The Convergence Coefficient and the Heart of an Election: An Application to Recent Elections in Canada

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Abstract

Formal work on the electoral model often suggests that parties or candidates should locate themselves at the electoral mean. Recent research has found no evidence of such convergence. In order to explain non-convergence, the stochastic electoral model is extended by including estimates of electoral valence. We introduce the notion of a convergence coefficient, $c$. It has been shown that high values of $c$ imply that there is a significant centrifugal tendency acting on parties. We used a electoral survey for the 2004 election in Canada to construct a stochastic valence model of the election. The survey allows us to estimate partisan constituency positions for the parties and to model the relationship between party position and vote share in Québec and in the rest of Canada.

We find that the Nash equilibrium for the election outside Québec is one where the centrifugal tendency dominates. We suggest that this is due to the regional characteristic of the Canadian polity.

This result was compared with other empirical results for countries with proportional electoral systems, namely Israel, Turkey and Poland, where the centrifugal tendency is very high, and with the majoritarian polities of the United States and Great Britain, where the centrifugal tendency is very low.

Key words: stochastic vote model, valence, local Nash equilibrium, convergence coefficient, the heart.

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1 Introduction

Work on modeling elections has often assumed that the policy space was restricted to one dimension, or that there were at most two parties (Downs, 1957; Riker and Ordeshook, 1973). The extensive formal literature on electoral competition has typically been based on the assumption that parties or candidates adopt positions in order to win, and has inferred that parties will converge to the electoral median, under deterministic voting in one dimension, or to the electoral mean in stochastic models.

In this paper we offer a formal stochastic model of elections that emphasizes the importance of the idea of valence, and use this notion to provide an explanation why vote maximizing political leaders in Canada will not adopt convergent policy positions at the mean of the electoral distribution. In the standard spatial model, candidate positions matter to voters. However, as Stokes (1963, 1992) has emphasized, the non-policy evaluations, or valences, of candidates by the electorate are just as important.

Being a country with large economic and political differences across regions Canada has parties representing different constituencies elected to the House of Commons. Clarke (1991 [1970]) argues that these differences did not surface in the Canadian system because it was too dangerous for parties to put forth distinct ideological positions. In spite of this lack of mandate Canadians have consistently elected more than two parties to the House of Commons since 1921.

Over the last three decades, regional discontent with the Federal government motivated regional discontented political leaders, activists and interest groups. These regional differences have periodically led to the birth of new parties and the disappearance of others. Since the 1980s, three new parties came into existence: the Reform Party of Canada, the Bloc Québécois (BQ) and the Green Party of Canada (GPC).

Two of these parties represented regional interests. Reform came into existence to voice the discontent of Albertans with the policies adopted in Ottawa. Prior to (but also after) the National Energy Program in 1980, Albertans have wanted to have full control over the revenue from the natural resources, in particular from the oilsands. After the failure of the Meech Lake Accord in 1987 and during the talks leading to the Charlottetown Accord in 1992 the Bloc Québécois emerged as a party that represented the interest of those Québécers who wanted to separate from Canada. Constitutional agreements were supposed to bring “Québec back into the Constitution” but due to failure to reach an

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1See also Clarke, Sanders, et al. (2005, 2009), Scotto et al (2010), and Clarke,Scotto et al. (2011).
2The Canadian regions include the large provinces (Ontario, Québec and British Columbia), the Western provinces (Alberta, Saskatchewan and Manitoba) and the Atlantic provinces (Nova Scotia, New Brunswick, Newfoundland and Labrador and Prince Edward Island). Since the Northwest Territories, Yukon and Nunavut are not yet provinces they are not considered in our analysis.
3The National Energy Program implemented by prime minister Pierre Trudeau in 1980 transferred some of Alberta’s oil revenue to the rest of Canada.
4Until 1982, Canada was governed by a constitution that was a British law and could be
agreement, Québec did not sign the 1982 Constitution.

Ever since inception, both Reform and the BQ have been represented in the Commons. Reform got its first member of Parliament in 1989. The Bloc Québécois, formed by elected members of Parliament, had representatives from its inception. The birth of these parties was an indication that regional differences had become sufficiently strong to set politicians, activists and voters in motion in spite of every one being aware that these new parties had no chance at winning the next election. Their mandates led them to explicitly adopt policies that represented their partisan constituencies in the Commons. This paper argues, contrary to the finding of Clarke (1991 [1970]) on Canadian politics prior to 1980, that starting in the eighties, parties anticipated the electorate's mood and positioned themselves in such a way as to represent their partisan constituencies.

The main purpose of this paper is to examine the parties' electoral positions in response to their partisan constituencies after taking into account the anticipated electoral outcome and the positions of other parties. We do so by studying in detail the 2004 Canadian Federal election. This election is of particular interest because it was the first election since the early eighties that the governing Liberals faced a united right. Unable to make a break through in Eastern Canada, Reform rebranded itself as the Canadian Reform Alliance Party of Canada. Alliance was also unable to appeal to Eastern Canadians. After long deliberations Alliance and the Progressive Conservatives merged in 2003 to form the Conservative Party of Canada. Furthermore issues of Québec were prominent after the failed Constitutional agreements between Québec and the rest of Canada.

The newly formed Conservative Party (CP) needed to gain the support of Eastern Canadians to win the election. It took the Conservative Party three elections (2004, 2008 and 2011) to gain a parliamentary majority. In 2011 the CP's position finally convinced enough Eastern Canadians that it was no longer a western right-wing protest party.

In order to have a chance at winning the 2004 election, the newly merged CP had to increase its vote share relative to that of its predecessors, Alliance and the Progressive Conservatives. The Bloc Québécois, a separatist party that only fields candidates in Québec, had the objective of maximizing its representation in the Commons in order to further the interests of Québec. The governing Liberals faced various scandals and needed to maintain their vote share to stay in office. The Green Party (GPC), starting from a very low base wanted to increase their vote share. However, the 2004 campaign did not run smoothly for the two major parties, the Liberals and the new Conservatives. These two parties took turns at leading in the pre-election polls.

Since parties, particularly new ones, had specific mandates from their core supporters and activists, our estimate of party positions is based on the notion of partisan constituencies. That is, we assume parties position themselves at the mean of their supporters’ preferred policy positions. The logic of this notion changed only by an Act of the British Parliament. The Constitution of 1982 changed this.
is that party leaders can fairly easily, through party membership, conventions, polls and data bases, obtain information about the policy positions of their supporters, and each can respond by advocating policies that are close to the mean preferences of their supporters. That is, parties respond to their base. This assumption of “responsible parties” has been shown by Bernhardt et al. (2009) to enhance electoral welfare under some circumstances. On the other hand, the standard Downsian (1957) model of political competition is that of “opportunistic,” office seeking parties. Each voter is assumed to choose the party whose policy position is closest while parties are assumed to maneuver so as to gain as many votes as possible. The usual Downsian spatial models suggest that convergence to the electoral mean is to be expected. In contrast, Roemer (2001, 2011) has offered a hybrid model of political competition where each party comprises various groups with different agendas: Downsian “opportunists” who simply want to maximize their party’s vote share and “guardians” who champion the interests of the party’s core constituency.

