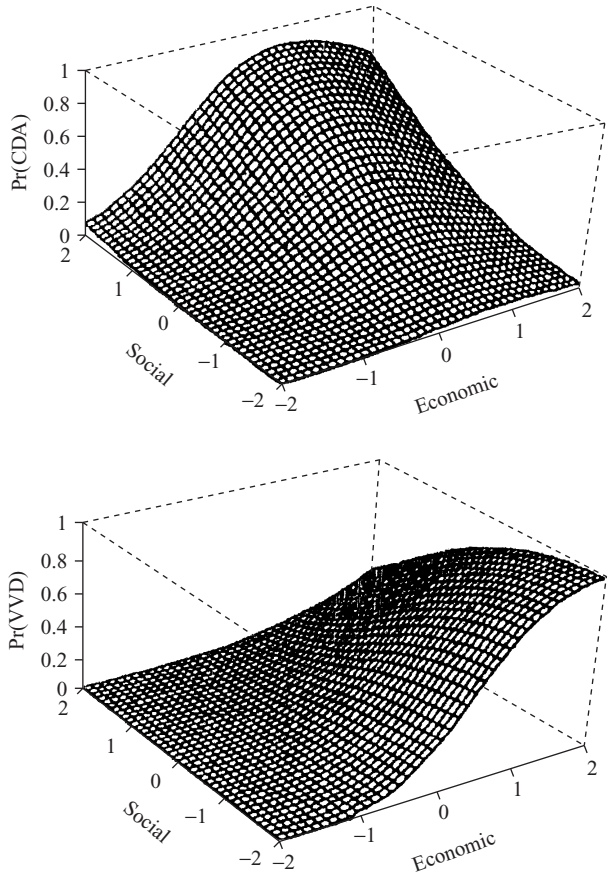


Figure A6.2. Estimated probability functions for voting for the CDA and VVD. (Source: Schofield, Martin, Quinn, and Whitford, 1998.)

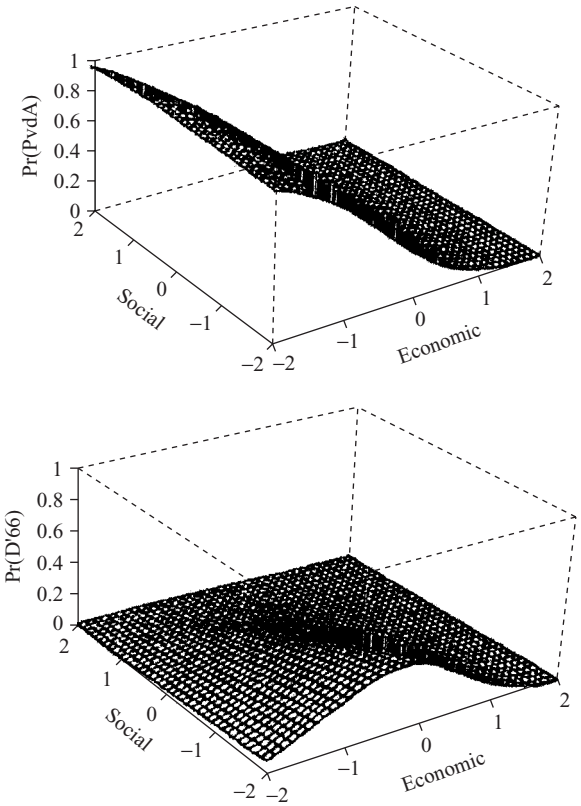


Figure A6.3. Estimated probability functions for voting for the Pvda and D'66. (Source: Schofield, Martin, Quinn, and Whitford, 1998.)

7

Elections in Britain, 1979–2005

The previous chapters on the proportional electoral systems of Israel, Italy, and the Netherlands have considered the hypothesis that the policy positions of parties were chosen not simply to maximize vote-shares, but incorporated strategic concerns over the effect of position on the probability of joining a government coalition. However, this coalition consideration is generally not present in the plurality electoral system of Britain. (For convenience we use the term Britain for the United Kingdom). We can therefore use our electoral model for this polity to determine the degree to which simple vote-maximization characterizes policy choices. We first discuss the MNP model used by Quinn, Martin, and Whitford (1999) to study the election of 1979 in Britain, and then extend the analysis to MNL models of the 1992 and 1997 elections. In all three cases the estimated parameters give low convergence coefficients. Theorem 3.1 then implies that convergence to the electoral center should have occurred under vote-share maximization.

Since there is no evidence of convergence by the major parties in Britain (Alvarez, Nagler, and Bowler, 2000) we develop the activist valence model mentioned in the previous chapter. We now allow the contributions of activists to indirectly enhance the valence of the party leader. The principal result we offer shows that there is a tradeoff to be made between the leader's "exogenous" valence and this "indirect" valence induced by the activists for the party.

We suggest that the valence of the Labour Party, under Tony Blair, increased in the period up to 1997. As a consequence of the relative decline of the Conservative Party leader's valence, the Conservative Party was obliged to depend increasingly on activist support, forcing it to adopt a more radical position. Conversely, Blair's high valence weakened

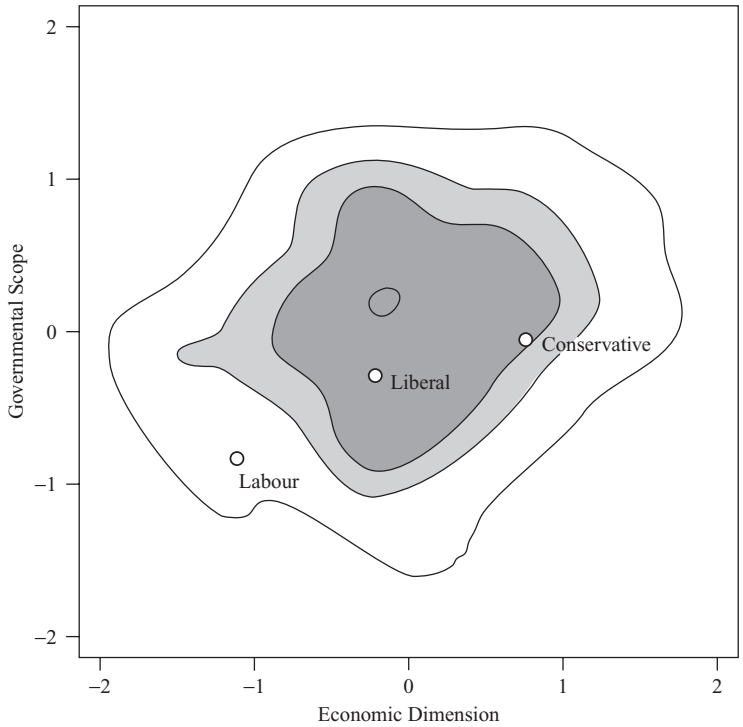


Figure 7.1. Distribution of voter ideal points and party positions in Britain in the 1979 election, for a two-dimensional model, showing the highest density contours of the sample voter distribution at the 95%, 75%, 50%, and 10% levels.

his dependence on activists and allowed him to adopt a more centrist, election-winning position.

We propose the following hypothesis:

Hypothesis 7.1: *If policy choices in a plurality electoral system appear to conflict with vote-maximization in the simple exogenous valence model, then this is due to the influence of activists for the party.*

7.1 THE ELECTIONS OF 1979, 1992, AND 1997

We now examine this indirect role played by activists in determining the policy decisions of parties in Britain. To set the scene, Figure 7.1 presents the estimated positions of the party principals of the three major parties at the election of 1979, in a two-dimensional policy space.

Just as in the case of the Netherlands, the estimation used the middle level Elites Study (ISEIUM, 1983) coupled with the Rabier-Inglehart (1981) Eurobarometer study (see Quinn, Martin, and Whitford, 1999, and Schofield, 2005b for further details.) The electoral variances were 0.605 on the first axis and 0.37 on the second, giving a total variance of 0.975. For the MNL model incorporating sociodemographic characteristics, the spatial coefficient was $\beta = 0.27$ and the convergence coefficient

Table 7.1. *Elections in Britain in 2005, 2001, 1997, and 1992.*

2005	Seats	Seats %	Vote %
Party			
Labour Party (LAB)	356	55.2	35.3
Conservative Party (CON)	198	30.7	32.3
Liberal Democrats Party (LIB)	62	9.6	22.1
Total: major parties	615	95.2	89.7
Scottish National Party (SNP)	6	1.0	1.5
Plaid Cymru (PC)	3	0.5	0.6
Independent, Respect, KHHC	3	0.5	0.3
Northern Ireland in total, comprising:	18	2.8	2.5
Ulster Unionists (UU)	1		0.5
Democratic Union	9		0.9
SDLP	3		0.5
Sinn Fein	5		0.6
Total	646		

2001	Seats	Seats %	Vote %
Party			
Labour Party (LAB)	412	62.6	40.1
Conservative Party (CON)	166	25.1	31.7
Liberal Democrats Party (LIB)	52	7.8	18.6
Total: major parties	630	97.5	90.4
Scottish National Party (SNP)	5	0.8	1.8
Plaid Cymru (PC)	4	0.6	0.7
Independent	1		
Northern Ireland in total, comprising:	18	2.8	
Ulster Unionists (UU)	6		
Democratic Union	5		
SDLP	3		
Sinn Fein	4		

(*continued*)

Table 7.1. *(continued)*

1997	Seats	Seats %	Vote %
Party			
Labour Party (LAB)	419	63.6	44.4
Conservative Party (CONS)	165	25.0	31.4
Liberal Democrats Party (LIB)	46	6.9	17.2
Total: major parties	630	95.5	93.0
Scottish National Party (SNP)	6	0.9	2.0
Plaid Cymru (PC)	4	0.6	0.5
Independent	1		
Northern Ireland in total, comprising:	18	2.7	
Ulster Unionists (UU)	10		
UK Unionists	1		
Democratic Union	2		
SDLP	3		
Sinn Fein	2		

1992	Seats	Seats %	Vote %
Party			
Labour Party (LAB)	271	41.6	34.5
Conservative Party (CON)	336	51.6	41.9
Liberal Democrats Party (LIB)	20	3.1	17.9
Total: major parties	627		94.3
Scottish National Party (SNP)	3	0.5	1.9
Plaid Cymru (PC)	4	0.6	0.5
Northern Ireland	17	2.6	3.3

was calculated to be 0.26. The eigenvalues of the Hessians of all parties at the origin were then negative. With the MNP model the coefficient was even smaller—0.08—and the eigenvalues of all parties close to −1.0. As in the previous example from the Netherlands, the origin was an LSNE. Indeed, the estimation suggests that the origin was a PSNE. This inference conflicts with the estimated positions of the parties given in Figure 7.1.

To pursue this contradiction further, we now consider more recent elections. Table 7.1 gives details on the elections of 1992, 1997, 2001, and 2005 in Britain. As usual with plurality electoral rules, small gains in vote-share lead to large gains in seat share. British National Election Surveys (British Election Studies, 1992, 1997) were used to construct a single-factor model of the voter distribution (see Table 7.3 for the survey questions). We shall call this factor the *economic dimension*. Note that

Table 7.2. *Factor weights from the British National Election Survey for 1997.*

Britain (without Scotland), 1997		
Issue		Factor weights
1.	Unemployment and Inflation	0.265
2.	Taxation and Services	0.223
3.	Nationalization	0.225
4.	Redistribution	0.318
5.	European Community	0.087
6.	Women's Rights	0.149
Scotland, 1997		
Issue		Factor weights
1.	Unemployment and Inflation	0.127
2.	Taxation and Services	0.104
3.	Nationalization	0.156
4.	Redistribution	0.580
5.	European Community	0.008
6.	Women's Rights	0.137
7.	Scottish Nationalism	0.101

Table 7.3. *Question Wordings for the British National Election Surveys for 1997.*

1. Do you feel that the government's top priority should be getting people back to work, keeping prices down, or somewhere in between?
2. Do you feel the government should raise taxes and spend more money on health and social services, or do you feel they should cut taxes and spend less on these services?
3. Do you feel the government should nationalize or privatize more industries?
4. Do you feel the government should be more concerned with equalizing people's incomes, or less concerned?
5. Do you feel Britain should unite with the European Union or protect its independence from the European Union?
6. Do you feel women should share an equal role in business, industry, and government, or do you feel a woman's place is in the home?
7. Do you feel Scotland should a) become independent, separate from the UK and the European Union, b) become independent, separate from the UK, but a part of the European Union, c) remain part of the UK, with its own elected assembly, with taxation and spending powers, or d) remain as it is?

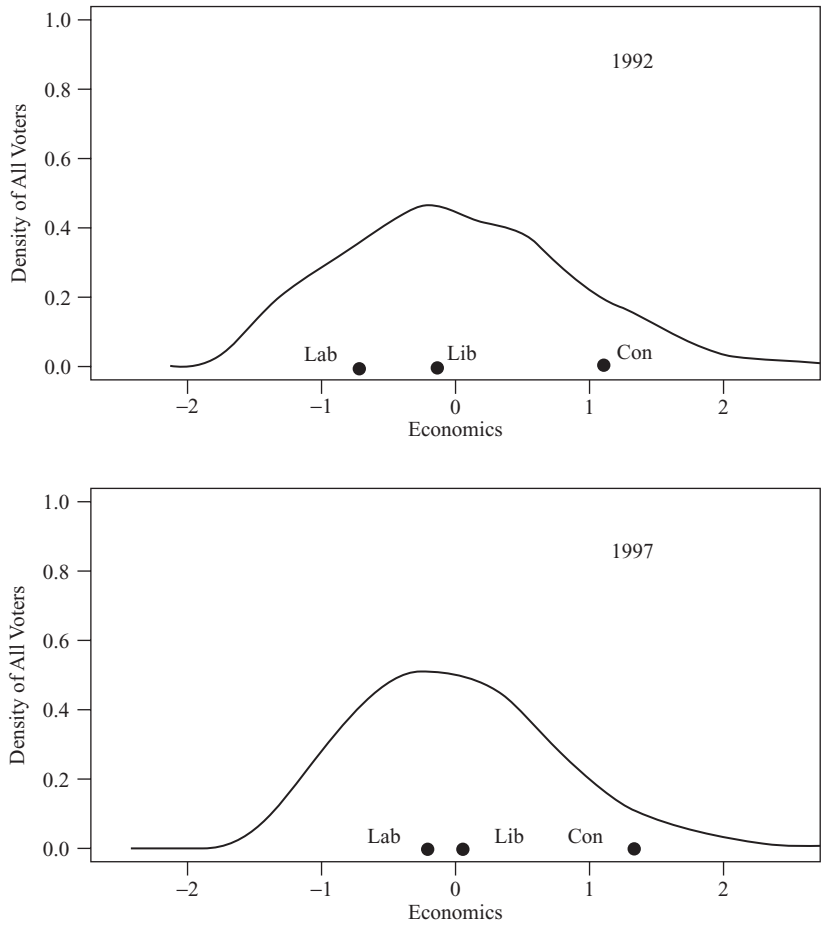


Figure 7.2. Estimated party positions in the British Parliament in 1992 and 1997, for a one-dimensional model (based on the National Election Survey and voter perceptions) showing the estimated density function (of all voters outside Scotland).

Scottish Nationalism is, of course, an issue in Scotland but not in the rest of the country.

Table 7.2 gives the factor weights for 1997 for Britain (subdivided into Britain without Scotland, and Scotland alone). The 1992 weights were very similar. Figure 7.2 presents the estimated distribution of voter ideal points (for voters outside Scotland), on the basis of this single economic dimension. The voter distribution in Scotland was somewhat similar, though less symmetric, and skewed to the left. The party positions for the Labour Party (Lab), Liberal Democrat Party (Lib), Conservative

Table 7.4. *Sample and Estimation Data for Elections in Britain, 1992–1997.*

1992	Sample %	Coefficients	Confidence interval	Correct prediction %
UK without Scotland		$\beta = 0.56$	[0.50, 0.63]	50.0
Conservative	49.3	$\lambda = 1.58$	[1.38, 1.75]	62.9
Labour	32.4	$\lambda = 0.58$	[0.40, 0.76]	47.4
Liberals	18.2	$\lambda = 0.00$		19.7
Scotland		$\beta = 0.50$	[0.31, 0.67]	35.6
Conservative	30.2	$\lambda = 1.68$	[1.18, 2.21]	48.6
Labour	31.4	$\lambda = 0.91$	[0.38, 1.51]	37.2
SNP	25.9	$\lambda = 0.77$	[0.26, 1.30]	29.5
Liberals	12.3	$\lambda = 0.00$		12.8
1997	Sample %	Coefficients	Confidence interval	Correct prediction %
UK without Scotland		$\beta = 0.50$	[0.44, 0.56]	45.7
Conservative	31.9	$\lambda = 1.24$	[1.03, 1.44]	45.2
Labour	49.6	$\lambda = 0.97$	[0.85, 1.07]	56.1
Liberals	18.5	$\lambda = 0.00$		19.0
Scotland		$\beta = 0.50$	[0.40, 0.64]	40.5
Conservative	14.2	$\lambda = 0.92$	[0.58, 1.24]	33.7
Labour	52.7	$\lambda = 1.33$	[1.10, 1.57]	56.3
SNP	20.4	$\lambda = 0.42$	[0.16, 0.72]	21.5
Liberals	12.5	$\lambda = 0.00$		12.8

Party (Con), and Scottish National Party (SNP) were inferred by taking average voter perceptions of the locations of these parties.

The positions Lab, Lib, and Con in the two election years (for voters outside Scotland) were given by the vectors

$$\mathbf{z}_{92} = (z_{lab}, z_{lib}, z_{con}) = (-0.65, -0.11, +1.12) \tag{7.1}$$

$$\mathbf{z}_{97} = (-0.2, +0.06, +1.33). \tag{7.2}$$

See Figure 7.2.

In 1992 the SNP position was perceived to be $z_{snp} = -0.3$, and in 1997 it was $+0.14$.

Using these data, MNL models were constructed for the four cases in 1992 and 1997, for Scotland and the rest of the country. These models allowed us to estimate the exogenous valence terms, as in Table 7.4.

The estimated coefficients in the two elections were

$$(\lambda_{con}, \lambda_{lab}, \lambda_{lib}, \beta)_{1997} = (+1.24, 0.97, 0.0, 0.5) \tag{7.3}$$

$$(\lambda_{con}, \lambda_{lab}, \lambda_{lib}, \beta)_{1992} = (+1.58, 0.58, 0.0, 0.56) \tag{7.4}$$

These estimates are compatible with extensive survey research which demonstrates the relationship between positive attitudes to party leaders and voting intentions (King, 2002; Clarke et al., 2004). Notice that the Conservative Party valence fell, while that of the Labour Party rose. These changes in valences are presumed to be independent of the apparent *perceived move away from the electoral center* by the Conservative Party, and the *perceived move toward the electoral center* by the Labour Party.

The empirical model was relatively successful, in the sense that the model prediction success rate was approximately 50 percent. As Table 7.4 indicates, the 95 percent confidence intervals for the valences of the Labour Party and Conservative Party exclude zero. We infer that the valence differences between the Liberal Democrats and both Labour and the Conservatives are non-zero. The log marginal likelihood of the 1997 MNL model with valence was −531, giving a Bayes' factor of 75 over the MNL model without valence. For Britain without Scotland in 1997 we can use the results of Chapter 3 to compute the convergence coefficient for these two elections. Because the model is MNL we use the formal model based on the Type I extreme value distribution. Since the model is one-dimensional, the electoral variance on the single axis is normalized to be 1.0. Because the valence of Lib is normalized to be zero, we find that for 1997 the eigenvalue of the Hessian of the Liberal Democrat Party at the joint origin is −0.28. A similar value of −0.18 was obtained for 1992.

The results of Chapter 3 thus imply convergence for the formal model. Even using the upper estimated bound of the parameters, we obtain similar estimates for the eigenvalues. Thus, on the basis of the formal model, we can assert with a high degree of certainty that the low-valence party, the Liberal Democrats, can be located at an LNE at the origin if all other parties also locate there. According to the model, the vote-share of the Liberal Democrat Party would have been 13 percent in 1992, or 14 percent in 1997, had the other two parties located at the origin in these elections. Because the two major parties did not locate at the origin, the actual vote-share of 17 percent to 18 percent for the Liberals is compatible with these estimates.

Thus, under the assumptions of fixed or exogenous valence, vote-maximization, and unidimensionality, a version of the mean voter theorem

should have been valid for the British elections of 1992 and 1997. Although Figure 7.2 indicates that a position close to the center was adopted (or seen to be adopted) by the Liberal Democrats in 1992 and 1997, this was not so obvious for the Labour Party, and was clearly false for the Conservative Party. Indeed, for both subsets of the electorate (within Scotland and outside), the Labour Party was perceived to approach closer to the center between 1992 and 1997, but the Conservative Party was perceived to become more radical.

7.2 ESTIMATING THE INFLUENCE OF ACTIVISTS

In an attempt to account for the obvious disparity between the conclusions of the vote-maximization model and party location, we considered the hypothesis that party location was determined by party elites. As we proposed in the discussion of the Netherlands, the location of the delegates or elite positions can be used to determine the position of maximum activist support for each party. This, in turn, will determine the precise equilibrium location of each party. While activists contribute time and money and affect overall political support for the party, the activist locations will tend to be more radical than the average voter. This presents the party leader with a complex "optimization problem." We use the activist valence argument to offer a conjecture about how party leaders deal with this problem by choosing differing policy positions to present to the electorate (Robertson, 1976).

Figure 7.3 gives the estimated voter distribution in the British election of 1997, based on the British National Election Survey but using the two dimensions obtained from factor analysis. (See Table 7.2 for the factor weight associated with this second "European" dimension.) Positions of MPs of each party were estimated on the basis of an MP sample response to the British National Election questionnaire. For each party, the average of the party MP positions was used as an estimate of the position of each party "principal." The estimated positions of individual MPs in the survey are given in Figure 7.4.

A considerable difference among ideal points of MPs within parties can be observed. The second, "vertical," axis in Figure 7.3 is determined by "pro-Europe" versus "pro-British" (or "anti-Europe") attitudes. The party principal positions of Labour (LAB) and Conservatives (CONS) are separated on both axes, but more so on the Europe axis. The small number of Ulster Unionists (UU) appeared to be similar to other Conservatives, but more extreme on the pro-British axis. The single sampled MP

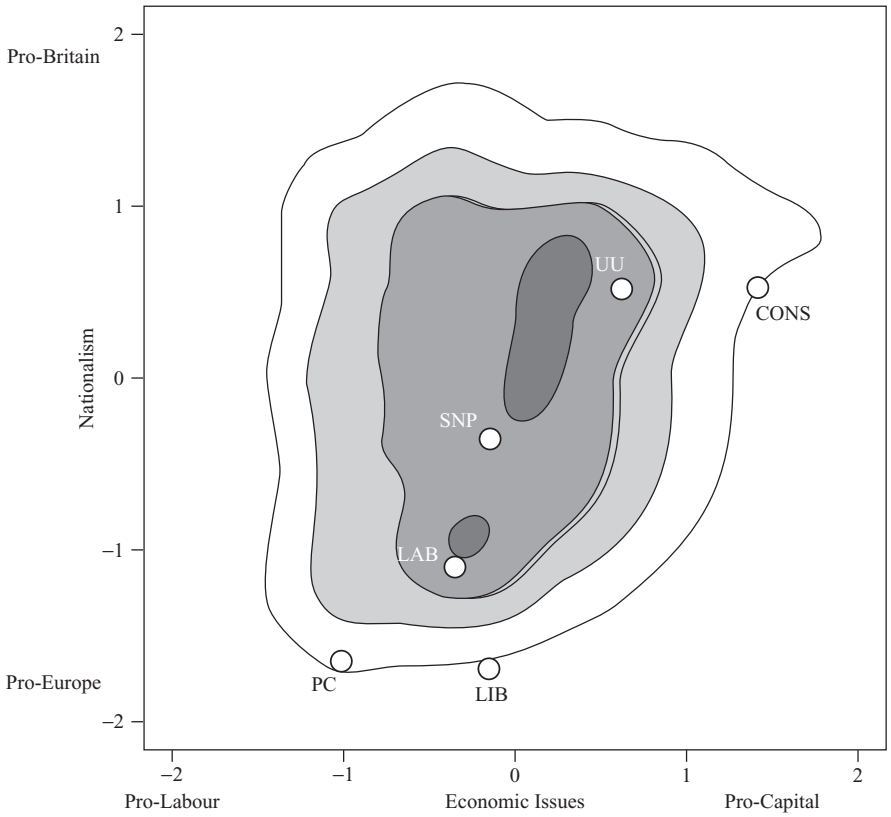
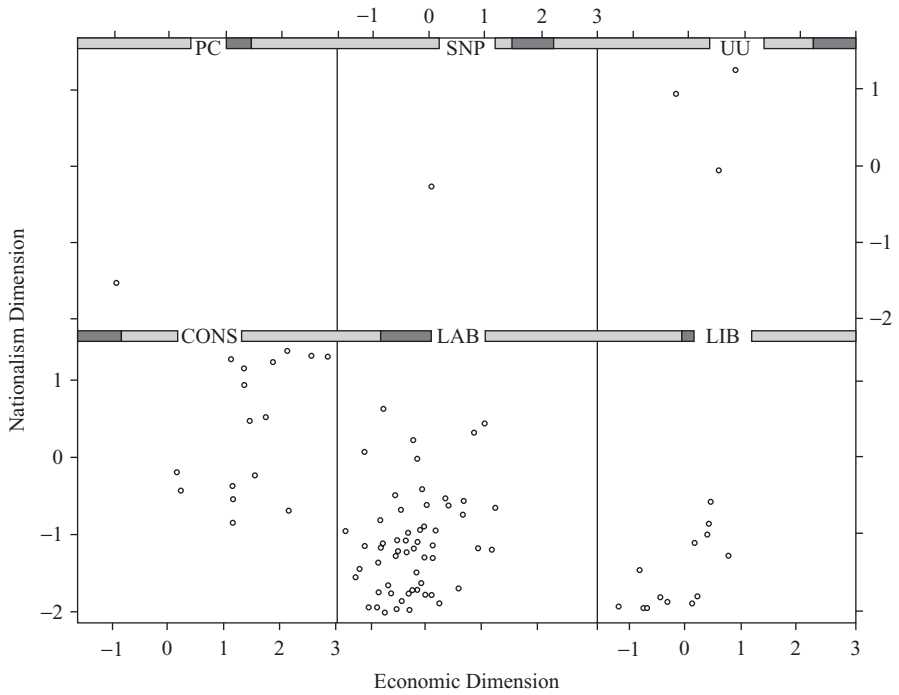


Figure 7.3. Estimated party positions in the British Parliament for a two-dimensional model for 1997 (based on MP survey data and the National Election Survey) showing highest density contours of the voter sample distribution at the 95%, 75%, 50%, and 10% levels.

for Plaid Cymru (PC, from Wales) was similar to other left, pro-Europe Labour MPs, while the single sampled member of the SNP (from Scotland) also resembled other Labour MPs who were less pro-Europe. The fifteen sampled Liberal Democrats (with principal position denoted LIB) were all somewhat left of center, and very pro-Europe.

The empirical estimates presented above, and those based on the one-dimensional model, suggest that the Labour valence had increased from 1992 to 1997. In terms of this empirical model, this increase was independent of the greater voter support induced by the party moving closer to the electoral center under Tony Blair. We now consider the following hypothesis.



Key

PC:	**Plaid Cymru (Welsh Nationalist Party)**
SNP:	**Scottish Nationalist Party**
UU:	**Ulster Unionists**
CONS:	**Conservative Party**
LAB:	**Labour Party**
LIB:	**Liberal Democrat Party**

Figure 7.4. Estimated MP positions in the British Parliament in 1997, based on MP survey data and a two-dimensional factor model derived from the National Election Survey.

Hypothesis 7.2: *The apparent move by the Labour Party toward the electoral center between 1992 and 1997 was a consequence of the increase of the valence of the leader of the party, rather than a cause of this increase.*

To develop this hypothesis, we shall assume that the party principal positions given in Figure 7.3 do indeed represent in some sense the average location of party activists. We then attempt to model the influence of activists on optimal party position.

Note first that the positions perceived by the electorate in 1997 and given by the vector $\mathbf{z}_{97} = (-0.2, +0.06, +1.33)$ are very close indeed to the projections of the positions of the party principals onto the economic axis in Figure 7.3. This leads us to infer that the party principal positions do influence perceived party positions. Just as we did in Chapter 6, we can examine whether the party principal positions can be a local equilibrium to a simple vote-maximizing game. The Technical Appendix to this chapter shows that when we include the second European axis then the Liberal Party eigenvalue on this axis is positive. This calculation is based on zero electoral covariance between the two axes, and the greater electoral variance on the second "Europe" axis. In other words, if all three parties were at the electoral center, then the positive eigenvalue of the Hessian on the second dimension would give the Liberal Democrat Party leader an incentive to change position, but only on the second axis. We may infer that the average preferred position of the party MPs would induce the party leader to adopt a pro-Europe position. If the Liberal Party were to adopt a pro-Europe position as indicated by its principal's position, then the logic of vote-maximization would induce the Labour Party leader to make a similar move. Thus, the positions LAB and LIB are compatible with the simple vote-model with exogenous valence.

This conclusion still leaves unexplained the perceived location of the Conservative Party (CONS).

Under the assumptions of the fixed valence model, the Conservative Party should have adopted a vote-maximizing position closer to the origin than did the Labour Party. We suggest that the Conservative Party did not converge on the mean because of the subtle interrelationship between exogenous valence and activist valence. Blair's increasing valence in the period up to 1997 resulted in a decrease in the importance of the activists in the party (Seyd and Whiteley, 2002). This led to a more centrist vote-maximizing strategy by Labour, associated with a larger "electoral sphere of influence." In contrast, decreasing Conservative leader valence led to an increase in the importance of activists. To maintain grassroots support, the Conservatives were forced to adopt quite radical positions, both on the question of Europe and on economic issues.

Schofield (2003b, 2004, 2005a,b) presents a formal analysis of these differing valence effects. It is consistent with this more general model that all parties at the election of 1997 were at vote-maximizing positions. We now turn to this extension.

In essence, the model we propose suggests that if the leader of one party benefits from increasing valence, then the party's optimal strategy

will be to move toward the political center in order to take advantage of the electoral benefits. In contrast, a party, such as the Liberal Democrat Party, whose leader is unable to take advantage of high valence, cannot expect to gain commanding electoral support, even when the party adopts a centrist position.

In the following section, we present the underlying formal electoral model that we use, and state the constraint on the model parameters, which is sufficient for concavity and thus for existence of a noncentrist pure strategy Nash equilibrium. Indeed we show that the joint vote-maximizing positions will generally not be at the voter mean. We briefly discuss the optimality condition when both "exogenous" valence and "activist" valence are involved, and indicate why activists become more relevant when leader valence falls.

7.3 A FORMAL MODEL OF VOTE-MAXIMIZING WITH ACTIVISTS

We return briefly to the model we introduced in Chapter 3 so that we can extend it here to account for noncentrist political choice in the case of Britain.

In the model with valence, the stochastic element is associated with the weight given by each voter i to the perceived valence of the party leader. We now allow valence to be indirectly affected by party position.

Definition 7.1 ***The formal model*** $M(\boldsymbol{\lambda}, A, \boldsymbol{\mu}; \boldsymbol{\Psi})$

In the general valence model, let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of policy positions. Given $\mathbf{z}$, each voter i is described by a vector $\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$, where the utility of voter i, at the party declaration vector z, is given by

$$u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - A_{ij}(x_i, z_j) + \varepsilon_j. \tag{7.5}$$

The term $A_{ij}(x_i, z_j)$ is derived from a general metric. The errors $\{\varepsilon_j\}$ are again assumed to be distributed by the Type I extreme value distribution, Ψ. For party j, the vote-share $\mathcal{E}_j$ is the expectation

$$\frac{1}{n} \sum_i \rho_{ij}.$$

For convenience, in terminology below we shall refer to the effect of candidate strategies on the expected vote-share function $\mathcal{E}_j$, through change in $\mu_j(z_j)$, as the *valence* component of the vote. Change in $\mathcal{E}_j$ through

the effect on the policy distance measure $A_{ij}(x_i, z_j)$ we shall refer to as the *nonvalence*, or *policy* component. We discuss this activist model below. One important modification of the pure spatial model that we make is that the salience of different policy dimensions may vary among the electorate. More precisely, we assume that

$$A_{ij}(x_i, z_j) = ||x_i - z_j||_i^2 \tag{7.6}$$

may vary with different i.

The term $\mu_j(z_j)$ is called the *activist valence* of the party. Notice that activist valence is now a function of the leader position z_j. To distinguish the two forms of valence, we call λ_j the *exogenous valence*.

We propose an extension of the model presented in Chapter 3 to include activist valence. In this new model the first-order condition for vote-share maximization is not satisfied at the mean. We now briefly sketch the procedure for determining the first-order condition. The choice of voter i now depends on the comparison *vector*

$$g_{ij}(\mathbf{z}) = (\ldots, \delta_{ik}^2 - \delta_{ij}^2 - \lambda_k + \lambda_j - \mu_k(z_k) + \mu_j(z_j), \ldots) : \text{for all } k \neq j), \tag{7.7}$$

where $\delta_{ij}^2 = ||x_i - z_j||_i^2$, etc.