The nature of political competition in Canada suggests that there will be a strong motivation for each party to attempt to maximize its vote share. That means that within each party there will be competition for control of the party agenda between opportunists and guardians.

We will test whether any party could gain votes by moving from the partisan constituency position to some other position. In formal voting theory it is usual to define a “Nash equilibrium” as a vector of party positions with the property that no party may make a unilateral move so as to increase its vote share. We use a variant of this concept, that of a “local Nash equilibrium” (LNE) where we consider only small moves from the position. We use $z_K$ to denote the LNE of model $K$ where $K$ denotes one of the various models that we consider. One of the standard results in formal theory is the mean voter theorem, that the “Nash equilibrium” of a spatial voting game under vote maximization is one where all parties position themselves at the electoral mean. We call such a vector the joint electoral mean.

In order to study each party’s best response to the electoral situation they face, we use the formal results presented in Schofield (2007). This research has developed an analytical framework appropriate for a probabilistic model of elections that allows us to determine whether parties would maximize their vote shares by locating close to, or far from, the electoral center.

Using a pure spatial model, Schofield (2007) defined a convergence coefficient, denoted $c$, that depends on various parameters of the model, particularly the exogenous valences of the party leaders. Using $\{j \in P\}$ to denote the parties, the valences are denoted $\lambda = \{\lambda_j : j \in P\}$. The valence of party $j$ essentially measures the electoral perception of the “quality” of $j$, that is the

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5See also Huber and Bingham Powell (1994), Adams (2001) and Cox and McCubbins (2005) on the congruence between citizens’ preferences and the positions of responsible parties.

6Roemer (2011) focuses on tax policy and only considers a two party model, but does show the existence of a non-centrist equilibrium.

7The electoral mean is simply the average of the voter preferred points, dimension by dimension. For variants of the theorem see Enelow and Hinich (1982, 1984, 1989).
voters’ overall common evaluation of the ability of a party leader to provide good governance. Thus, valences are independent of the party’s position. This intrinsic or exogenous valence can be estimated as the intercept term in the appropriate stochastic model of the voter utility function. As Sanders et al. (2011) comment, valence theory is based on the assumption that “voters maximize their utilities by choosing the party that is best able to deliver policy success.” These valence terms measure the bias in favor of one or other of the party leaders (McKelvey and Patty, 2006).

The convergence coefficient, $c \equiv c(\lambda, \beta)$ also depends on a parameter $\beta$ which measures the weight that voters give to the policy differences they have with the various parties in the model. Lastly, $c$ depends on the variance/covariance matrix of the electoral distribution. By its construction, $c$ is dimensionless, and is independent of the units of measurement of the various parameters. The coefficient can be used to compare results across models and countries.

The convergence coefficient is a summary measure that provides an estimate of the centrifugal or centripetal forces acting on the parties. The valence theorem is presented in Section 4.8 It shows that if the policy space is two-dimensional and if $c(\lambda, \beta) < 1$, then the “local Nash Equilibrium” (LNE) is one where all parties adopt the same position at the electoral mean. On the other hand, if $w$ is the dimension of the policy space and $c(\lambda, \beta) > w$, then the LNE will be one where at least one opportunistic party will have an incentive to diverge away from the electoral mean. In essence, a low empirical convergence coefficient for the model is a convenient measure of the electoral incentive of a small, or low valence, party to move from its position at its core constituency to the electoral mean. Conversely, we can interpret a high value of the convergence coefficient as a measure of the centrifugal tendency exerted on parties pulling them away from the electoral mean. The convergence coefficient is therefore a convenient, simple and intuitive way to examine whether parties will have an incentive to locate close to, or far from, the electoral mean.

In addition to exogenous valence, the formal model can incorporate sociodemographic valences. Whereas exogenous valence measures a common bias across all voters, sociodemographic valences allow these perceptions of the party leaders to vary across relevant sociodemographic categories. We cannot use the results of the valence theorem for the spatial sociodemographic model but must determine the LNE by simulation. We can then compare the LNE of the spatial model with that of the spatial model with sociodemographics.

To apply the formal model, we first construct a multinomial logit model of the election in 2004, based on data collected by Blais et al. (2006). The economic and political differences across provinces (including the emphasis on economic outcomes in Alberta and on identity and culture in Québec) lead us to use a multidimensional policy space in our analysis. We used the survey together with principal component analysis to construct a two dimensional policy space,

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8See Schofield (2007) for the proof of this result.
9The set of such local Nash Equilibria contains the set of Nash Equilibria and in our analysis the two sets are identical.
10These data are available at at http://www.ces-eec.umontreal.ca/surveys.html.
one dimension being socio-economic, the other defined by decentralization.\textsuperscript{11}

Using the estimated parameters of the probabilistic vote model, we can then use simulation to determine the LNE vector of parties’ positions and compute the convergence coefficient of the election. Comparing the LNE with the electoral mean allow us to estimate if all parties positions are determined by “opportunists” who simply want to maximize their parties vote share, as suggested by Downs (1957) or closer to their “partisan constituency” position that reflect the preferences of its “guardians” (Roemer, 2011).

In order to determine whether a party has an incentive to move to a position that will increase its vote share, we suggest that all parties have to take into account the stochastic nature of the election. Consequently there is some risk that by changing position the party’s vote share may decrease rather than increase. To find whether the move is worth the risk, we take the low estimate (given by the lower 95% bound of the estimated vote share at the LNE) and then define the \textit{low vote margin} of the party to be the difference between the low vote share given by the LNE and the sample vote share. We implicitly assume that the sample vote share gives a relatively riskless estimate of the party vote share at the election. If the low vote margins of the low valence parties are positive then this is an indication of their incentive to move their policy positions to the LNE position. We therefore say a LNE is a \textit{stable attractor} if the low vote margins of the small parties are positive. Here we address a theoretical problem with the Nash equilibrium concept. The logic of equilibrium is that if the parties are located at a Nash equilibrium then no party should deviate. However, it need not be the case that the parties will be attracted to the equilibrium from a random vector of positions. The idea of a stable attractor is one way to deal with the problem. In particular if the low vote margin is positive then we suggest that the opportunists can win the argument and persuade the party leader to attempt to maximize votes. On the other hand, if the low vote margin is negative then the guardians of the party will have a strong argument to maintain position at the core constituency position. We also suggest that activists for the party will provide further support for constituency position.