Section 7.6.2 in the appendix to this chapter shows that the first-order solution z_j^* is given by the balance equation

$$z_j^* = \frac{1}{2}\frac{d\mu_j}{dz_j} + \sum_{i=1}^{n} \alpha_{ij} x_i. \tag{7.8}$$

In this equation, the coefficients α_{ij} depend on $\{\lambda_k, \lambda_j, \mu_k(z_k), \mu_j(z_j)\}$ and are increasing in $\{\lambda_j, \mu_j(z_j)\}$ and decreasing in $\{\lambda_k, \mu_k(z_k) : k \neq j\}$. The actual coefficients will depend on the distribution assumption made on the errors. For convenience let us write

$$\sum_i \alpha_{ij} x_i = \frac{d\mathcal{E}_j^*}{dz_j}. \tag{7.9}$$

Then we can rewrite the balance equation as

$$\left[\frac{d\mathcal{E}_j^*}{dz_j} - z_j^*\right] + \frac{1}{2}\frac{d\mu_j}{dz_j} = 0. \tag{7.10}$$

The bracketed term on the left of this expression is the *marginal electoral pull* and is a gradient vector pointing toward the *weighted electoral mean*. This weighted electoral mean is simply that point at which the electoral pull is zero. In the case $\mu_j = 0$ for all j, then for each party j, it is

obvious that all α_{ij} are identical, so $z_j^* = \frac{1}{n}\Sigma x_i$. This gives, as in the proof of Theorem 3.1, the point where the marginal electoral pull is zero.

The vector $\frac{d\mu_j}{dz_j}$ "points toward" the position at which the party's activist valence is maximized. We may term this vector the *(marginal) activist pull*. When this marginal or gradient vector $\frac{d\mu_j}{dz_j}$ is increased, then the equilibrium is pulled away from the weighted electoral mean, and we can say the "activist effect" is increased. On the other hand, if the activist valence functions are fixed, but λ_j is increased, or the terms $\{\lambda_k : k \neq j\}$ are decreased, then the vector $\frac{d\mathcal{E}_j^*}{dz_j}$ increases in magnitude, the equilibrium is pulled toward the weighted electoral mean, and we can say the "electoral effect" is increased.

When the first-order condition is satisfied for all parties at the vector z* then say z* is a *balance solution* or satisfies *the balance condition*.

Moreover, if the activist effect is concave, then the second-order condition (or the negative definiteness of the Hessian of the "activist pull") will guarantee that a vector z* that satisfies the balance condition will be an LSNE. Schofield (2003b) proved this result for iind errors. These observations then give the following theorem.

Theorem 7.1 *Consider the vote-maximization model $M(\boldsymbol{\lambda}, A, \boldsymbol{\mu}; \Psi)$ based on a disturbance distribution Ψ and including both exogenous and activist valences. The first-order condition for $\mathbf{z}^*$ to be an equilibrium is that it satisfies the balance condition. Other things being equal, the equilibrium position z_j^* will be closer to a weighted electoral mean the greater is the party's exogenous valence, λ_j. Conversely, if the activist valence function μ_j is increased, due to the greater willingness of activists to contribute to the party, the nearer will z_j^* be to the activist-preferred position. If all activist valence functions are highly concave, in the sense of having negative eigenvalues of sufficiently great magnitude, then the balance solution will be a PNE.*

The proof of this result is given in Section 7.6.2 in the Technical Appendix.

Figure 7.5 illustrates this result, in a two-dimensional policy space derived from the data as presented in Figure 7.3. We have observed that overall Conservative valence dropped from 1.58 in 1992 to 1.24 in 1997, while the Labour valence increased from 0.58 to 0.97. These estimated valences include both exogenous valence terms for the parties and the activist component. Nonetheless, the data presented in Clarke et al., (1998,

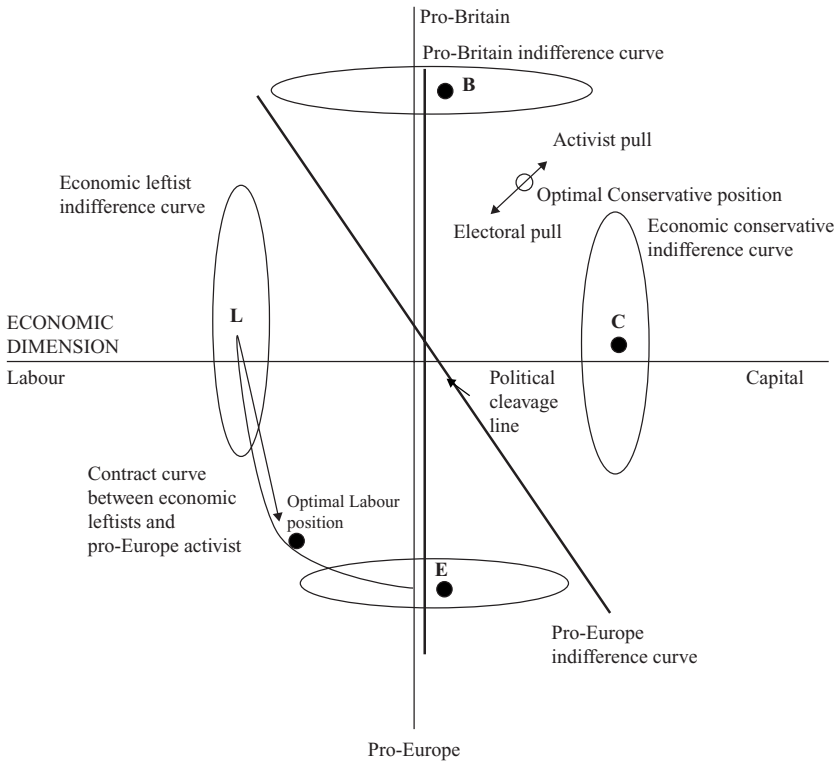


Figure 7.5. Illustration of vote-maximizing party positions of the Conservative and Labour leaders for a two-dimensional model.

2004) suggest that the Labour exogenous valence λ_{lab}, due to Blair, rose in this period. Conversely, the relative exogenous term λ_{con} for the Conservatives fell. Since the coefficients in the equation for the electoral pull for the Conservative Party depend on $\lambda_{con} - \lambda_{lab}$, the effect would be to increase the marginal effect of activism for the Conservative Party, and pull the optimal position away from the party's weighted electoral mean.

Indeed, it is possible to include the effect of two potential activist groups for the Conservative Party: one "pro-British," centered at the position marked B in Figure 7.5 and one "pro-Capital," marked C in the figure. The optimal Conservative position will be determined by a version of the balance equation, but one that equates the "electoral pull" against the two "activist pulls." Since the electoral pull fell between the elections, the optimal position z_{con}* will be one that is "closer" to the locus of points that generates the greatest activist support. This locus is where the joint

marginal activist pull is zero. This locus of points can be called the *activist contract curve* for the Conservative Party.

Note that in Figure 7.5, the indifference curves of representative activists for the parties are described by ellipses. This is meant to indicate that preferences of different activists on the two dimensions may accord different saliences to the policy axes. The activist contract curve given in the figure, for Labour, say, is the locus of points satisfying the activist equation $\frac{d\mu_{lab}}{dz_{lab}} = 0$. This curve represents the balance of power between Labour supporters most interested in economic issues concerning labour (centered at L in the figure) and those more interested in Europe (centered at E). The optimal positions for the two parties will be at appropriate positions that satisfy the balance condition. In other words, each optimal position will lie on a locus generated by the respective activist contract curves and the party's weighted electoral mean point where the electoral pull is zero. As the theorem states, because the coefficients of the weighted electoral mean for Labour depend on $\lambda_{lab} - \lambda_{con}$, we would expect a rise in this difference to pull the party "nearer" the electoral origin.

In Chapter 8 we apply this model and show that the equation for this contract curve from the preferred pro-European point (s_E, t_E) to the pro-labour point (s_L, t_L) is given by the equation

$$\frac{(y - t_E)}{(x - s_E)} = S\frac{(y - t_L)}{(x - s_L)} \tag{7.11}$$

where

$$S = \frac{b^2}{a^2} \cdot \frac{e^2}{f^2}. \tag{7.12}$$

Here, $\frac{b}{a} > 1$ measures the degree to which activists for the Labour Party are more concerned with economics rather than with Europe, while $\frac{e}{f} > 1$ measures the opposite ratio for Europe activists. Obviously, with identical saliences, $S = 1$, and the contract curve is linear.

The "political cleavage line" in the figure is a representation of the electoral dividing line if there were only the two parties in the election. The weighted electoral mean should lie on the intersection of the political cleavage line and the line connecting the two party positions.

As Theorem 7.1 indicates, when the relative exogenous valence for a party falls, then the optimal party position will approach the activist contract curve. Moreover, the optimal position on this contract curve will depend on the relative intensity of political preferences of the activists of each party. For example, if grassroots "pro-British" Conservative Party

activists have intense preferences on this dimension, then this feature will be reflected in the activist contract curve and thus in the optimal Conservative position.

For the Labour Party, it seems clear that two effects are present. Blair's high exogenous valence gave an optimal Labour Party position that was closer to the electoral center than the optimal position of the Conservative Party. Moreover, this affected the balance between pro-Labour or "old left" activists in the party and "New Labour" activists, concerned with modernizing the party through a European style "social democratic" perspective. This inference, based on our theoretical model, is compatible with Blair's successful attempts to bring "New Labour" members into the party (See Seyd and Whiteley, 2002, for documentation). To relate this analysis to the idea of a party principal offered in earlier chapters, we may say that the both parties are characterized by competition between opposed party principals, located at L and E for Labour, and at C and B for the Conservative Party.

7.4 ACTIVIST AND EXOGENOUS VALENCE

Our purpose in introducing the notions of "exogenous valence" and "activist valence" has been to explore the possibility that the relationship between the party and the potential party activists will be affected by the exogenous valence of the leader. Party leaders can either exploit changes in their valence, or become victims of such changes. The theoretical framework that we have offered is intended to provide an explanation for the seemingly radical policy choices of the Labour Party during the period of Conservative government from 1979 until about 1992. By *radical* we mean simply that the party adopted positions that appeared to be far from the electoral center. In recent years, the Conservative Party appears to have adopted policy choices that are radical, but on the European axis. According to the model just presented, these policy choices are perfectly rational in that they are designed to maximize votes. A similar argument will be applied in the next chapter to apparently radical policy choices in the Republican–Democrat electoral competition in the United States.

Although the elections of the 1980s are not examined here, we conjecture that, during this period, the electorate, in general, viewed Margaret Thatcher as more competent than her rival Neil Kinnock. In the model that we have proposed, Thatcher's perceived degree of competence, or exogenous popularity valence, was relatively independent of the particular policies that she put forward for the party. It is, of course, a simplification

to assume that the perception of her competence was independent of the policy preferences, or the sociodemographic characteristics, of individual voters. In principle it would be possible to refine the above model by examining optimal party positions with respect to these variables.

The simple model presented above suggests that the low average perception of Kinnock's competence, in comparison to Thatcher's, obliged him to pay great weight to the activists within the Labour Party. As a consequence, both Labour and Conservative Parties adopted vote-maximizing, but relatively radical, positions far from the electoral center. Even though the Liberal or Liberal Democrat Party adopted a centrist position, its low exogenous valence kept it in the third party position.

It is possible that Thatcher was deposed from the leadership of the Conservative Party precisely because her falling personal valence led to greater electoral weight for powerful activist elements in her party. Indeed, the party mandarins may have understood the nature of the balance condition, although Thatcher probably denied it. We have, somewhat simplistically, characterized the optimal activist intraparty balance in terms of a contract curve. In fact, the process by which the party leader is selected by the competing party principals can be expected to be highly contentious.

During Major's tenure as leader of the Conservative Party, the debacle over the value of sterling and the change to John Smith as the Labour Party leader led to a transformation in the relative exogenous valences of the two parties. Clarke, Stewart, and Whiteley (1998) note the rapid change in voter intentions in favor of Labour when John Smith took over from Kinnock, in July of 1992, and again when Blair took over the leadership of the party in July of 1994. Time-series analyses of voter intentions show quite clearly how these are determined by perceptions of government competence in dealing with economic problems (see also Seyd and Whiteley, 1992; Clarke et al., 1995, 1997, 1998; Clarke, Sanders et al., 2004).

In addition, however, voting intentions will be affected by judgments about the presumed "fitness" of the party leaders. Our estimates of these average electoral judgments suggest that Tony Blair was perceived to be much more fit than earlier Labour Party leaders to head the government. By themselves, however these changes in electoral judgments would not have given the Labour Party such a clear majority in 1997. The model that we propose suggests that Blair's enhanced valence made it possible for him to persuade the "Old Labour" activists of the party that it was in the best interests of the party to move to a much more centrist policy position. This transformation of the party was electorally credible, and led to the overwhelming Labour Party victory in 1997.

Up until 2003, the Conservative Party leaders, William Hague and Iain Duncan Smith, seemed to have been deemed by the electorate to have low exogenous valence. One way to estimate exogenous valence of a leader is to take as a proxy the difference between the proportion of the electorate who are satisfied with the leader and those who are not. The valence proxy for Blair in 1997 was about 0.5, whereas the valence proxy for Hague was about −0.2. In 2002, the valence proxy for Duncan Smith was about −0.1. Consistent with our model and with the estimations given above, Conservative Party activists have exerted their power to move the party further from the electoral origin. This led, first of all, to the Conservative Party defeat in 2001, and to the struggle inside the party over which activist group would construct the party policy in the future. The leadership contest was won by Michael Howard in October 2003. By the election of 2005, the proxies of both Howard, the Conservative Party leader, and Blair, were similar at about −0.2.

Recent international events, and Blair's responses to them, appear to have decreased his personal valence. As Table 7.1 indicates, in May 2005 the Labour Party lost nearly sixty seats that it had won in 2001. The drop of nearly 6 percent of the popular vote would appear to be entirely due to the increased electoral mistrust caused by Blair's handling of the Iraq situation. There has been a move to force Blair to resign in favor of Gordon Brown, but at the time of writing, it is fairly subdued.

While the number of seats for the Conservatives increased by thirty over the 2001 figure, the popular vote-share hardly increased over the levels for 1997 and 2001. This was obviously the reason that Howard announced his resignation "sooner rather than later" from the party leadership immediately after the election. In December, 2005, the party chose a new leader, David Cameron. It remains to be seen whether the electorate will judge him to be a competent leader.

The model proposed here suggests that this change in Blair's valence from 2001 to 2005 may induce conflict inside the Labour Party, between economic activists, on the one hand, and pro-Europe social democrats on the other. Indeed, a third axis of political choice, concerned with the Middle East, may have come into existence recently.

7.5 CONCLUSION

We have presented the electoral model for Britain in order to contrast the political configurations of party positions that are possible in a polity

whose electoral system is based on plurality rule with those in polities such as Israel, Italy, and the Netherlands, based on proportional representation.

We contend that the result on the formal model presented in Theorems 3.1 and 7.1, together with the empirical analysis, indicate that the vote-maximizing principle (with valence), together with the simple structure of the stochastic vote-model, accounts for party divergence in particular and party behavior more generally. The analysis also suggests that party activism is an essential component of any electoral model.

It has been argued that proportional rule and plurality lead to very different political patterns (Riker, 1953, 1982; Duverger, 1954; Taagepera and Shugart, 1989). Although Theorem 7.1 of this chapter (together with Theorem 3.1) is based on the simple assumption of vote-maximization it should be possible to extend it to deal with seat maximization, under different electoral rules. This could provide a theoretical explanation for different configurations observed in multiparty polities.

The various spatial maps that we presented here and in the chapters on Israel, Italy, and the Netherlands, demonstrate considerable variety. One conclusion that can be drawn from the two electoral theorems is that centrifugal and centripetal forces will both be relevant. This follows because activist coalitions will typically occur on the electoral periphery. An argument to this effect can be seen as the basis for Duverger's contention that the "centre does not exist in politics" (Duverger, 1954: 215; Daalder, 1984). In line with this assertion, Theorems 3.1 and 7.1 suggest, contrary to the mean voter theorem, that a crowded political center is highly unlikely.

Under plurality rule, the two principal parties, if their valences are sufficiently close, will compete over the center, but in such a way that their "spheres of influence" are disjoint. In addition, activists will tend to pull parties to the periphery, as suggested by Figure 7.5.