Given that the Bloc Québécois only contested the election in Québec, we divided the sample into two: those in Canada outside Québec (C/Q) and in Québec (Q). In the model outside Québec, we found the convergence coefficient, to be equal to 1.94. We then show that the joint origin could not be a LNE for the pure spatial model, and thus could not be a stable equilibrium. Moreover, when we examine the LNE for the joint model, including sociodemographics, outside Québec, we also show that the low vote margin of the low valence party, the GPC, is negative implying that this party had no incentive to move from its partisan constituency position.

According to the logit model for Québec for 2004, the results suggest that the Bloc had the highest valence and that the electoral origin was a stable LNE. However, since Québec has only one quarter of the Canadian population and the

\textsuperscript{11}\textit{Decentralization, or the separation of parts of Canada, from Ottawa is of fundamental importance in Canada.}
LNE is not stable for the model outside Québec, we infer that the LNE could not be stable for the overall election for the whole of Canada. We then show that the LNE for the joint model with sociodemographics is very different from the set of partisan constituency positions. Since the parties have no incentive to move to the LNE positions predicted for the conditions outside Québec, we are also led to the inference that the activists for each party provide inducements to the party to remain close to the partisan constituency position. We use the difference between the equilibrium positions and the partisan constituency positions to provide an estimate of the influence activist had on the parties.

In extending the results for Canada, in a concluding section we also discuss the estimates of the convergence coefficients for a number of polities in order to offer a classification of various electoral systems. We contrast the results for Canada with other work on the very highly fragmented polities of Poland, Israel and Turkey, where the convergence coefficients lie in the range from 4.0 to 7.0. At the opposite pole, the presidential electoral system of the United States is extremely majoritarian with the convergence coefficients for recent elections estimated to be in the range [0.45, 1.1]. Other empirical work has shown that the convergence coefficient for the 2005 and 2010 elections in Great Britain are 0.8 and 0.98, respectively. We suggest that the centrifugal tendency in the majoritarian polities like the United States and the United Kingdom is very low. In contrast, the centrifugal tendency in Israel, Turkey and Poland is very high. The convergence coefficient for Canada lies between these two extremes suggesting that the centrifugal tendency is only moderate, being generated to some extent by the regional nature of Canadian politics.

We can illustrate political configuration by a graphical representation of the party locations and strengths known as the “heart.”12 In a polity like Canada, with a relatively low convergence coefficient and with a small number of parties, the possible winning coalitions are easy to determine, and the heart is fairly simple. However, in a fragmented polity like Poland with a high convergence coefficient and many small parties, the heart is a very complex star shaped figure that illustrates the large number of possible coalition configurations. The effective vote number or effective seat number,13 can measure fragmentation, and gives an indication of the difficulty inherent in inter-party negotiation over government. These measures, however, do not address the fundamental aspect of democracy, namely the electoral preferences for policy. Since convergence involves both preferences, in terms of the electoral covariance matrix and the effect of the electoral system, the convergence coefficient allows us to classify various polities and political systems.

In the next section we discuss recent Canadian political history, while section 3 gives the background to the 2004 election. Section 4 presents the formal model

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12 The heart is just a convenient way to represent the different possible coalition governments that can form.

13 Fragmentation can be identified with the effective number. That is, let \( H_v \) (the Herfindahl index) be the sum of the squares of the relative vote shares and \( env = H_v^{-1} \) be the effective number of party vote strength. In the same way we can define \( ens \) as the effective number of party seat strength using shares of seats. See Laakso and Taagepera (1979).
and applies it to the election. A brief subsection 4.6 discusses events after 2004. Sections 5 compares the models for different polities while section 6 concludes.

2 The emergence of partisan constituencies

In recent history, Canadians have consistently elected at least three parties to the Federal legislature. In the current Parliament we have the governing Conservative Party of Canada (CP), the New Democratic Party (NDP) as official opposition, Liberal Party of Canada (LPC) and the separatist Bloc Québécois (BQ). The Green Party of Canada (GPC) is a relatively new party whose support steadily rose up to an including the 2008 election then falling in the 2011 election. However, since only the first-past-the-post candidate in each riding gets elected to the legislature, the Green Party only managed to elect its first member of Parliament in the 2011 election. Other parties, with fewer votes, have been unable to gain seats in Parliament.

[Insert Table 1 here]

Traditionally, Liberals and Conservatives have fought each other to form the government. In the 2011 election, the Conservatives finally managed to win a majority after presiding over consecutive minority governments. The NDP became the official opposition for the first time since its inception in 1960. The Liberals were relegated to third place for the first time in their history. Table 1 gives the election results in Canada for 2004, 2006, 2008 and 2011.

2.1 The Birth of New Parties

The western-based Reform Party of Canada came into existence during the first term of Brian Mulroney’s Progressive Conservative (PC) government (1984-1988). Westerners, who long felt alienated from Ottawa, became angry with Mulroney’s government as they believed that he favored Québec, that his government lacked fiscal responsibility, and that he had failed to institute an elected Senate. Preston Manning joined discontented Western interest groups to create the Reform Party in May, 1987. Reform elected its first member of parliament in 1989 and gained prominence in the 1993 election replacing the PC as the leading right-wing party (the PC won only 2 seats with 16% of the vote). The 1993 election marked the emergence of a regionally fractured opposition.

The 1993 and 1997 elections made it clear to Reformers that Canadians living east of Manitoba saw them as an extreme right-wing western protest party. Between 1997 and 1999, Manning tried to reunite Reform and the PC, finally launching the "unite-the-Right" campaign in January of 2000. The process culminated instead in rebranding Reform as the Canadian Alliance Party.

The highly public in-fighting within the Liberal Party, in the period leading up to the 2000 election, gave Alliance and Progressive Conservatives (who had somewhat recovered from the 1993 election) hope of winning the next election. After long deliberations, on 15 October 2003, the new PC leader Peter MacKay
and Alliance leader Stephen Harper announced their merger agreement. Ratification by the two parties lead to the creation of the Conservative Party (CP) on December 7, 2003. Some prominent red PC members refused to join the new party. Harper became the new CP leader on March 20, 2004.

The emergence of Bloc Québécois was Québécizers, response to the constitutional uncertainty created by the failure of the Meech Lake in 1987 and to the talks leading to the Charlottetown Accord in 1992. In 1991, Québec Tories led by Lucien Bouchard severed their connections with the PC party and in conjunction with some Québec Liberals on June 15, 1992 formed the Bloc Québécois, a pro-sovereigntists party focused on independence for Québec. Even though running candidates only in Québec, the Bloc has obtained a significant number of members to the Commons as the high concentration of its support base allowed Bloc candidates to be the first-past-the-post in many ridings. From 1997 to 2000, the Bloc became the Official Opposition after narrowly winning the second largest number of seats in 1997 election.\(^\text{14}\)

The Green Party of Canada (GPC) was founded in 1983. Even though its support has steadily increased up to and including the 2008 election, it fell in the 2011 election. However, in 2011 the GPC managed to gain its first member of Parliament.

2.2 Political Change at the Ballot
Over the last thirty years, Canadians have also expressed their discontent at the ballot. The 1984 election marked the last time a party won more than fifty percent of the vote. The next nine years saw Brian Mulroney’s Progressive Conservatives (PCs) hold two consecutive parliamentary majority governments. The 1993 election brought the Liberal Jean Chrétien to office. Chrétien won three consecutive parliamentary majorities (1993, 1997 and 2000). In spite of these successes, and his popularity, Chrétien was replaced by his finance minister and long time rival Paul Martin as party leader at the Liberal convention on November 14, 2003. Martin was sworn as prime minister on December 12, 2003.

On February 10, 2004, the Sponsorship Scandal broke.\(^\text{15}\) The Liberals’ ratings plummeted, specially in Québec, but were still above those of the new CP (Clarke, Kornberg, \textit{et al.} 2005). In May 2004, the governing Ontario Liberal party reneged on their campaign promise not to raise taxes. This hurt the Federal Liberals as Ontarians had been their major support base in the 1993, 1997 and 2000 elections. On May 22, Martin was forced to call an election for June 28, 2004, and faced Stephen Harper, the leader of the new Conservative Party.

\(^{14}\) However, Québécizers only elected 4 members of the Bloc to Parliament in the 2011 election.
\(^{15}\) The Auditor General’s report released on February 10, 2004 on the Federal government’s Sponsorship Program seriously hurt the Liberals. The program started in 1996 under Chrétien was to promote the federalist view after the close defeat of 1995 Referendum on Québec’s separation in order to offset the Parti Québécois government’s effort to promote Québec independence. The Auditor General report indicated that she saw little evidence of any work done under the sponsorship contracts, that many of the agencies had Liberal ties, and that about $100 million of the $250 million spent under the Program was unaccounted for.
During the 2004 electoral campaign, pre-election polls showed the Liberals and Conservatives neck-and-neck. By mid-campaign the CP was slightly ahead of the Liberals, and it was thought that a minority CP government was possible. The CP, however, made two major mistakes. They accused prime minister Martin of being soft on child pornography. Ralph Kline, the PC premier of Alberta, announced that his government was considering a two-tier health care system. The Liberals and many Canadians reacted strongly against both issues. The Liberals’ campaign portrayed Harper as an extreme rightwing Conservative and encouraged NDP-supporters to vote strategically. By the last week of the campaign the Liberals were ahead of the Conservatives.

3 The 2004 Canadian election

Table 2 shows the 2004 election results by province. The Liberals (LPC) under Martin won the 2004 election with 135 (44%) seats out of 308, down 37 from the 2000 election. Martin formed the first minority government since 1979 and was informally supported by the NDP. Relative to the 2000 election, the Liberals lost votes in Ontario and Québec winning 75 out of 106 Ontario seats in 2004 (down from 100 out of 103 in 2000) and 21 out of 75 Québec seats in 2004 (down from 36 out of 75 in 2000). They held onto the 14 seats they had in the Western provinces since 2000, gaining in British Columbia and losing in Manitoba.

The Conservatives won the second largest number of seats, winning more seats (99) than both of its two predecessors in 2000 (Alliance 66 and PC 12). Its vote share (30%) was, however, lower than that of its predecessors combined (Alliance 26% and PC 12%). Support for the CP came mainly from Western Canada and in spite of making some progress in Ontario, gaining 24 seats, they failed to make in roads in Québec and the Atlantic Provinces. Clearly, the Conservatives were still seen by many as mainly representing western interests.

Support for the Bloc Québécois soared in 2004 as almost half (49%) of Québécers voted for them, thus winning 54 out of 75 Québec seats with 12.4% of the national vote. The NDP not only almost doubled its vote share relative to 2000, it managed to add 6 members to its caucus mostly in Ontario and British Columbia. Support for the Green Party also increased relative to 2000 but since it started from a very low base, it did not gain any seats in the Parliament.

3.1 Policy Dimensions and Sociodemographic Data

To study the 2004 Canadian election we used the survey data for Canada collected by Blais et al. (2006). Because Bloc Québécois only contested the election in Québec, we divided the sample into those outside Québec (C/Q) and in Québec (Q). The votes and shares in these two regions are given in Table 3 and the sample vote shares in Table 4. Table A1 in Appendix A lists the survey questions used in our analysis.
The factor analysis performed on the voters’ responses in the survey questions led us to conclude that there were two factors or policy dimensions: one “social,” the other “decentralization.”

The social dimension is a weighted combination of voters’ attitudes towards (1) the gap between poor and rich, (2) helping women, (3) gun control, (4) the war in Iraq and (5) their position the left-right scale. We coded the social dimension such that lower values imply higher interest in social programs so as to have a left-right scale along this axis. The decentralization dimension included voters’ attitudes towards (1) the welfare state, (2) their standard of living, (3) inter-jurisdictional job mobility, (4) helping Québec and (5) the influence of Federal versus Provincial governments in their lives. A greater desire for decentralization implies higher values on this axis.

Using the factor loading given in Table A2 in Appendix A, we computed the value for each voter along the social and decentralization dimensions. As shown in Table A3 in Appendix A, the mean and median values of voters’ positions along these two dimensions in Canada are at the electoral origin, \((0, 0)\). To illustrate, a voter who thinks that more should be done to reduce the gap between rich and poor would tend to be on the left of the Social axis \((x - \text{axis})\), while a voter who believes that the federal government does a better job of looking after peoples’ interests would have a negative position on the D axis \((y - \text{axis})\), and could be regarded as opposed to decentralization. Figures 1 and 2 give smoothings of the electoral distribution in these two regions.