Under proportional representation, as our discussion of Israel illustrated, high-valence parties such as Labour and Likud may position themselves close to the electoral center. In the absence of a core party, coalition formation requires the assistance of smaller, low-valence parties. These parties will tend to locate at the periphery, either because of the logic of vote-maximization, or again, because of the influence of party activists.

Theorem 7.1 does not necessarily imply that all parties will avoid the electoral center. Our analysis has shown that there are centrist parties in Israel, Italy, the Netherlands, and Britain. However, though their policy

positions would suggest that they should be candidates for government leadership, their low valence may make this difficult.

At a more general level, the spatial theory offered here could be used to construct a theory of party formation. The exogenous valences may be assumed to be random initially. High-valence parties will jockey at the electoral center as described above. Severe competition will generate nonconcavities in voter response and force some parties to retreat from the electoral center. Small, low-valence parties may emerge at the periphery and activist coalitions will form to generate support for their chosen policies. As these activist coalitions become more efficient, the party vote-functions may become increasingly concave (as the eigenvalues of the relevant Hessians become large and negative). This has the effect of stabilizing party positions. This suggests to us why it is that there is, on the one hand, such great variation in party configurations, and on the other, considerable stability within each political system.

7.6 TECHNICAL APPENDIX

7.6.1 *Computation of Eigenvalues*

(i) The one-dimensional model for 1992, with $\beta = 0.56$ gives:

$$\rho_{lib} = \frac{e^0}{e^0 + e^{1.58} + e^{0.58}} = \frac{1}{7.36} = 0.13$$
$$A_{lib} = \beta(1 - 2\rho) = 0.41$$
$$C_{lib} = 0.82 - 1 = -0.18.$$

(ii) The one-dimensional model for 1997, with $\beta = 0.5$ gives:

$$\rho_{lib} = \frac{e^0}{e^0 + e^{1.24} + e^{0.97}} = \frac{1}{7.08} = 0.14$$
$$A_{lib} = \beta(1 - 2\rho) = 0.36$$
$$C_{lib} = 0.72 - 1 = -0.28.$$

(iii) The two-dimensional model for 1997 gives:

$$C_{lib} = (0.72)\begin{pmatrix} 1.0 & 0 \\ 0 & 1.5 \end{pmatrix} - I = \begin{pmatrix} -0.28 & 0 \\ 0 & +0.8 \end{pmatrix}.$$

7.6.2 *Proof of Theorem 7.1*

To simplify the proof, we consider the case with $A_{ij}(x_i, z_j) = ||x_i - z_j||^2$. For the extreme value distribution Ψ we have

$$\rho_{i1}(x_i, z_1) = \left[1 + \Sigma_{j=2}\left[\exp(f_j)\right]\right]^{-1},$$
$$\text{where } f_j = \lambda_j + \mu_j(z_j) - \lambda_1 - \mu_1(z_1) + ||x_i - z_1||^2 - ||x_i - z_j||^2$$

is the comparison function used by i in evaluating party j in contrast to party 1. We then obtain

$$\frac{d}{dz_1}[\rho_{i1}] = -\left[2(z_1 - x_i) - \frac{d\mu_1}{dz_1}\right]\left[1 + \sum_{j=2}\exp(f_j)\right]^{-2}\left[\sum_{j=2}\exp(f_j)\right]$$
$$= \left[2(x_i - z_1) + \frac{d\mu_1}{dz_1}\right][\rho_{i1}][1 - \rho_{i1}].$$

Thus

$$\sum_i \frac{d}{dz_1}[\rho_{i1}] = \sum_i \left[2(x_i - z_1) + \frac{d\mu_1}{dz_1}\right][\rho_{i1}][1 - \rho_{i1}] = 0,$$

or

$$\left[2z_1 - \frac{d\mu_1}{dz_1}\right]\sum_i [\rho_{i1}][1 - \rho_{i1}] = \sum_i 2x_i[\rho_{i1}][1 - \rho_{i1}],$$

so

$$z_1 - \frac{1}{2}\frac{d\mu_1}{dz_1} = \sum_i \alpha_{i1}x_i,$$

where

$$\alpha_{i1} = \frac{[\rho_{i1}][1 - \rho_{i1}]}{\Sigma_i[\rho_{i1}][1 - \rho_{i1}]}.$$

Clearly the coefficient α_{i1} is increasing in μ_1 and λ_1, and decreasing in λ_j, μ_j for $j \neq 1$. An identical argument holds for each party j giving an equilibrium at a weighted electoral mean satisfying, for all j, the balance equation:

$$\left[\frac{d\mathcal{E}_j^*}{dz_j} - z_j^*\right] + \frac{1}{2}\frac{d\mu_j}{dz_j} = 0. \tag{7.13}$$

To examine the second-order condition, note that now the Hessian of party 1 is given by

$$\frac{1}{n}\sum_i \frac{d^2\rho_{i1}}{dz_1^2} = \frac{1}{n}\sum_i \left[\rho_{i1} - \rho_{i1}^2\right]\left[4[1-2\rho_{i1}][\nabla_i^*] + \left[\frac{d^2\mu_1}{dz_1^2} - 2I\right]\right].$$

Here, $[\nabla^*] = \frac{1}{n}\Sigma_i\ [\nabla_i^*]$ is the total electoral covariance matrix taken about the point $z_1 - \frac{1}{2}\frac{d\mu_1}{dz_1}$. Even though the matrix term involving ∇^* may have positive eigenvalues, if the eigenvalues of $\frac{d^2\mu_1}{dz_1^2}$ are negative, and of sufficiently large modulus, then the Hessian will also have negative eigenvalues.

Obviously, this can give a PSNE. □

Note that for a general spatial model with $A_{ij}(x_i, z_j) = ||x_i - z_j||_i^2$ involving different coefficients in different dimensions, the only change will be in the definition of the weighted electoral mean. It is also worth mentioning that the model can be developed with the Cartesian norm:

$$A_{ij}(x_i, z_j) = \sum_{r=1}^{w} |x_{ir} - z_{jr}|.$$

Instead of a weighted *electoral mean* the first-order condition will give a weighted *electoral median*.

8

Political Realignments in the United States

8.1 CRITICAL ELECTIONS IN 1860 AND 1964

This chapter will develop the idea of activist influence in elections presented in the previous chapter, but will apply the model to the transformation of electoral politics that has seemed to occur in recent elections in the United States. Indeed we shall use the model to suggest that a slow transformation has occurred in the locations of Republican and Democrat presidential candidates, and as a consequence, pattern of majorities for the two parties in the States of the Union have shifted. In our account, this is because the most important policy axes have slowly rotated. We ascribe this to the shifting balance of power between different activist groups in the polity.

Just to illustrate the idea, Table 8.1 shows the shift in state majorities for the two-party candidates between 1896 and 2000, whereas Table 8.2 shows the similarity between the two elections. It is clear that there is a strong tendency for states that voted Republican in 1896 to vote Democrat in 2000, and vice versa. Aside from the fact that a number of states had been formed out of the territories in the period from 1860 to 1896, there is little substantive difference between the pattern of Democrat and Republican states in 1860 and 1896. However, as Table 8.1 suggests, the states that voted Republican for Lincoln in 1860, or for McKinley in 1896, tended to vote Democrat in 2000.

Prior to 1856 of course, there was good reason to believe that the Democrat Party had almost become the permanent majority, by controlling almost all southern and western states. Schofield (2006) argues that the Democrat Party was intersectional, with support in both North and South. Riker (1980, 1982) had suggested that this predominance of the

Table 8.1. *Presidential State Votes, 1896 and 2000.*

	DEM 1896	REP 1896		
DEM 2000	*Washington*		**Connecticut** **Delaware** **Illinois** **Maryland** **Massachusetts** **Michigan** **Minnesota** **N. Jersey** **N. York** **Pennsylvania** **Rhode Island**	*California* *Iowa* *Maine* *Oregon* *Vermont* *Wisconsin*
REP 2000	*Colorado* *Florida* *Idaho* Kansas *Kentucky* *Montana* *Nebraska* *S. Dakota* *Tennessee* *Utah* *Virginia* *Wyoming*	**Alabama** **Arkansas** **Georgia** **Louisiana** **Mississippi*** **Missouri** **Nevada** **N. Carolina** **S. Carolina** **Texas**	**W. Virginia**	*Indiana* *N. Hampshire* *N. Dakota* *Ohio*

Italicized states are those that were Republican in 1960.
Bold states are those that were Democratic in 1960.
* Although Kennedy outpolled Nixon in Mississippi (31% to 25%), a plurality voted for electors who cast their electoral votes for segregationist Harry Byrd.

Democrat Party was broken by Lincoln in the election of 1860, as a result of his ability to bring the issue of slavery to the forefront.

To seek the causes of this recent electoral realignment we can start with the election of 1860. In that election, Abraham Lincoln, the Republican contender, won the presidential election by capturing a majority of the popular vote in fifteen northern and western states. The Whig or "Conservative Union" candidate, Bell, only won three states (Virginia, Kentucky, and Tennessee), while the two Democrat candidates, Douglas and Breckinridge, took the ten states of the South. (See Schofield, 2006a: Table 5.2)

From 1836 to 1852, Democrat and Whig vote shares had been roughly comparable (Ransom, 1989), with neither party gaining an overwhelming

Table 8.2. *Simple Regression Results between the Elections of 1896, 1960, and 2000, by State.*

Dependent Variable	% Democrat vote 1960	% Democrat vote 2000	% Democrat vote 2000
Independent Variable	% Democrat vote 1896	% Democrat vote 1960	% Democrat vote 1896
coef. (st. error)	−0.049 (0.046)	0.762 (0.198)	−0.266 (0.053)
t statistic	−1.06	3.84	−4.98
r^2	0.02	0.23	0.37

preponderance in the North or South. However, in 1852, the Democrat Pierce won 51 percent of the popular vote, but because of its distribution, the plurality nature of the electoral college gave him 254 electoral college seats out of 296. Similarly, in 1856, the Democrat, Buchanan, won 45 percent of the popular vote, and took 174 electoral college seats out of 296. Fremont, the candidate for the Republican Party, did well in the northern and western states, but still lost 62 electoral college votes in these states to Buchanan. The Whig, Fillmore, only won 8 electoral college votes in the border states.

Thus, between 1852 and 1860, the American political system was transformed by a fundamental "realignment" of electoral support. (See Schofield, 2006a: Table 5.1)

The sequence of presidential elections between 1964 and 1972 also has features of a political transformation, where race and civil rights again played a fundamental role. Except for President Eisenhower, Democrats had held the presidency since 1932. The 1964 election, in particular, had been a landslide in favor of Lyndon Johnson. By 1972, this imbalance in favor of the Democrats was completely transformed. The Republican candidate, Nixon, took 60 percent of the popular vote, while his Democrat opponent, McGovern, only won the electoral college votes of Massachusetts and Washington, D.C.

In between, of course, was the three-way election of 1968, between Humphrey, Nixon, and Wallace. In some respects, this election parallels the 1856 election between Buchanan, Fremont, and Fillmore. Nixon won about 56 percent of the vote in 1968, but Humphrey had pluralities in seven of the northern "core" states, as well as Washington D.C., Hawaii, and West Virginia. The Southern Democrat, Wallace, with only about 9 percent of the popular vote, won six of the states of the old Confederacy. (See Schofield, 2006a: 197–199, for the results of the elections of 1964 to 1972).

It is intuitively obvious that, in some sense, Humphrey and McGovern can be likened to Fremont and Lincoln, at least in terms of the "civil rights" policies that they represented, while Wallace and Goldwater resemble Breckinridge. It is equally clear that the elections of 1968 and 1972 were critical in some sense since they heralded a dramatic transformation of electoral politics that mirrored the changes from 1856 to 1860. In both cases, parties increasingly differentiated themselves on the basis of a civil rights dimension, rather than the economic dimension. This raises the question about why Republican policy concerns circa 1860 should be similar to Democrat positions circa 1972.

When Schattschneider (1960) first discussed the issue of electoral realignments, he framed it in terms of strategic calculations by party elites. For example, in discussing the election of 1896, Schattschneider argued that the Populist, William Jennings Bryan, instigated a radical agrarian movement which, in economic terms, could be interpreted as anti-capital. To counter this, the Republican Party became aggressively pro-capital. Because conservative Democrat interests feared populism, they revived the sectional cleavage of the Civil War era, and implicitly accepted the Republican dominance of the North. According to Schattschneider, this "system of 1896" contributed to the dominance of the Republican Party until the later transformation of politics brought about in the midst of the Depression by F. D. Roosevelt.

Recently, Mayhew (2000, 2002), has questioned the validity of the concepts of a "critical election" and of "electoral realignment" as presented by Schattschneider (1960) and other writers (Key, 1955; Burnham, 1970; Sundquist, 1973; Brady, 1988; Abramowitz and Saunders, 1998; etc.). Indeed, it is true that one fundamental difficulty with this literature on realignment is that its principal analytical mode has been macropolitical, depending on empirical analysis of shifting electoral preferences. In general, the literature has not provided a theoretical basis for understanding the changes in political preferences. Electoral choices are, after all, derived from perceptions of party positions. Schattschneider implied that these party, or candidate, positions are, themselves, strategically chosen in response to perceptions of the party elite of the social and economic beliefs of the electorate.

Formally speaking, this implies that politics is a "game." Individual voters have underlying preferences that can be defined in terms of policies, and they perceive parties in terms of these policies. Party strategists receive information of a general kind, and form conjectures about the nature of aggregate electoral response to policy messages. Finally, given

the utilities that strategists have concerning the importance of policy, and of electoral success, they advise their candidates how best to construct "utility maximizing" strategies for the candidates.

In the previous chapters of this book we have proposed that the "game" takes place in a policy space X, say, which is used to characterize individual voter preferences. Each candidate j offers a policy position z_j to the electorate, chosen so as to maximize the candidate's utility. Typically, this utility is a function of the "expected" vote share of the candidate. It is also usually assumed that all candidates have similar utilities, in that each one prefers to win. While there are many variants of this model, the conclusion asserted by the mean voter theorem, for example, is that all candidates will adopt identical, or almost identical, policy positions in a small domain of the policy space, centrally located with respect to the distribution of voter-preferred points.

Any such formal model has little to contribute to an interpretation of critical elections or of electoral realignment. From the point of view of this literature, change can only come about through the transformation of electoral preferences by some exogenous shock. Even allowing for such shocks, the divergence of party positions observed by Schattschneider can only occur if perceptions of party strategists are radically different. This seems implausible.

In this chapter, we develop the model proposed in Chapter 7, in which rational political candidates attempt to balance the need for resources with the need to take winning policy positions. Voters choose among candidates for both policy and nonpolicy reasons. The policy motivations of voters pull candidates toward the center. However, centrist policies do little to earn the support of party activists, who are more ideologically extreme than the median voter, and who supply vital electoral resources. Candidates realize that the resources obtained from party activists make them more attractive, independent of policy positions. This implies that candidates must balance the attractiveness of activists' resources against the centrist tug of voters.

During most elections, there is a stable pattern of partisan cleavages and alliances. In such an environment, candidates can adopt equilibrium vote-maximizing positions that allow them to appeal to one set of partisan activists or another. But in certain critical elections, candidates realize that they can improve their electoral prospects by appealing to party activists on a new ideological dimension of politics. In the next section, we present a sketch of the possible repositioning of presidential candidates in the critical elections of 1860, 1896, 1932, and 1964–1968. We then develop

an overview of the model to focus on the nature of activists' choices. In the final two sections, we draw some further inferences with a view to providing a deeper understanding of recent political alignments.

8.2 A BRIEF POLITICAL HISTORY, 1860–2000

Before introducing the model, it will be useful to offer schematic representations of the "critical elections" between 1860 and 1968 in order to illustrate what it is we hope to explain.

For Schattschneider, the 1896 election was based on an attack by Bryan against the sectional cleavage of the Civil War and the Reconstruction. It is therefore consistent with this argument that the contest between the Republican, McKinley, and the Populist Democrat, Bryan, was characterized by policy differences on a "capital" dimension. It is also convenient to refer to this dimension as an "economic" dimension. McKinley clearly favoured pro-business policies, while Bryan made a case for soft-money (bimetallism) and easy credit, both attractive to hard-pressed agrarian groups of the time. The sectional conflict of the Civil War era had obviously been over civil rights, so we can describe this earlier conflict in terms of a "social" dimension. Another way of characterizing this dimension is in terms of labour, since policies that restricted the civil rights of southern blacks had significant consequences for the utilization of labour. To give a schematic representation of the election of 1860, we may thus situate Lincoln and Breckinridge in opposition on the social dimension, as in Figure 8.1. The Whig, Bell, may be interpreted as standing for the commercial interests, particularly of the northeast. In contrast, Douglas represented the agrarian interests of the West, and his support came primarily from states such as Iowa, Ohio, Indiana, Illinois, and so forth.