The survey gave voter support for the parties, so we estimated the partisan constituency positions by taking the mean of the party supporters. For the Bloc Québécois we used the average of the positions of Bloc supporters in Québec. The party’s positions along these two dimensions are given in Tables A4 and A5 in Appendix A, and are described by the vector

\[
\mathbf{z}^* = \begin{bmatrix} \text{Party} & \text{NDP} & \text{GPC} & \text{LPC} & \text{CP} & \text{BQ} \\
S & -0.78 & -0.63 & -0.17 & 1.27 & -1.48 \\
D & 0.05 & -0.13 & -0.38 & 0.32 & 0.23 \end{bmatrix}.
\]

Note that supporters of the Liberals (LPC) and the Greens (GPC) advocate far more social programs and less decentralization \((-0.17, -0.038)\) and \((-0.63, -0.13)\) respectively. Supporters of the New Democratic Party (NDP) at \((-0.78, 0.05)\) want more social programs but seem neutral on the decentralization scale. Supporters of the Bloc Québécois (BQ) preferred more social programs and more decentralization \((-1.48, 0.23)\). Finally, supporters of the Conservative Party (CP) preferred fewer social programs and more decentralization \((1.27, 0.32)\) than the other parties. The Liberals and Conservatives, the two major contenders in the 2004 election, were located in opposite quadrants. The Liberals had historically been longer in power and had a broad
national representation in the 2000 election. Their supporters preferred more social programs and more centralization than the Conservative Party. The CP was strongly supported by Albertans (see Table 2). Our estimates of the positions of the parties correspond fairly closely to the estimates of Benoit and Laver (2006), obtained by a sample of expert opinions. Their estimates were obtained for 2000 before the merging of the Alliance and the Progressive Conservatives. Just as in Figure 1, Benoit and Laver estimated both parties to lie in the upper right quadrant of the policy space, with the LPC located to the left.\footnote{We infer that our estimated party positions correspond quite closely to the perceived positions of the parties.}

\[ [\text{Insert Table 5 here}] \]

The full sample included 862 respondents. From Table 5, the \textit{electoral covariance matrix} for the entire sample is

\[
\nabla_C^0 = \begin{bmatrix}
\text{Soc} & 2.78 & 0.00 \\
\text{Soc} & 0.00 & 1.14
\end{bmatrix}.
\]

At the national level, the covariance between the social and decentralization dimensions is \( \sigma^2_{SD} = 0.0 \). The “total” variance is \( \sigma^2_C = \sigma^2_S + \sigma^2_D = 2.78 + 1.14 = 3.92 \) with an \textit{electoral standard deviation (esd)} \( \sigma_C = 1.98 \). The principal electoral component of \( \nabla_C^0 \) is given by the eigenvector \((-1, 0)\) with eigenvalue (variance) 2.78, while the minor eigenvector is \((0, -1)\), with eigenvalue (variance) 1.14.\footnote{The principal electoral component is given by the eigenvector with the larger eigenvalue.}

Notice that the variances on these two orthogonal axes are different. In the formal model below we show that a convergence coefficient \( c(\lambda, \beta) \) can be determined when we use different spatial parameters on the two dimensions. We examined models of this kind for the two regions but found no difference in our conclusions between the models with a single or multiple spatial parameters.

Table 5 shows that non-Québécers prefer fewer social programs and more centralization than Québécers. The median Québécer is slightly to the left of the mean Québécer in the decentralization dimension.

For Canada outside Québec (C/Q), the sample of \( n = 675 \) respondents gives an electoral covariance matrix of

\[
\nabla_{C/Q}^0 = \begin{bmatrix}
2.70 & 0.12 \\
0.12 & 1.18
\end{bmatrix}.
\]

The “total” variance is \( \sigma^2_{C/Q} = \sigma^2_S + \sigma^2_D = 3.88 \) with an esd \( \sigma_{C/Q} = 1.97 \). Note that along the social dimension the variance is smaller and along the decentralization higher in C/Q than in the national sample. Moreover, while there is no covariance between the social and decentralization dimensions in the national sample, but positive in the C/Q sample. The major electoral component
of $\nabla_0^{C/Q}$ is slightly different from that of $\nabla_0^C$ and is given by the eigenvector $(1.0, 0.08)$ with eigenvalue (variance) 2.7, while the minor eigenvector is $(-0.08, 1.0)$, with variance 1.19.

Québec, with a sample of $n = 187$, has an electoral covariance matrix

$$\nabla_0^Q = \begin{bmatrix} 1.48 & -0.57 \\ -0.57 & 0.98 \end{bmatrix}$$

(3)

whose “total” variance is $\sigma^2_Q = 2.46$ with esd $\sigma_Q = 1.57$. Note that in Québec the variances along the social and decentralization dimensions are much smaller than in all of Canada. Moreover, while in all of Canada and outside Québec the covariance between the social and decentralization dimensions is zero, or close to zero, for the Québec sample it is negative. The principal electoral component of $\nabla_0^Q$ for Québec is different from $\nabla_0^{C/Q}$ and is given by the eigenvector $(1.0, -0.66)$ with variance 1.85, while the minor eigenvector is $(0.66, 1.00)$ with variance 0.62. These differences in eigenvectors show that the electoral distributions in the two regions as presented in Figures 1 and 2 are slightly different.

Figures 1 and 2 together with Table 2 and the electoral covariance matrices suggest that there are significant regional differences. These differences are driven by Québec and Alberta who residents wanted greater decentralization but for different reasons. Québécers wanted to ensure the survival their culture, language, laws, and to control the composition of its population (immigration). Thus, due to its distinct nature, Québécers wanted decentralization for cultural reasons. Residents of Alberta wanted control over the regions vast natural resources, mainly its oil sands, and did not want to share its oil revenues with the rest of Canada. Thus, Alberta wanted economic decentralization.

The survey also collected data on age, gender and education. The descriptive statistics of these sociodemographic variables are given in Table A3 in Appendix A. While Table A3 shows that there are no major differences between respondents in C/Q and in Québec, there are differences in support base across parties. The mean Liberal supporter is older than that of other parties with the youngest mean supporter voting Green. More than half those voting Liberal, NDP and BQ were women with more than half of the men voting for the Conservatives and the Greens.

The differences between Québec and the rest of Canada provided an additional reason to study the 2004 election in these two regions separately.

4 Models of the 2004 Canadian election

Clarke, Kornberg et al. (2005) point out that in the last stages of the 2004 election campaign it was not clear which of the two front runners, the Liberals or Conservatives, would win as “the election was to close to call.” The polls indicated that neither party was expected to win a majority of votes and given that support for both parties was hovering around the 33-35% range, neither party expected to win a parliamentary majority (see Figure 1 in Clarke, Kornberg et al. 2005, p. 248). This promoted parties, voters and political commentators to
speculate which party would form a minority government. We infer that there would be some motivation each party to attempt to maximize its vote share.

4.1 The pure spatial model

To study the 2004 Canadian election we use an electoral model that incorporates both valence and spatial components as presented in Schofield (2007). Knowing vote intentions we constructed various multinomial logit (MNL) models of the election separately for outside Québec and in Québec using the survey data.