With two distinct dimensions and four candidates, it is immediately obvious that the policy space could be divided into four quadrants. Voters who had conservative preferences on both social and economic axes we may simply term *Conservatives*. In the 1860 election, such voters would have commercial interests and be pro-slavery. On the other hand, voters with commercial interests, but who felt strongly that slavery should be restricted we call *Cosmopolitans*. Voters opposed to both slavery and commercial interests, we call *Liberals*. (This term is clearly something of a misnomer in 1860 since such voters would, at the time, probably be "free soil" farmers in states such as Illinois, etc.). Agrarian, anticommercial interests who were conservative on the social axis, we term *Populists*. For

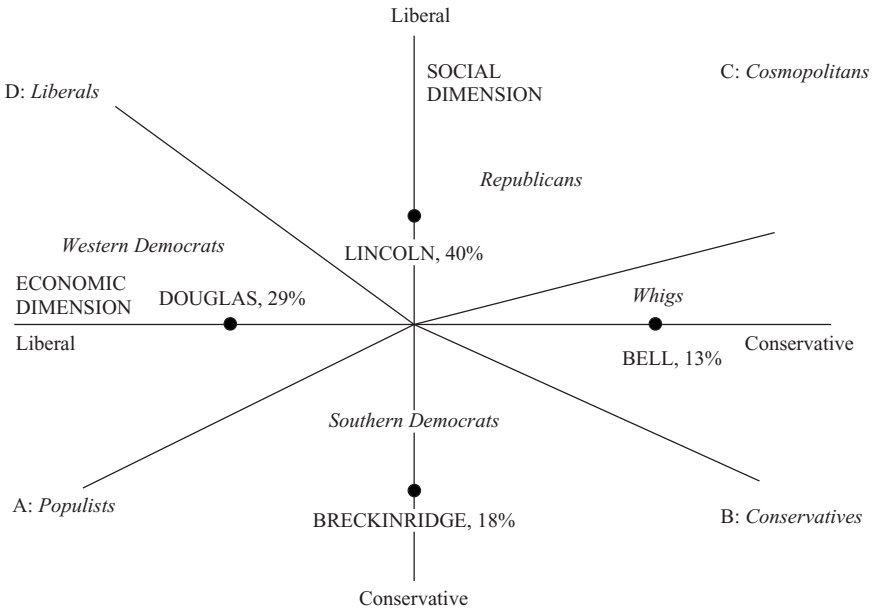


Figure 8.1. A schematic representation of the election of 1860 in a two-dimensional policy space.

convenience, we denote these four quadrants as A (Populists), B (Conservatives), C (Cosmopolitans), and D (Liberals).

The boundaries in Figure 8.1 indicate the division of the electorate into the supporters of the four presidential candidates in 1860. Figure 8.1 is intended to imply that each of the candidates in 1860 had to put together a coalition of divergent interests. Prior to 1852, the social or labour dimension played a relatively unimportant role, at least in presidential elections. How and why this dimension came into prominence in 1856 has been discussed at length elsewhere, using notions from social choice theory (Riker, 1982; Weingast, 1998; Schofield, 2006a). It is our contention that the economic and social dimensions are always relevant to some degree in U.S. political history. However, at various times, one or the other may become less important, for reasons we shall explore. After the Civil War, and the disappearance of the Whig Party (and of the distinct Western Democrat faction, represented by Douglas), political conflict between Republicans and Democrats focused on the social axis, as illustrated in Figure 8.2.

The horizontal "partisan cleavage line" is intended to separate the Republican and Democrat voters immediately after the Civil War. It is

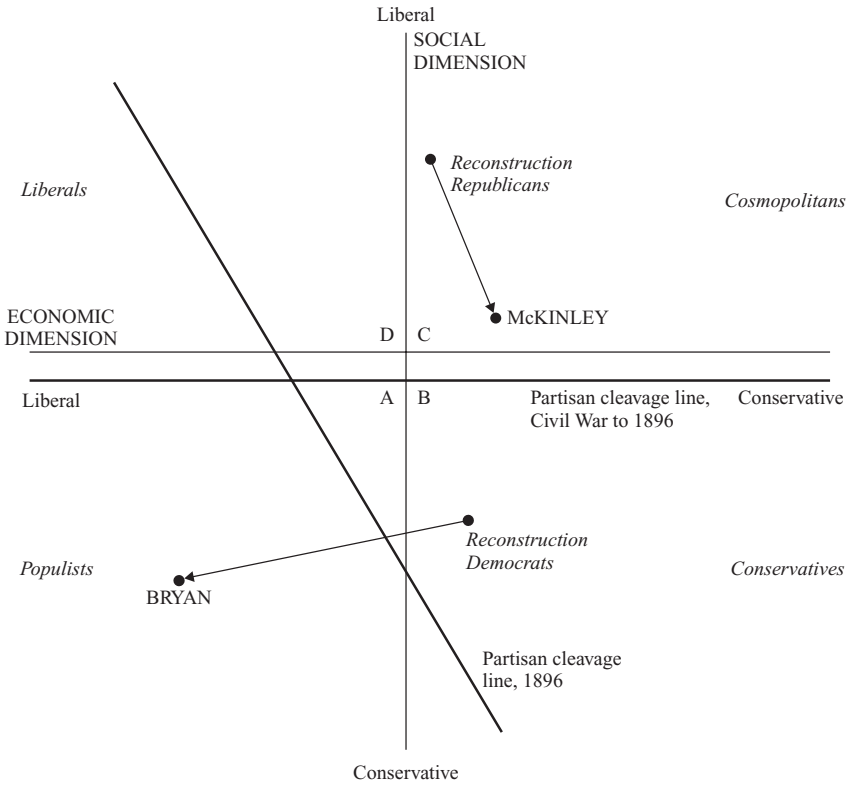


Figure 8.2. Policy shifts by the Republican and Democratic Party candidates, 1860–1896.

consistent with Schattschneider's interpretation of the election of 1896 that McKinley adopted a much more pro-business, or conservative, position on the economic axis, while Bryan took up a policy position in the populist quadrant (A). The 1896 partisan cleavage line in Figure 8.2 is used to distinguish between Republican and Populist Democrat voters. Figure 8.2 makes it intuitively clear why Bryan could not win the election. Moreover, support for conservative economic policies would lead to Republican predominance. As Schattschneider (1960: 85) observed, "the Democrat party carried only about an average of two states (outside of southern and border states) between 1896 and 1932." The increasing "degree of competition" between Democrat and Republican parties in 1932 can be represented by the positioning of F. D. Roosevelt and Hoover near the economic axis, as in Figure 8.3.

Note that the successful Roosevelt coalition comprised what we have called Populists and Liberals opposed to Conservatives and Cosmoplitans.

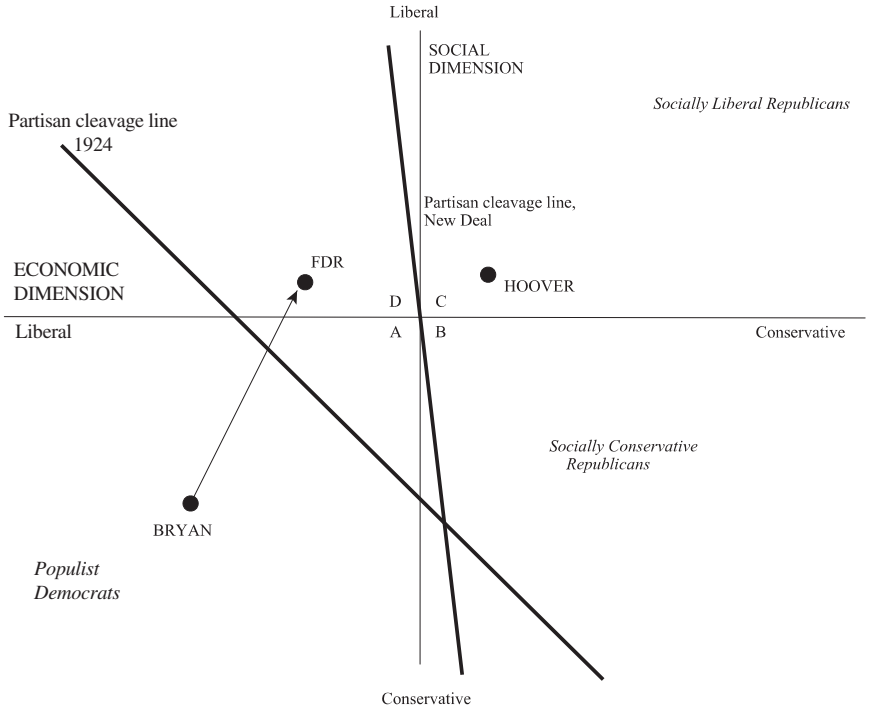


Figure 8.3. Policy shifts by the Democratic Party *circa* 1932.

The standard formal model (Downs, 1957) has tended to generalize from the location of party positions in the period from 1932 to 1960 and to infer that political competition is primarily based on the economic axis, and involves the coalition {A,D} against {B,C}. However, as Carmines and Stimson (1989) have analyzed in great detail, "race" (or "policy" on the social dimension) has become increasingly important since about 1960. Indeed, they present data to suggest that Republicans in the Senate tended to vote in a more liberal fashion on racial issues than Democrats prior to 1965.

Although L. B. Johnson may have had many of the characteristics of a Southern Democrat while he was Senate leader, he introduced, while President, the major policy transformation of the Great Society. Figure 8.4 presents a plausible policy position for Johnson in 1964, as well as presidential candidate positions for the period 1964 to 1980. The candidate positions for the elections of 1968 and 1976 are compatible with the empirical work of Poole and Rosenthal (1984: Figs 1, 3), while the positions

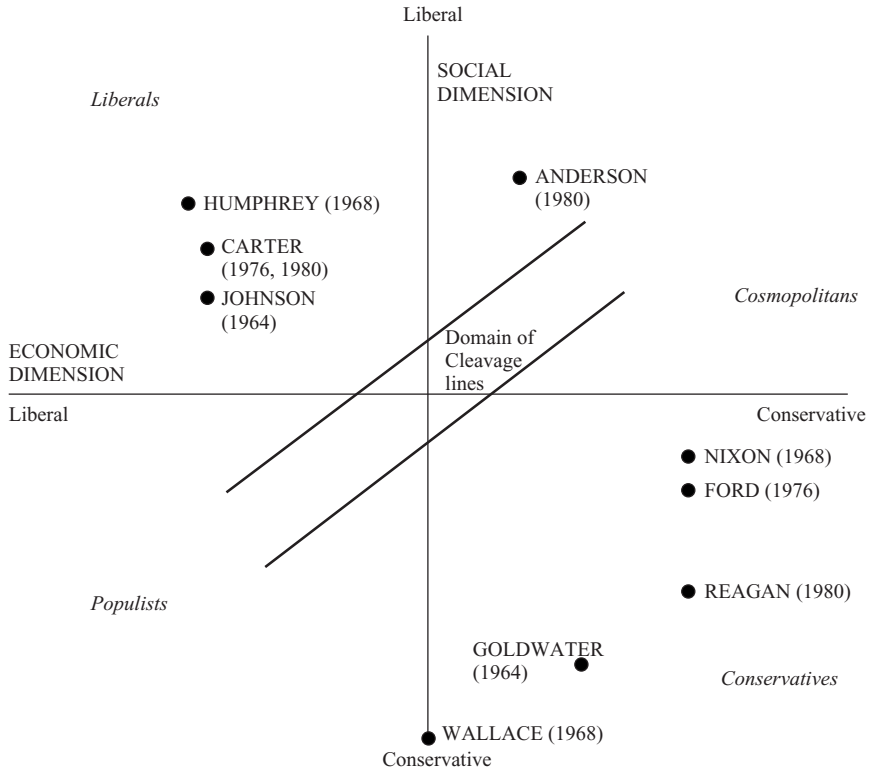


Figure 8.4. Estimated presidential candidate positions, 1964–1980.

for the elections of 1964 and 1980 are based on our analyses discussed below.

A number of comments are necessary to understand the significance of this figure. As in the previous two figures, a partisan cleavage line can be drawn in the policy space for each election, determined by the positions of the two principal presidential candidates. What we denote as the "Domain of Cleavage lines" in Figure 8.4 includes these partisan cleavage lines for the various elections. As our analysis (presented in Figure 8.5 below) suggests, the cleavage line for the 1964 election would fall below and to the right of the origin. Since the origin is at the mean of voter bliss points, this is meant to represent Johnson's successful candidacy for President.

The standard spatial model of candidate positioning implies that attempts by candidates to maximize votes draws them into the electoral center. It is apparent, however, that the estimates of candidate positions, presented in Figure 8.4, contradict this inference. Indeed, the positioning

of Republican and Democrat candidates in Figure 8.4 suggests that voters who can be described as Cosmopolitan (with preferences in the policy domain C) or Populists (in domain A) may find it difficult to choose between the candidates.

In the next section, we examine the standard spatial model to determine the basis for this inference, and then consider in somewhat more detail how empirical analysis suggests how the standard spatial model may be adapted to better account for candidate behavior. The principal goal of our modified activist voter model of elections is to provide the foundation for a theory of dynamic electoral change that can provide a formal account of the inferred transformation or "rotation" in the policy space presented in Figures 8.1 through 8.4.

8.3 MODELS OF VOTING AND CANDIDATE STRATEGY

As we have discussed in the previous chapters, the formal model of voting assumes that voter utility is given by the expression

$$\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p)) \in \mathbb{R}^p. \tag{8.1}$$

Here, $\mathbf{z} = (z_1, \ldots, z_p)$ is the vector of strategies of the set P of political agents (candidates, parties, etc.). The point z_j is the position of candidate j in the space X. Previously we assumed that

$$u_{ij}(x_i, z_j) = \lambda_j - A_{ij}(x_i, z_j) + \theta_j^{\mathrm{T}} \eta_i + \varepsilon_j, \tag{8.2}$$

where A_{ij} was the symmetric Euclidean metric and $\theta_j^{\mathrm{T}} \eta_i$ gave the effect of the sociodemographic characteristics of voter i on vote probabilities.

As we have seen, both the MNL and MNP models typically provide an excellent account of voter choice. For example, the MNL two-dimensional voter model of Poole and Rosenthal (1984) for the 1968 and 1976 elections had success rates for voter choice of over 60 percent. Their estimates of the 1968 and 1976 candidate locations closely correspond to the positions of candidates indicated in Figure 8.4. As Poole and Rosenthal (1984: 287) suggest, "the second dimension captures the traditional identification of southern conservatives with the Democratic party."

Our own analyses, presented in Figures 8.5 and 8.6, suggest that the second dimension is, in fact, a long-term factor in U.S. elections. Each circle in these figures represents the ideal point of a voter in a factor space derived from the National Election Surveys of 1964 and 1980, respectively.

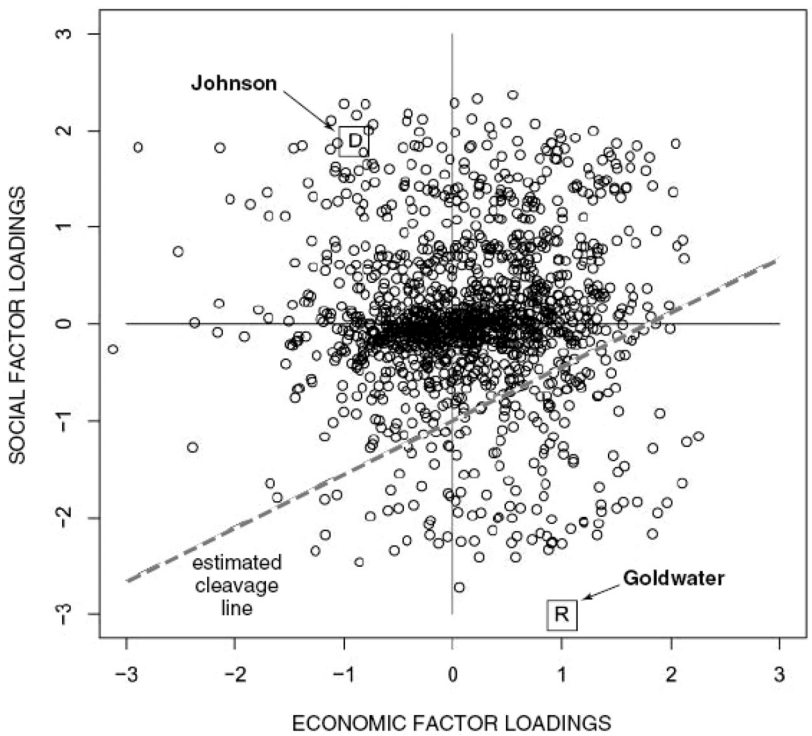


Figure 8.5. The two-dimensional factor space, with voter positions and Johnson's and Goldwater's respective policy positions in 1964, with linear estimated probability vote functions (log likelihood = −617).

A standard confirmatory factor analysis was used to estimate the factor space. Standard hypothesis tests suggest that a two-factor model was appropriate. A pure linear spatial probit model was used to estimate the probability $\rho_{i,dem}$ that a voter i would choose the Democratic candidate. Thus, instead of basing the model on voter utility based on Euclidean distance, as in earlier chapters, we assumed that

$$\rho_{i,dem} = \lambda_{dem} + ax_i + by_i, \tag{8.3}$$

where (x_i, y_i) are the coordinates of the ideal point of voter i in the two dimensions.

The "estimated cleavage lines" in these two figures gives the boundary $\rho_{i,dem} = \frac{1}{2}$. (North of this line, the voters choose the Democratic candidate, and south they choose the Republican.) The cleavage lines were estimated using a probit model, with the factor scores on each dimension used as

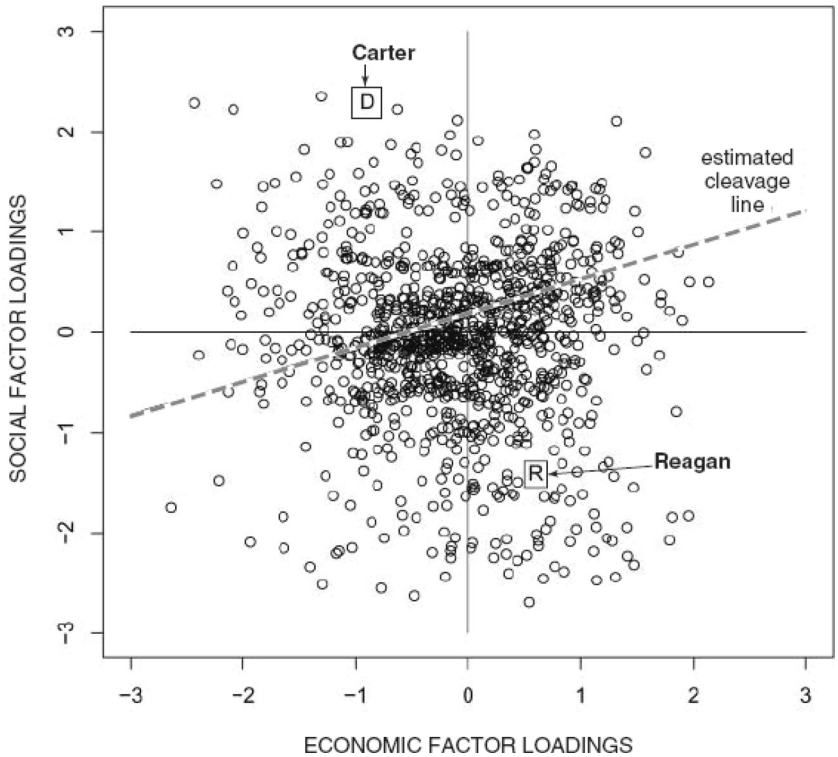


Figure 8.6. The two-dimensional factor space, with voter positions and Carter's and Reagan's respective policy positions in 1980, with linear estimated probability vote functions (log likelihood = −372).

covariates. In both the 1964 and 1980 models, the estimated coefficients were highly statistically significant ($p < 0.001$ in all cases). Both models classify reasonably well; the McKelvey and Zavoina R^2 for 1964 is 0.2000 and for 1980 is 0.465. (See Schofield, Miller and Martin, 2003, for further details.) Given the estimated probabilities, it is possible to infer the location of the two candidates. For example, for 1964, the symbol R is used to indicate our estimation of the position of Goldwater and D that of Johnson. Comparing the results for 1964 and 1980 suggests that Carter was just as "liberal" on economic issues as was Johnson, but slightly more liberal on social issues. Notice that in 1964, the cleavage line $\rho_{i,dem} = \frac{1}{2}$ passes "south" of the origin, so that a clear majority of the voter sample is assigned a probability greater than $\frac{1}{2}$ of voting for Johnson. In contrast, in 1980, the cleavage line passes "north" of the origin, giving Reagan the advantage.

In 1964, the total electoral variance on the two axes was 1.28, while in 1980 the variance was very similar at 1.365. Since the linear probability model is different from the one used in our previous analyses, we cannot use the convergence coefficient directly. In terms of the linear probit model, it is evident that Goldwater in 1964 and Carter in 1980 had lower exogenous valences than their respective competitors. The above analyses suggest that all candidates were indeed positioned some distance from the electoral origin.

Figures 8.5 and 8.6 buttress the remark made by Poole and Rosenthal (1984: 288) that their analysis "is at variance with simple spatial theories which hold that the candidates should converge to a point in the center of the [electoral] distribution" (namely, the origin in Figures 8.5 and 8.6). Poole and Rosenthal suggest that this "party stability," of divergent candidate locations, is the result of the need of candidates to appeal to a support group to be nominated. Our earlier results suggest that the divergent positions were consistent with vote maximization.

To see this, note that in their estimation of the vote function for 1968, the intercept, or valence λ for Humphrey and Nixon was 3.416, while for Wallace, it was 7.515. Moreover, the coefficient β was 5.260 for Humphrey and Nixon, but 7.842 for Wallace. In other words, the underlying valence, or innate attractiveness of Wallace was high, but voter support dropped rapidly as the distance between the voter ideal point and the Wallace position increased. In their analysis of the 1980 election, the β coefficient for the third independent, National Union candidate, John Anderson, was 1.541. Anderson took only 6.6 percent of the national vote, and this is reflected in his estimated λ coefficient of -0.19, in contrast to $\lambda = 3.907$ for Carter and Reagan.

We now develop the model proposed in Chapter 7, where valence comprises two components. For candidate j, there is an "innate" or exogenous valence whose distribution is characterized by the stochastic error term ε_j. As before, the expectation of the valence term for candidate j is identified with the average valence λ_j, of j in the electorate. The second component, μ_j, is affected by the money and time that activists make available to candidate j. Essentially, this means that this second valence component μ_j is a function of the policy choices of candidates. We can ignore the exogenous valence terms since they have been examined above. Concentrating on activist valence gives the following expression for voter utility:

$$u_{ij}(x_i, z_j) = \mu_j(z_j) - A_{ij}(x_i, z_j) + \varepsilon_j. \tag{8.4}$$

For convenience, in the discussion below we refer to the effect of candidate strategies on the expected vote-share function $\mathcal{E}_j$, through change in $\mu_j(z_j)$, as the *valence* component of the vote. Change in $\mathcal{E}_j$ through the effect on the policy distance measure $A_{ij}(x_i, z_j)$ we refer to as the *nonvalence,* or *policy* component. We discuss this "activist" model in the next section. One important modification of the pure spatial model that we make is that the salience of different policy dimensions varies among the electorate. More precisely, we assume that

$$A_{ij}(x_i, z_j) = ||x_i - z_j||_i^2. \tag{8.5}$$

Here, $||\cdots||_i$ is an "ellipsoidal" norm giving a metric whose coefficients depend on x_i. We make this assumption clearer in the following section, where we assume that activists, motivated primarily by one policy dimension or the other, may choose to donate resources that increase their candidate's valence. We argue that it is the candidate's attempt to position himself with respect to different types of activists that accounts for the partisan realignment.

8.4 A JOINT MODEL OF ACTIVISTS AND VOTERS

We adapt a model of activist support first offered by Aldrich (1983a,b) and introduced in the previous chapter.

Essentially the model is a dynamic one based on the willingness of voters to provide support to a candidate. Given current candidate strategies $\mathbf{z}$ let

$$C(\mathbf{z}) = (C_1(\mathbf{z}), \ldots, C_p(\mathbf{z})) \tag{8.6}$$

be the current level of support to the various candidates. The candidates deploy their resources, via television, and other media, and this has an effect on the vector $\mu(\mathbf{z}) = (\mu_1(z_1), \ldots, \mu_p(z_p))$ of candidate-dependent valences. We assume that each μ_j is in fact a function of $C_j(z_j)$.

At this point, a voter i may choose to add his own contribution c_{ij} to candidate j as long as

$$c_{ij} < \mu_j(z_j) - A_{ij}(x_i, z_j) + \varepsilon_j. \tag{8.7}$$

The total contributions to candidate j is then Σc_{ij}. Aldrich considered an equilibrium of this dynamic process between two candidates, 1 and 2, where the candidate's position, z_j, was defined to be the mean of the ideal points of all activists who supported this candidate. The existence

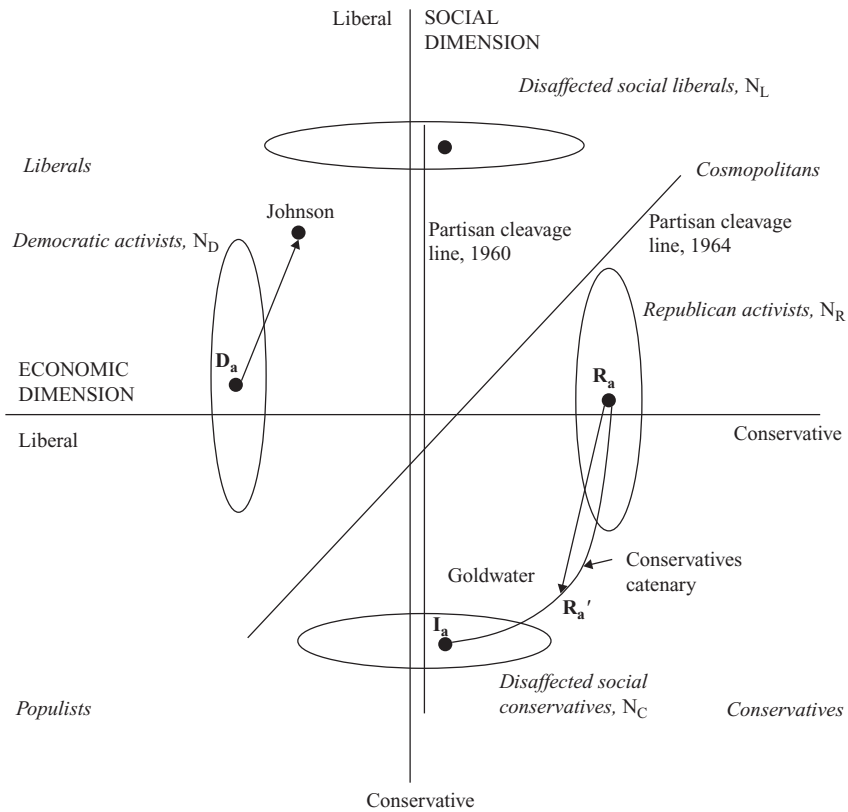


Figure 8.7. Illustration of vote-maximizing positions for a Republican presidential candidate (Goldwater) facing a Democratic candidate (Johnson).

of such a candidate equilibrium can be seen most easily with reference to Figure 8.7. (This figure is adapted from Miller and Schofield, 2003.)

Consider a group of Republican "economic" activists. The Republican candidate j is situated at the position $R_a = (x_j, y_j)$, while the activist has an ideal point (s_i, t_i). Then the activist's utility at a position (x, y) given by

$$u_{ij}((s_i, t_i), (x, y)) = \mu_j(z_j) - A_{ij}((s_i, t_i), (x, y)), \tag{8.8}$$

where

$$A_{ij}((s_i, t_i), (x, y)) = \frac{(x - s_i)^2}{a^2} - \frac{(y - t_i)^2}{b^2} \tag{8.9}$$

is induced from an ellipsoid metric. The activist contributes some amount, $c_{ij} < u_{ij}((s_i, t_i), (x_j, y_j))$, to the Republican candidate.

Because this activist is most concerned about economic issues, it is natural to assume that $a < b$. If the activist actually had an ideal point $(s_i, t_i) = R_a$, then his indifference curve would be given by the "ellipsoid" centered at R_a, say. Depending on various parameters, there will exist a domain N_R, say, in X, with the property that every voter whose ideal point (s_i, t_i) belongs to N_R is a contributor to the Republican candidate. For purposes of illustration, we may take N_R to be represented by the "ellipsoid" of ideal points centered on R_a, as in Figure 8.7. It is natural to assume that there is an opposing Democratic candidate, whose position is at D_a, say, and an opposing set N_D of Democratic activists whose ideal points belong to a similar set but are centered on D_a. Aldrich showed, essentially, that these conditions could be satisfied, such that R_a was given by the mean point of the set N_R, while D_a was the mean of the set N_D. It is obvious that for such an activist equilibrium to exist, it is necessary that each μ_j, regarded as a function of campaign contributions, is concave (or has diminishing returns) in contributions to candidate j. (A more refined model could naturally include voter income.)

As Figure 8.7 indicates, a typical socially conservative voter would regard Democrat and Republican candidates as equally unattractive, and tend to be indifferent. Let us now suppose that such a voter, g, has bliss point (s_g, t_g), say, near the position I_a, with utility function

$$u_{gj}((s_g, t_g), (x, y)) = \mu_j(z_j) - \left(\frac{(x - s_g)^2}{e^2} + \frac{(y - t_g)^2}{f^2}\right). \quad (8.10)$$

Let N_C be the set of such "disaffected" social conservatives who would be willing to contribute to a candidate as long as this candidate adopted a policy position close to $I_a = (x_k, y_k)$. We suggest that such social conservatives regard social policy to be of greater significance and so $e > f$ in this equation. Unlike Aldrich, we now suppose that the Republican candidate adopts a position, not at the position R_a, but at some compromise position R'_a between R_a and I_a. It is easy to demonstrate that the "contact curve" between the point (s_i, t_i) and the point (s_g, t_g) is given by the equation

$$\frac{(y - t_i)}{(x - s_i)} = S\frac{(y - t_g)}{(x - s_g)}, \quad (8.11)$$

where

$$S = \frac{b^2}{a^2}\frac{e^2}{f^2}. \quad (8.12)$$

This contract curve between Republican economic activists and disaffected social activists is labeled the "Conservatives catenary" in Figure 8.7. If the candidate moves on this catenary then the resulting number of activists will be of the order of $qN_R + (1-q)N_C$, where q is some constant (<1) dependent on the position taken by the candidate. Because of the asymmetry involved, the total number of activists may increase, thus increasing overall contributions to the Republican candidate. Clearly, there are plausible conditions under which μ_{rep} increases as a result of such a move, thus potentially increasing the vote share of the Republican candidate.

Determination of existence of a candidate PNE in this modified model depends, as before, on continuity and concavity of the candidate utility (or vote-share) functions $\{U_j\}$. While each U_j will be a function of $\mathbf{z}$, its dependence on $\mathbf{z}$ will be more complex than the simple relationship implicit in the model we have studied above. It is important to note that this proposed model involves differing voter utility functions. To preserve continuity of voter response, it is necessary that the coefficients of voter policy loss vary continuously with the voter-preferred policy. With these assumptions, candidate vote-share functions $\{U_j\}$ will be continuous in candidate strategies. Concavity of the candidate utility functions and thus existence of PNE will then follow from assumptions about the concavity of the activist valence functions. It is worth emphasizing that the greater the relative saliencies, b/a and e/f, the greater will be S and thus, the more significant will be the attraction of using activist groups to enhance electoral support.

Chapter 3 has shown that LNE will generally exist, and will be locally isolated, for almost all games of the kind considered here, as long as the game is smooth. As we have seen in the chapter on Britain, this allows us to assume that equilibria do exist in these complex activist games.

Computation of LNE will generally depend on the factors we have specified: the elasticity of response of the disaffected, potential activists, and the effect of contributions on the valence functions $\{\mu_j\}$. If the contribution term is very significant, then adopting a position to maximize contributions is clearly rational. For example, let us use R and D to denote the positions adopted by the two candidates (as in Figure 8.7). Then it is, in principle, possible to estimate the contributions and thus the valence function. As the results of Chapter 7 indicated, a move by either candidate toward the origin will increase the *nonvalence* component of the electoral vote, but at the same time, decrease contributions, and thus decrease the *valence* component of the vote. It is the optimal balance of valence

and nonvalence vote components that is encapsulated in the notion of LNE.