When the positions in the policy space $X$ of the $p$ parties are given by the vector $z = (z_1, \ldots, z_j, \ldots, z_p) \in X^p$ and if voter $i$’s ideal policy is given by $x_i \in X$, then the voter’s utility is given by the expression

$$u_{ij}(x_i, z_j) = \lambda_j - \beta \|x_i - z_j\|^2 + \epsilon_j = u^*_j(x_i, z_j) + \epsilon_j. \tag{4}$$

Here $u^*_j(x_i, z_j)$ is the observable component of utility. The exogenous valence of candidate $j$ is $\lambda_j$, and the exogenous valence vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ is assumed to satisfy $\lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1$. Note that $\lambda_j$ is the same for all voters and provides an estimate of the “quality” of party $j$ or its ability to govern. The term $\|x_i - z_j\|$ is simply the Euclidean distance between voter $i$’s position, $x_i$, and candidate $j$’s position $z_j$. The coefficient $\beta$ is the weight given to this policy difference. We assume that the error vector $\epsilon = (\epsilon_1, \ldots, \epsilon_j, \ldots, \epsilon_p)$ has a Type I extreme value distribution, as appropriate for a multinomial logit (MNL) estimation (Train, 2003). The variance of $\epsilon_j$ is fixed at $\frac{\pi^2}{6}$. Note that $\beta$ has dimension $\frac{1}{L^2}$, where $L$ is whatever unit of measurement is used in $X$.

Thus, voter behavior is modeled by a probability vector. The probability that voter $i$ chooses party $j$ at the vector $z$ is

$$\rho_{ij}(z) = \Pr[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l), \text{ for all } l \neq j]$$

$$= \Pr[\epsilon_l - \epsilon_j < u^*_j(x_i, z_j) - u^*_l(x_i, z_j), \text{ for all } l \neq j].$$

Here $\Pr$ stands for the probability operator generated by the distribution assumption on $\epsilon$. Thus, the probability that $i$ votes for $j$ is given by the probability that $u_{ij}(x_i, z_j) > u_{ij}(x_i, z_l)$, for all $l$, i.e., that $i$ gets a higher utility from $j$ than from any other party.

Given the extreme value distribution, the probability $\rho_{ij}(z)$ has a MNL specification and can be estimated. For each voter $i$ and party $j$, the probability that voter $i$ chooses party $j$ at the vector $z$ is then given by

$$\rho_{ij}(z) = \frac{\exp[u^*_j(x_i, z_j)]}{\sum_{k=1}^{p} \exp[u^*_k(x_i, z_k)]}. \tag{5}$$

Voters decisions are stochastic in this framework.\textsuperscript{18} Even though parties cannot perfectly anticipate how voters will vote, they can estimate the expected

\textsuperscript{18}See for example the models of McKelvey and Patty (2006). Note that there is a problem
vote share of party \( j \) as the average of these probabilities:

\[
V_j(z) = \frac{1}{n} \sum_{i \in N} \rho_{ij}(z).
\]  

We assume that parties can estimate how their vote shares would change if they marginally move their policy position. The Local Nash Equilibrium (LNE) is that vector \( z \) of party positions so that no party may shift position by a small amount to increase its vote share. More formally a LNE is a vector \( z = (z_1, \ldots, z_j, \ldots, z_p) \) such that each \( V_j(z) \) is weakly locally maximized at the position \( z_j \).

To avoid problems with zero eigenvalues we also define a SLNE to be a vector that strictly locally maximizes \( V_j(z) \). We typically denote an LNE by \( z_K \) where \( K \) refers to the model we consider. Using the estimated MNL coefficients we simulate these models and then relate any vector of party positions, \( z \), to a vector of vote share functions \( V(z) = (V_1(z), \ldots, V_p(z)) \), predicted by the particular model with \( p \) parties.

Using this pure spatial model we can examine whether parties will locate at the electoral mean. We call this vector the joint electoral mean.

**Definition:** The Convergence Coefficient of the Pure Spatial Model when the space \( X \) has dimension \( w \).

(i) Define

\[
\rho_1 = \left[ 1 + \sum_{k=2}^{p} \exp \left( \lambda_k - \lambda_1 \right) \right]^{-1}.
\]  

This is the theoretical probability, obtained from the pure spatial model that a generic voter chooses the lowest valence party, namely party 1, when all parties are at the electoral mean.

(ii) Let the space \( X \) be endowed with an orthogonal system of coordinate axes \((1, \ldots, s, \ldots, t, \ldots, w)\).

For each coordinate axis let \( \xi_t = (x_{1t}, x_{2t}, \ldots, x_{nt}) \in \mathbb{R}^n \) be the vector of the \( t^{th} \) coordinates of the set of \( n \) voter ideal points. Let

\[
\chi_t = \frac{1}{n} \sum_{i=1}^{n} x_{it}
\]

be the electoral mean on the \( t^{th} \) axis and \( \chi = (\chi_1, \ldots, \chi_t) \in X \) be the electoral mean. The joint electoral mean \( z_0 \) is the vector

\[
z_0 = (\chi, \ldots, \chi) \in X^n
\]

where each party adopts the same position at the electoral mean.\(^{19}\)

---

\(^{19}\)We can readily renormalize so that \( \chi = 0 \), the electoral origin.
Let $\xi_t^0 = \xi_t - \chi_t$ and let $(\xi_s^0, \xi_t^0) \in \mathbb{R}$ denote scalar product. The covariance about the mean, between the $s^{th}$ and $t^{th}$ axes is denoted $(\sigma_s, \sigma_t) = \frac{1}{n}(\xi_s^0, \xi_t^0)$ and $\sigma_s^2 = \frac{1}{n}(\xi_s^0, \xi_s^0)$ is the electoral variance on the $s^{th}$ axis.

(iii) The symmetric $w \times w$ electoral covariance matrix $\nabla_0$ about the electoral mean is defined to be $\frac{1}{n}[(\xi_s^0, \xi_t^0)]_{s=1}^{w}.$

(iv) The total electoral variance is

$$\sigma^2 = \sum_{s=1}^{w} \sigma_s^2 = \frac{1}{n} \sum_{s=1}^{w} (\xi_s, \xi_s) = \text{trace}(\nabla_0).$$

(v) The $w$ by $w$ characteristic matrix, of party $j$ is given by

$$C_j = 2\beta(1 - 2\rho_j)\nabla_0 - I.$$  \hspace{1cm} (8)

where $I$ is the $w$ by $w$ identity matrix.