8.5 THE LOGIC OF VOTE MAXIMIZATION

The simple probabilistic voter model suggests that it is relatively easy for voters to identify attractive candidates, and for candidates to learn about voter response (McKelvey and Ordeshook, 1985). For candidates, the use of opinion polls can be used to indicate how small changes in policy objectives should affect support. The earlier theories of Hinich (1977) and Enelow and Hinich (1984) mentioned in Chapter 3 argued that candidates will gain most electoral support at the centre. The fact that candidates do not act in this way suggests that these theories need serious revision. One extreme response is to propose that voter support is independent of candidate declarations. As suggested before, this is equivalent to supposing that the spatial component of the model is irrelevant. Indeed, earlier sociological or psychological models (Berelson, Lazerfield, and McPhee, 1954; Campbell et al., 1960) essentially made this assumption. The sociological model regarded voter choice simply as a function of "party identification." It is clear enough that if one fundamental cleavage is dominant, and party candidates adopt fixed positions on this cleavage (as in Figure 8.7) then voters will find candidate choice relatively easy. Over a sequence of elections, it is plausible to believe that voters will tend to identify with one party or the other. From one election to another, voter saliencies will vary, and this will affect activist support, and thus candidate vote shares. It is this phenomenon that Aldrich's activist model analyzed (Aldrich, 1983a,b; Aldrich and McGinnis, 1989; Aldrich, 1995). If we are correct in our interpretation of U.S. electoral politics, then an optimal response by a Democrat candidate to the socially conservative Republican position is to adopt a more economically centrist position. It is possible that Clinton's successes in 1992 and 1996 were a results of such a strategy.

Over the years there will be slow transformations in the principal electoral cleavage, and it is the change from one cleavage, or electorally perceived dimension, to another that constitutes an electoral "realignment."

The beginning of the last transformation of the principal cleavage in U.S. politics from an economic dimension to a social, or civil rights, dimension is generally understood to have been triggered by the Civil Rights Act of 1964 (Huckfeldt and Kohfeld, 1989; Edsall and Edsall, 1991; Mann, 1996). This political event eventually brought about the electoral shifts

that we described earlier. The evidence suggests that the degree of party identification dropped from 1964 to 1980 (from about 35 percent of the electorate to 20 percent: see Clarke and Stewart, 1998). During the period from 1960 to 1972, attitudes of Democrat and Republican activists became increasingly polarized over civil rights issues (Carmines and Stimson, 1989). There is therefore no doubt that both voter perceptions and activist attitudes began to change rapidly in the 1960s. The model presented in the previous section suggests that these changes were due to strategic calculations on the part of candidates. To amplify this inference, let us consider how such calculations can be made.

Unlike candidate choice in the simple spatial model, strategic calculation in the proposed activist model is dependent on uncertain outcomes. Consider the strategy of L. B. Johnson to push through the Civil Rights Act of 1964. Clearly, it appealed to those voters designated "disaffected social liberals" in Figure 8.7. The argument presented above suggests that the total number of Democrat activists could increase as a consequence of this policy initiative. The resources made available could, moreover, increase Johnson's overall valence. At the same time, voters, particularly in the Southern states, who traditionally identified with the Democrat party, would suffer a utility loss. Such disaffected social conservatives would then more readily switch to the Republican party. However, the tradeoff between the valence and policy components of voter response is intrinsically difficult to make. For L. B. Johnson, the calculation may well have been that the Democrat coalition of southern social conservatives and economic liberals was unstable. A second, but contradictory, possibility, apparent from 1957 onwards, was that the Republican Party could also move to attract social liberals and create a winning coalition. The actions undertaken by Johnson, first as leader of the Senate in the late 1950s, and then as President after J. F. Kennedy's assassination in 1963, all suggest that he was extremely shrewd in estimating electoral and congressional support, but also capable of extreme risktaking. In 1957, for example, he persuaded the Southern Democratic senators not to deploy their traditional filibuster, but to accept the Voting Rights Bill (Caro, 2002). Indeed, Johnson's maneuvers in the Senate can be characterized as "heresthetic" (to use the term invented by Riker [1982]).

After Kennedy was elected President in 1960 (by a very narrow margin of victory against Nixon), he delayed sending a Civil Rights Bill to Congress, precisely because of the possible effect on the South. To push through the Civil Rights Act in July 1964, Johnson effectively created, with Hubert Humphrey's support, an unstable coalition of liberal Northern

Democrats and moderate Republicans, with sufficient votes in the Senate to effect "cloture," to block the southern Democratic filibusters (Branch, 1998). This was the first time since Reconstruction that the southern veto was overwhelmed. The danger for Johnson in the election of 1964 was that a Republican candidate could make use of the fact of Republican Party support for civil rights to attract disaffected social liberals. Traditional Republican Party activists were thus in an electoral dilemma, but resolved it by choosing the southern social conservative, Goldwater.

Once L. B. Johnson initiated the policy transformation, the strategic calculation of Republican candidates, whether Nixon, Ford, Reagan, or Bush, became much easier. The knowledge of the existence of a set of disaffected social conservatives meant that such voters would appear increasingly attractive to Republican candidates. This in turn created an electoral dilemma for Democrats, as they attempted to maintain support of both economic and social liberals. As economic competition decreased in importance and class became less relevant as an indicator of voter choice, activist support for Democrat candidates from the remnant of the New Deal coalition would fall. One possible response for a Democrat would be to seek new potential activists among the Cosmopolitans—the economically conservative social liberals. Obviously, this would create conflict within the Democrat Party elite. A natural response by the Republican Party was to move their policy choices into quadrant A, the Populist domain. President G. W. Bush's initiatives in 2002, over protection for the steel industry and farm subsidies, indicate that this could, indeed, be his strategy. Support for G. W. Bush in the 2004 election would seem to support this hypothesis. (See also Fiorina, 2005.)

We suggest that the initial policy move by Johnson in 1964 had a basis in rational electoral calculation. Obviously the model proposed here ignores the element of attitude to risk, and this clearly must have been relevant to Johnson's motivation, so this feature is worth adding to the model. At a general level it does appear that the resulting moves and countermoves by Democrat and Republican candidates were in local equilibrium at each election, while the equilibria themselves appear to have slowly changed over the last forty years. We could say that the entire process of political realignment over this period was in "dynamic equilibrium."

8.6 DYNAMIC LOCAL EQUILIBRIA

Under plurality rule, or winner-takes-all electoral methods, it is obvious that presidential candidates, if they hope to win, must attempt to create

majority coalitions of disparate interests (Schlesinger, 1994). The historical record suggests that stable equilibria can occur, but these will be based on one or the other of the two principal cleavages—economic and social—that characterize beliefs in the society. Any such equilibrium, by definition, will create two groups of disaffected, and opposed, voters. Either one of these groups of voters can become a political force once they realize their potential. This depends, of course, on their ability to successfully signal to a candidate, such as L. B. Johnson, that they would be willing to contribute time and money. Although we have suggested that an equilibrium will exist in this activist-voter model, we have not attempted an analysis of the complexities of the signaling game between possible presidential candidates and potential activists.

It should also be evident, from the structure of the activist model presented here, that the willingness of voters to become activists depend on the salience ratios (denoted by b/a and e/f for the economically and socially concerned voters, respectively). These ratios may change within the electorate as a result of exogenous shocks. In turn, this will affect the activist response to candidate positions and thus the positional valences of the candidates. The standard spatial model has principally depended on using data based on voter-preferred policies to estimate electoral support. To estimate the more complex activist model proposed here, it would be necessary to explore the variation of cleavage saliencies within the electorate.

Although we still view the political process as a "game" involving rational utility–maximizing voters and candidates, we suggest that this game is much more complex than previous models have suggested. We believe that the model proposed here can be developed so as to offer a more empirically relevant theory of electoral dynamics.

A task that still remains is to develop a macropolitical account of the long run transformations that can be observed in U.S. politics. We can only offer a very tentative outline of such a theory at present. We have suggested above that these electoral changes are based on new configurations of "factor" coalitions, where *factor* refers to the classic dimensions of capital, labor, and land power. In the 1896 election, the twenty-two states that voted for the Republican, McKinley, all had significant industrial working class populations. Because of the growth of the economic power of the United States, there existed a natural expansionist coalition based on capital and industrial labor (Rogowski, 1989). The hard-money policy of the Republicans naturally affected the agrarian interests (who tended to be indebted), as argued by Bardo and Rockoff (1996). This

is an old theme in U.S. politics (Beard, 1913). The twenty-three states that voted for the Populist-Democrat, Bryan, were all basically agrarian but lacked sufficient population and electoral college votes to upset the capital–labor coalition.

In the 1930s, economic decline broke the capital–industrial labor coalition. By the 1960s, the Democrat coalition comprised half of Bryan's southern Populist states and half of McKinley's commercial Republican coalition. By the 2000 election, the transformation was complete. The remainder of Bryan's coalition became Republican, and the remainder of McKinley's became Democrat. The decline of agriculture and the growth of modern industries in the southern and western states gave them the population and electoral college votes just sufficient for a Republican presidential victory. Clearly, the knife-edge result of the 2000 election indicates that voters in states such as Wisconsin, Michigan, Minnesota, Pennsylvania, and Iowa could be persuaded by G. W. Bush's populist strategies to join the Republican activist coalition. Such continuing transformation maintains the dynamic equilibrium of U.S. politics.

8.7 APPENDICES

(i) The Civil Rights Act.

Date: 02 JUL 64

88th Congress, H. R. 7152

An Act

To enforce the constitutional right to vote, to confer jurisdiction upon the district courts of the United States to provide injunctive relief against discrimination in public accommodations, to authorize the Attorney General to institute suits to protect constitutional rights in public facilities and public education, to extend the Commission on Civil Rights, to prevent discrimination in federally assisted programs, to establish a Commission on Equal Employment Opportunity, and for other purposes.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That this Act may be cited as the "Civil Rights Act of 1964."

(ii) Amendment XIV of the Constitution.

Section 1. All persons born or naturalized in the United States and subject to the jurisdiction thereof, are citizens of the United States and of the State wherein they reside. No State shall make or enforce any law which shall abridge the privileges or immunities of citizens of the United States; nor shall any State deprive any person of life, liberty, or property,

without due process of law; nor deny to any person within its jurisdiction the equal protection of the laws.

Section 2. Representatives shall be apportioned among the several States according to their respective numbers, counting the whole number of persons in each State, excluding Indians not taxed. But when the right to vote at any election for the choice of electors for President and Vice President of the United States, Representatives in Congress, the Executive and Judicial officers of a State, or the members of the Legislature thereof, is denied to any of the male inhabitants of such State, being twenty-one years of age, and citizens of the United States, or in any way abridged, except for participation in rebellion, or other crime, the basis of representation therein shall be reduced in the proportion which the number of such male citizens shall bear to the whole number of male citizens twenty-one years of age in such State.

Section 3. No person shall be a Senator or Representative in Congress, or elector of President and Vice President, or hold any office, civil or military, under the United States, or under any State, who, having previously taken an oath, as a member of Congress, or as an officer of the United States, or as a member of any State legislature, or as an executive or judicial officer of any State, to support the Constitution of the United States, shall have engaged in insurrection or rebellion against the same, or given aid or comfort to the enemies thereof. But Congress may by a vote of two-thirds of each House, remove such disability.

Section 4. The validity of the public debt of the United States, authorized by law, including debts incurred for payment of pensions and bounties for services in suppressing insurrection or rebellion, shall not be questioned. But neither the United States nor any State shall assume or pay any debt or obligation incurred in aid of insurrection or rebellion against the United States, or any claim for the loss or emancipation of any slave; but all such debts, obligations and claims shall be held illegal and void.

Section 5. The Congress shall have power to enforce, by appropriate legislation, the provisions of this article.

The Fourteenth Amendment of the Constitution was passed by the House and Senate on the 8th of June and the 13th of June, 1866.

9

Concluding Remarks

9.1 ASSESSMENT OF THE MODEL

We briefly conclude with an assessment of the model presented in this book, together with some remarks on how the work can be extended.

The essence of democracy is that voters respond to the past acts and promises of party leaders. It has been traditional to use manifestos as measures of promises, and to gauge the distance between the preferred policy of the voter and the promise of the party leader as the "disutility" of the voter. In addition, of course, the voter may not trust the party leader. Valence is one very simple way to model the judgment of the voter about the degree to which the party leader can be trusted. This addition to the standard spatial model of voting changes one of the principal results of the model, namely the mean voter theorem. As the main theorem of Chapter 3 shows, it is no longer necessarily the case that all parties will converge to an electoral center. Instead, each election will be characterized by a convergence coefficient. Empirical analysis associated with the election can be used to give a list of valence coefficients for each of the party leaders. If the valences are similar, then the convergence coefficient will be low, and symmetry will induce all parties to converge to the center.

The formal model shows that this convergence coefficient is an increasing function of the valence differences, the electoral variance, and the spatial parameter. Moreover, with many parties with differing valences, the coefficient will tend to be high. When the coefficient exceeds the dimension of the policy space, then the lowest-valence party will vacate the electoral mean in order to increase its vote share. One way to interpret this finding is that in such a circumstance the low-valence party can guarantee itself more votes by seeking the electoral periphery rather than by competing directly with high-valence parties.

It is often thought that radical parties gain few votes because they are radical. The theory presented here suggests that when policy is important, and when electoral preferences are heterogeneous, then only parties that are generally trusted by the electorate can approach the electoral center. The movement away from the electoral center by a low-valence party will be determined by the principal component of the electoral distribution. Asymmetry in the nature of electoral preferences will result in a principal electoral dimension, and it is this dimension that is the "eigenspace" for a low-valence party. Once the lowest-valence party leaves the electoral mean, then so will others. In general, all parties will adopt positions on the principal component of the voter distribution, but higher-valence parties will choose positions nearer the mean.

This stochastic spatial model does not specify whether parties move up or down the principal electoral component. The theory assumes that the final vector of party positions is in "local equilibrium." Given a system of electoral parameters, there may be multiple local equilibria. Nonetheless, it is plausible that the particular local equilibrium that occurs is "path dependent." That is, the previous historical situation generates a set of political niches wherin parties typically reside. Notice that the differing local equilibria may be associated with very different political configurations. It may also be the case that small changes in the electoral parameters can dramatically transform the local political equilibrium, destroying political niches and leading to the creation of new parties and new niches.

9.2 PROPORTIONAL REPRESENTATION

When the electoral system is based on some form of proportional representation, it is generally impossible for one party to gain a majority. Because coalitions involve compromise over policy, rational party principals should nominate leaders for the party who are in a position to bargain effectively with other party leaders. For low-valence parties occupying niches far from the electoral center, this motivation may cause the party elite to consider party leaders who have preferred positions that are even more radical than the vote-maximizing position. For high-valence parties, whose niches are close to the electoral center, there is an added centripetal tendency. If the party can position itself centrally and gain enough seats to be dominant, then it can occupy the "structurally stable core" position, which will allow it, under some circumstances, to control government, and perhaps to construct a minority government. Small

changes in electoral support can therefore make a big difference whether a party is dominant or not.

9.2.1 *The Election of September 2005 in Germany*

Though we have not considered a formal model of German elections, the recent election in Germany on September 18, 2005, can be used to illustrate some of the logic of coalition. The CDU/CSU (Christian Democrats, under Merkel) took 35.2 percent of the popular vote (and 225 seats in the 613-seat Bundestag). The SPD (Social Democrats, led by Schroeder) took 34.3 percent (and 222 seats) against 8.1 percent (and 51 seats) for the Greens, 8.7 percent (and 54 seats) for the Left Party, including PDS (Party for Democratic Socialism), and 9.8 percent (with 61 seats) for the FDP (Free Democrats). The previous SPD–Green coalition had 47.1 percent of the vote but 306 seats out of 603 in the Bundestag (slightly more than proportional because the PDS had failed to gain 5 percent of the vote). The CDU–CSU–FDP coalition thus had 286 seats (not a majority), while the SPD–Green coalition had 273 seats. Figure 9.1 suggests that the PDS lies on a separate axis (labelled East/West) from the other four parties. In which case it is obvious that the PDS (mostly excommunists from East Germany) would be crucial in creating a majority with either the SPD–Green or the CDU–CSU–FDP coalition. It may be able to select a coalition partner and effectively pivot in the coalition game. Notice that there are at least three very different coalitions that can come into being, and these together define the *heart* of the Bundestag, namely the convex hull of the party positions, as in Figure 9.1.