(vi) The convergence coefficient of the pure spatial model is

$$c \equiv c(\lambda, \beta) = 2\beta|1 - 2\rho_1|\sigma^2.\hspace{0.5cm} (9)$$

Observe that the $\beta$-parameter has dimension $L^{-2} = \frac{1}{\tau^2}$, so that $c$ is dimensionless. We can therefore use $c$ to compare different models.

The following result is proved in Schofield (2007).

Valence Theorem.

(i) The joint electoral mean $z_0$ satisfies the first order condition to be a LNE for the pure spatial model.

(ii) The necessary and sufficient second order condition for SLNE at $z_0$ is that $C_1$ has negative eigenvalues.\(^{20}\)

(iii) A necessary condition for $z_0$ to be a LNE for the model is that $c(\lambda, \beta) \leq w$.

(iv) A sufficient condition for $z_0$ to be a LNE in the two dimensional case is that $c(\lambda, \beta) < 1$. \[\blacksquare\]

The expression for $C_j$ has a simple form because of the assumption of a single distance parameter $\beta$. We can also use a model with different coefficients $\beta = \{\beta_1, \beta_2, ..., \beta_w\}$ on each dimension. That is we assume the spatial component of the utility of voter $i$ has the form

$$- \sum_{k=1}^{w} \beta_k(x_{ik} - z_{jk})^2$$

In this case the characteristic matrix for $j$ can readily be shown to be

$$C_j = 2(1 - 2\rho_j)\beta\nabla_0\beta - \beta, \hspace{0.5cm} (10)$$

\(^{20}\)The condition for an LNE is that the eigenvalues are negative semi-definite. The case where all eigenvalues are zero is non-generic and has to be examined by itself.
where $\beta$ is the diagonal matrix of the the $\beta$ coefficients, while $\beta \nabla_0 \beta$ is the covariance matrix where each axis is weighted by the coefficients $\beta = (\beta_1, \beta_2, \ldots, \beta_w)$. The necessary condition in this case is that \[ \text{trace}(C_1) < 0, \] or \[ 2(1 - 2 \rho_1) \text{trace}(\beta \nabla_0 \beta) < \beta_1 + \beta_2 + \ldots + \beta_w. \]

and we can take \[ c(\lambda, \beta) = \frac{2(1 - 2 \rho_1) \text{trace}(\beta \nabla_0 \beta)}{\frac{1}{w}(\beta_1 + \beta_2 + \ldots + \beta_w)} \tag{11} \]
again giving the necessary condition of $c(\lambda, \beta) \leq w$, for a LNE.

The sufficient condition in the two-dimensional case is that \[ d(\lambda, \beta) = 2(1 - 2 \rho_1)[\beta_1 \sigma_1^2 + \beta_2 \sigma_2^2] < 1. \]

The convergence coefficient (9) determines the nature of the Hessian of the vote share function of the low valence party, namely party 1. The Hessian of the vote share function of any party is the $w$ by $w$ second derivative and can be shown to be given by the characteristic matrix of the party given in (8). When all parties are at the mean and $c(\lambda, \beta) > w$, then this Hessian of the vote share function of the lowest valence party cannot be negative definite and thus cannot correspond to a maximum of the vote share function. To see this note that that from (8), \[ \text{trace}(C_1) = c(\lambda, \beta) - w \] so if $c(\lambda, \beta) > w$ then the trace of $C_1$ will be positive and one eigenvalue of $C_1$ must be positive. This implies that $z_0$ cannot be a LNE, since at least one party (namely 1) will diverge from the electoral mean in order to increase vote share. In particular, the Hessian of this low valence party at the joint mean must be a generalized saddlepoint or a minimum.

In these cases, the low valence party can shift position by moving away from the mean, either up or down the eigenvector associated with the largest positive eigenvalue. In the case all eigenvalues are positive, the low valence party can move along any eigenvector to increase vote share. The resulting LNE can be found by simulation. As observed above, the case $c(\lambda, \beta) = w$ is indeterminate. The trace can be zero, so all eigenvalues could be zero.

In arbitrary dimension, $w$, if $c(\lambda, \beta) \leq 1$ then \[ \text{trace}(C_1) < 0. \] In the two-dimensional case, in particular, if $c(\lambda, \beta) < 1$ then \[ \text{det}(C_1) \] must be positive, implying that both eigenvalues of $C_1$ are negative. It then follows that all \{C_j\} have negative eigenvalues, giving a SLNE and thus an LNE. \(^{21}\)

The convergence coefficient in (9) increases as the weight given by voters to the policy differences, $\beta$, increases. It also increases in the electoral variance $\sigma^2$ and as the probability of voting for the lowest valence party, $\rho_1$, decreases. From (7) we can see generally that $\rho_1$ decreases if the valence differences between $\lambda_1$ and the other parties \{\lambda_2, \ldots, \lambda_p\} increase. If the valence differences are sufficiently large, then vote maximizing parties will not converge to the electoral mean.

Intuitively, the Valence Theorem asserts that if $c(\lambda, \beta) > w$ then party with the lowest valence has an incentive to move away from the electoral mean. Other

\(^{21}\)This result follows from the application of the triangle inequality to the determinant. A parallel result can be obtained in dimension above 2.
low valence parties may then also find it advantageous vacate the center. The convergence coefficient, together with the analysis of the Hessians of the low valence parties, allows us to identify which parties have an incentive to move away from the electoral mean.

In the next subsection, we use this result for the model for C/Q, to compute the probability of voting for the lowest valence party, GPC. This is given by (15).

In order to compute error bounds on the convergence coefficient, we also estimate the standard error on \( \rho_j \), and thus on \( c(\lambda, \beta) \), by using Taylor’s Theorem, which asserts that

\[
\rho_j(\lambda_j \pm h) = \rho_j(\lambda_j) \pm h \frac{d\rho_j}{d\lambda_j} = \rho_j(\lambda_j) \pm h\rho_j(1 - \rho_j) \tag{12}
\]

\[
= \rho_j(\lambda_j)[1 \pm h(1 - \rho_j)] \tag{13}
\]

We now apply this MNL pure spatial model to the elections outside Québec and in Québec. When estimating the models for the two regions, we use the Liberal Party as the base party, so that the coefficients of the models are measured relative to that of the Liberals.

4.2 Outside Québec

Using the Canada outside Québec sample, we built a pure spatial model of the 2004 election in Canada without Québec. Table 5\(^{22}\) gives the following coefficients from the MNL estimation:

\[
\begin{align*}
\lambda_{C/Q}^{NDP} &= -0.56, & \lambda_{C/Q}^{CP} &= -0.02, & \lambda_{C/Q}^{GPC} &= -2.23, & \lambda_{C/Q}^{LPC} &= 0.0 \\
\beta^{C/Q} &= 0.27.
\end{align*}
\tag{14}
\]

The exogenous valence, \( \lambda_j \), corresponds to the intercept term in the regression and measures the common perception of the quality of party \( j \) among the sample. Note that \( \lambda_j \) is the non-policy component in the voter’s utility function in (4).