For multiparty polities in which there is no majority party, we have argued that there are essentially two qualitatively different post-election decisive structures. One is where Parliament is characterized by a structurally stable core, or, in more traditional terms, a central dominant party. The second is one in which no such dominant core party emerges and the core of the legislature is empty. In the case of a structurally stable core, the policy that Parliament is expected to implement is the policy position of the dominant party. If the core is empty, the policy that Parliament will adopt is bound to shift constantly but remain within what we termed the *heart* of Parliament. If Figure 9.1 gives an approximate estimate of party positions after the September 2005 election in Germany, then there is obviously no core party. In the full theory of coalition formation presented in Chapter 3, we suggest that, in the absense of a core, the *cheapest minimal winning* coalition will tend to form, where this term refers to the

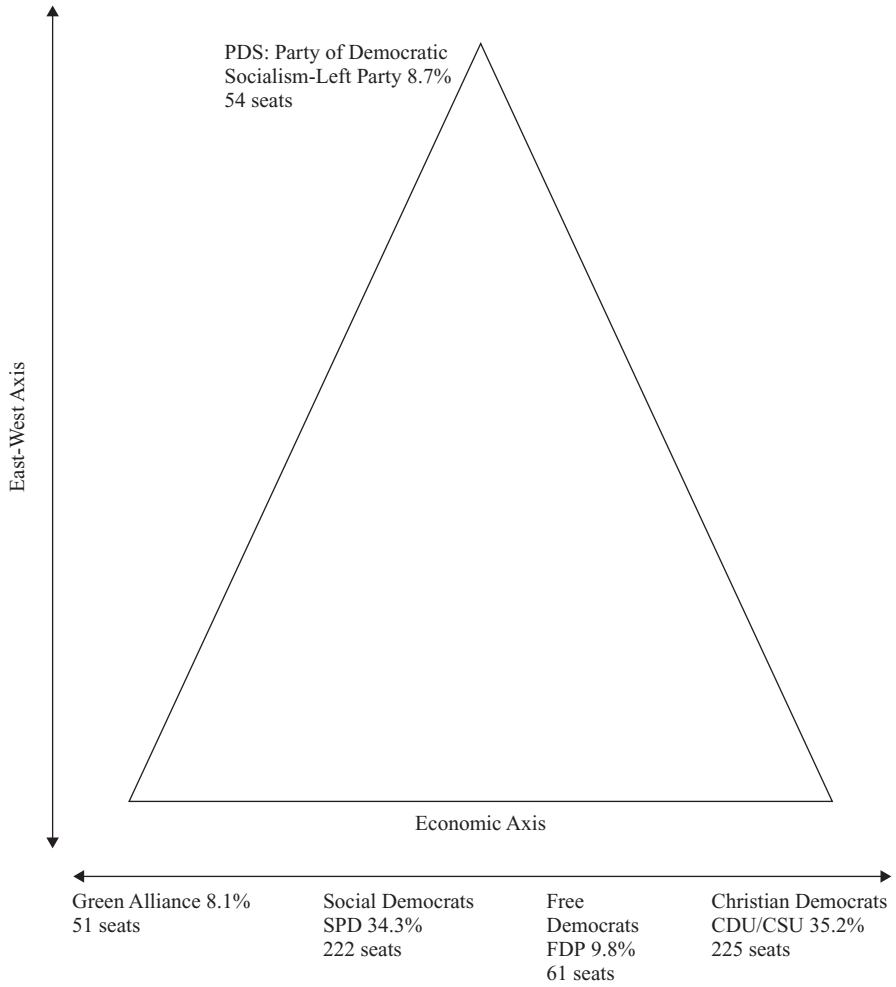


Figure 9.1. Schematic representation of party positions in the Bundestag in Germany, September 2005.

"smallest average policy distance" between the coalition members. The CDU-SPD coalition that finally formed in November, 2005, is an example of such a coalition.

9.2.2 *Recent Changes in the Israel Knesset*

We may also make some comments with regard to recent changes in Israel. Figure 9.2 provides a schematic representation of the 2003 Knesset

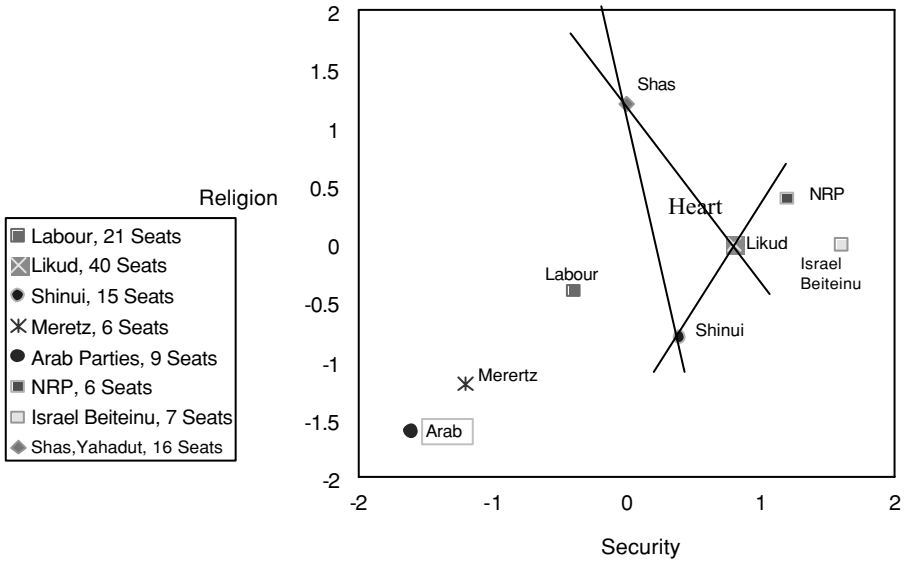


Figure 9.2. A schematic representation of the configuration of the Knesset in 2003.

based on the party positions estimated in Chapter 3 and seat allocations given in Table 2.1. The figure shows Labour with 21 seats, after Am Ehad, with 2 seats, joined Labour in 2003, while Likud has 40 seats after being joined by Olim, with 2 seats. Although Barak, of Labour, became Prime Minister in 1999, he was defeated by Ariel Sharon, of Likud, in the election for prime minister in 2000. The set denoted the heart in this figure represents the coalition possibilities open to Sharon after 2003.

The figure can be used to understand the consequences after Sharon seemingly changed his policy on the security issue in August, 2005, by pulling out of the Gaza Strip. First, the Likud party reacted strongly against this change in policy. In the first week of November, 2005, Amir Peretz, a union activist, and leader of Am Ehad, won the election for leader of the Labour Party. This event can be seen as an illustration of the argument in Chapters 7 and 8 above, where we suggest that a party that fails to attract voters because of a low relative valence will eventually be controlled by party activists. From this perspective, the low valence of Labour vis-à-vis Sharon was the reason the Labour members chose Peretz.

Many observers regarded the change in the leadership of Labour as a critical transformation in the political map of Israel. However, as Figure 9.3 suggests, the shift to the left by Labour under Peretz had no effect

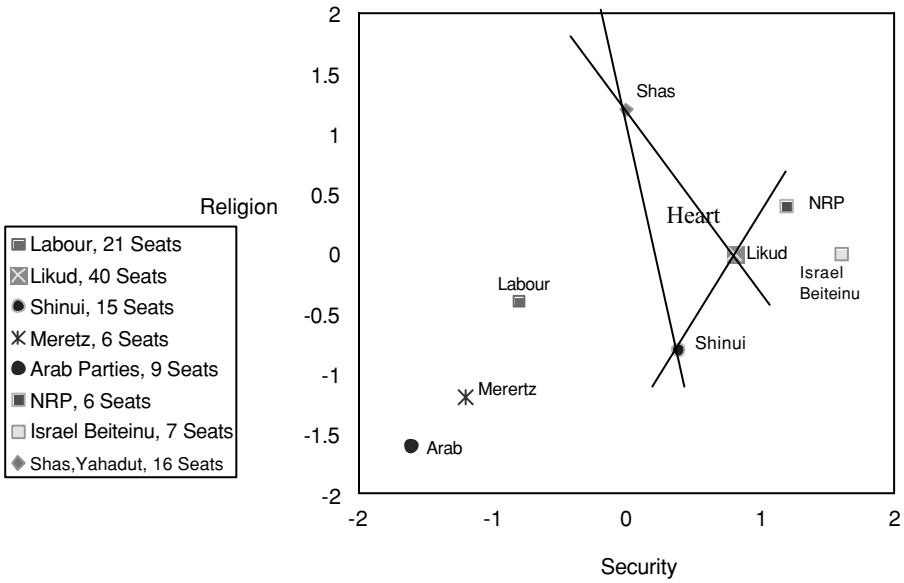


Figure 9.3. The configuration of the Knesset after Peretz becomes leader of Labour.

on the heart of the Knesset. According to our model, there would be no effect on party bargaining.

However, the move by Labour did have indirect consequence. In a highly publicized move, Sharon left the Likud Party and signaled a strong move to the left by allying with Shimon Peres, the former leader of Labour and the author of the Oslo accords. Together these two, with a number of other senior Labour Party members, formed the new party, *Kadima* ("Forward"). This move positioned Sharon at the origin of the electoral space at (0,0) as shown in Figure 9.4. In Chapter 3 the simulations of elections in Israel demonstrated that Likud was unable to "capture the core." By moving Labour to the left, Peretz created the opportunity for Sharon to out-maneuver him. Sharon could strategically move to a position that would increase the probability that he would control the core. Because Sharon's own party members would not support him in this move, he had to leave Likud and form Kadima.

However, Figure 9.4 suggests that the seat strengths were insufficient for Sharon to actually conrol the core in 2005. Indeed, the heart is bounded by the three median lines drawn in the figure. Since these lines do not intersect, the core is empty. Note however, that the new configuration

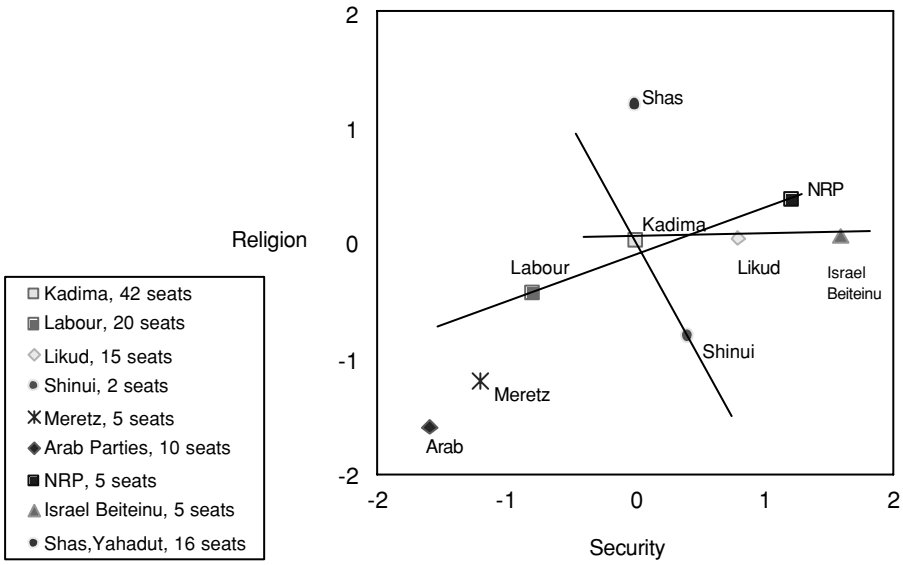


Figure 9.4. The effect of the creation of Kadima by Ariel Sharon on the configuration of the Knesset.

of the heart suggests a possible government coalition of Kadima with supporters from factions of either Likud or Labour.

In a less publicized move, Sharon took his political maneuvering a step further, obtaining the support of Uriel Reichman, founder of the Shinui party, for Kadima. On the face of it, this move seemed hard to explain. Although Reichman is a notable figure in Israel (currently the President of IDC, the largest and most successful private university in Israel), he has never held an elected office. In fact, Sharon promised Reichman the position of Minister of Education in a Kadima coalition government. The purpose of this contract is clear from Figure 9.5. By obtaining Reichman's support, Sharon made a small move "south" in the policy space towards the structurally stable core. Indeed this position is very close to the position previously held by the Labour Party, under the leadership of Rabin, at the 1992 election in Israel.

Figure 9.5 gives our estimates of party positions at the March 28, 2006, election to the Knesset. Because of Sharon's stroke in January, 2006, Ehud Olmert had taken over as leader of Kadima. Although his valence was presumably lower than that of Sharon, Kadima was still able to take 29 seats. Likud, together with religious parties, took 50 seats. One surprise of the

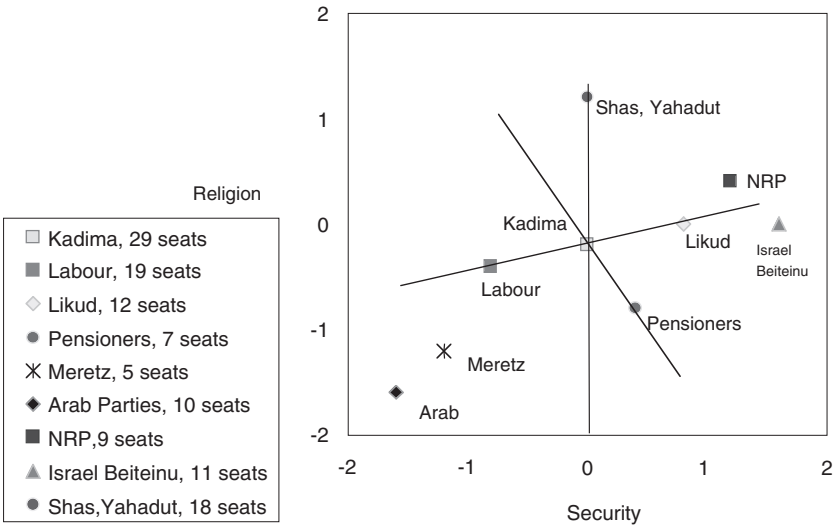


Figure 9.5. The configuration of the Knesset after the election of 28 March, 2006.

election was the appearance of a pensioners' party with 7 seats. However, this had no effect on coalition bargaining. Because a coalition between Labour and the religious parties is infeasible, we can infer that Kadima is located at the structurally stable core position (as indicated in Figure 9.5). It appears that Sharon's change of policy has led to a fundamental transformation in the political configuration, from the $\mathbb{D}_0$-coalition structure that had persisted since 1996, to a $\mathbb{D}_1$-structure associated with the new core party, Kadima.

9.3 PLURALITY RULE

Under plurality rule (in Britain or the United States, for example), we follow Miller and Schofield (2003) and suggest that coalition formation principally takes place inside the party rather than outside. In the United States, for example, parties are coalitions of disparate interests that are maintained only by bargaining between opposed activist groups. For Britain, we have refined this model, and have examined the effect on the "exogenous" valence of the party leader on the bargain made between the leader and the various activist support groups. Since exogenous valence is intrinsic to the party leader, calculating the marginal effects of

exogenous valence and *activist* valence allows us to conclude that increasing leader valence reduces the radical effect of activists.

Although the theory is most readily applied to the situation where there are disciplined parties in the Parliament, it is also applicable to the more complex situation of weakly disciplined parties in the U.S. Congress, with a President able to implement a veto. For presidential elections, we argue that the changing composition of activist support has generated a slow transformation in the positions of the candidates for the two parties. We suggest that this is the cause of what is sometimes called *political realignment*.

Although we have not examined the relationship between President and Congress in the U.S. case, we might expect a core to exist (or at least a restricted heart) when the presidential party controls both House and Senate. In this case, computation of the heart will depend on the particular distribution of policy preferences in both houses of Congress as well as the policies to which the President is committed.

9.4 THEORY AND EMPIRICAL EVIDENCE

The theory that we have presented in this volume came about because of the apparent empirical refutation of the generally accepted theories of elections. In our earlier work with our collaborators on Britain, Israel, the Netherlands, the United States, and, later, Italy, we found no evidence of a significant centripetal tendency to an electoral center. Initially, we did not see the significance of the intercept or valence terms in the electoral models. The simulations that we performed for Israel, however, made it clear that these terms, though they appeared to be trivial, had the effect of completely changing the nature of the equilibria. The "first-order theory" with exogenous valence that we developed gave quite a good approximation to the major party positions in Israel. Moreover, the formal result was bolstered by the correspondence between the theoretical finding and the simulation of the electoral model.

The "second-order theory" involving the "structurally stable core" and coalition bargaining, gave us a qualitative explanation for the location of pivotal parties, such as Shas in Israel, off the principal electoral axis.

The analysis of Italy and the Netherlands made it obvious that party activists should be brought into the electoral equation. The empirical work on Britain, based on the MP survey data, suggested that activists for each party would be quite heterogeneous, and this reinforced the inference made by Miller and Schofield (2003) that a party is itself a coalition.

This suggested the tradeoff between activist support and leader valence. The importance of activists in the U.S. party system, together with the changing nature of the intraparty bargain then naturally led us to the idea of the transformation of party candidate positions and the phenomenon of political realignment.

This "third-order theory" of activists is only sketched here, but we believe it will prove a fertile source of ideas about party competition.

We hope that this nested set of theories will be used in the future to construct an empirically relevant theory of democracy.

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