These results indicate that in 2004 voters regarded the GPC and the NDP (\( \lambda_{GPC}^{C/Q} = -2.23 \) and \( \lambda_{NDP}^{C/Q} = -0.56 \)) as parties with the lowest valences, that is, least able to govern, once policy differences were taken into account. The model suggests that both the GPC and NDP had a statistically significant lower valence than the Liberals.

Recall that we are interested in finding where the parties will locate in the policy space in order to maximize their vote share. Because the outcome of the election depends on these vote shares, we assume that parties use polls and other information at their disposal to form an idea of the anticipated election

\(^{22}\)In Tables 5 and 6 we include the Akaike (AIC) and Bayesian (BIC) Information Criterion. Lower values indicate better model performance.
outcome and then use this information to find their most preferred position taking into account their estimates of where other parties will locate.

One possibility is for all parties to locate at the electoral mean. Assuming this to be the case, then parties will differ only in their valence terms, i.e., on the party’s quality or ability to govern. Under this assumption, we can then use (5) and the above coefficients given in (14), to estimate the probability (when all parties are at the mean) that a typical voter outside Québec chooses the GPC:

$$\rho_{GPC}^{C/Q} = \frac{\exp[\lambda_{GPC}]}{\sum_{k=1}^{4} \exp[\lambda_k]} = \frac{e^{-2.23}}{e^{-2.23} + e^{-0.50} + e^{-0.02} + e^0} \simeq 0.041. \quad (15)$$

Thus, the predicted probabilities that a typical voter chooses each party are:

$$\rho^{C/Q} = (\rho_{CP}, \rho_{LP}, \rho_{NDP}, \rho_{GPC})^{C/Q} = (0.369, 0.375, 0.215, 0.041) \quad (16)$$

These estimates are close to the C/Q sample vote shares, given in Table 4 as

$$(s_{CP}, s_{LP}, s_{NDP}, s_{GPC})^{C/Q} = (0.378, 0.369, 0.213, 0.040) \quad (17)$$

and to the actual vote shares (between the four parties) given in Table 3, outside Québec

$$(v_{CP}, v_{LP}, v_{NDP}, v_{GPC})^{C/Q} = (0.373, 0.382, 0.197, 0.048)$$

We now show that for Canada outside Québec parties do not find it in their best interest to locate at the electoral mean. To do so, we apply the Valence theorem to determine whether the parties outside Québec converge to, or diverge from, the electoral mean. We can compute the convergence coefficient using (9) where $\nabla_0^{C/Q}$ is the electoral covariance matrix given in (2). Using (14) and (15) we can compute $2\beta^{C/Q}(1 - 2\rho_{GPC}^{C/Q}) = 2 \times 0.27 \times 0.92 = 0.50$ and from (2) we know that the trace($\nabla_0^{C/Q}$) = 3.88 so that the convergence coefficient is

$$c^{C/Q} \equiv 2\beta^{C/Q}(1 - 2\rho_{GPC}^{C/Q})\text{trace}(\nabla_0^{C/Q}) = 0.5 \times 3.88 = 1.94. \quad (18)$$

In Appendix B we show that this estimate of the convergence coefficient is significantly higher than 1 but not significantly different from 2. Consequently, to use the Valence theorem we examine whether the the party with the smallest valence, the GCP, has any incentive to locate at the origin.

The Hessian, or characteristic matrix of the vote share function of the GPC is given by

$$C_{GPC}^{C/Q} = 2\beta^{C/Q}(1 - 2\rho_{GPC}^{C/Q})\nabla_0^{C/Q} - \mathbf{I} \quad (19)$$

where $\nabla_0^{C/Q}$ is the electoral covariance matrix given in (2). Thus

$$C_{GPC}^{C/Q} = (0.50) \begin{bmatrix} 2.70 & 0.12 \\ 0.12 & 1.18 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} 0.35 & 0.06 \\ 0.06 & -0.41 \end{bmatrix} \quad (20)$$
We infer that we have a saddlepoint: the eigenvector with the positive eigenvalue (+0.35) is (1.00, 0.08), while the negative eigenvalue (−0.41) has eigenvector (−0.08, 1.00). Note however that the characteristic matrix \( C_{G/Q} \) is estimated from the coefficients given by a stochastic model. In Appendix B we also used Taylor’s Theorem to estimate the 95% probability bounds on the various parameters. We infer that, with probability exceeding 95%, the electoral mean cannot be an LNE for the pure spatial model.

Note also that the valence of the Conservatives given in (14) is not significantly different from zero (see Table 5), and thus not significantly different from that of the Liberals, the base party in the MNL estimation. However, since both the GPC and the NDP have significantly negative valences, the Liberals and Conservatives have significantly higher valences than the two low valence parties. By the Valence theorem these two large parties need not move from their partisan constituency positions. These are near the electoral origin and changing position would lower rather than increase their vote shares.

According to the theory, for the pure spatial model for the election outside Québec, we infer that some parties should position themselves away from the electoral origin.

Since the joint origin cannot be an LNE, it cannot be a stable LNE. Because the variances on the two axes were different we also analysed the models where the \( \beta \)-coefficients differed on the two axes (see Table A6 in Appendix A). We came to the same conclusion that the joint origin could not be an LNE for the pure spatial model. Below we compute the LNE for the spatial model. Before that however, we examine the model for Québec.

### 4.3 Québec

Table 6 presents the results of the pure spatial model for in Québec, again using the Liberals as the base party. The coefficients for the model in Québec are

\[
\begin{align*}
\lambda_{BQ}^Q &= 0.63, \quad \lambda_{NDP}^Q = -1.25, \quad \lambda_{CP}^Q = -0.34, \quad \lambda_{GPC}^Q = -2.33, \quad (21) \\
\lambda_{LPC}^Q &= 0, \quad \beta^Q = 0.23.
\end{align*}
\]

The \( \beta \)-coefficient and the valence estimates for the BQ, NDP, and GPC are significantly non zero. The valence of the Conservative Party is not significantly different from that of the Liberals. The significance levels of these valences imply that the Bloc has a greater valence than all the other parties and that both the GPC and the NDP have the lowest valences.

Using the coefficients in (21) the predicted probability, \( \rho_{GPC}^Q \), that a typical Québec voter chooses the lowest valence party (the GPC), when all parties locate at the joint mean, is

\[
\rho_{GPC}^Q = \frac{\exp[\lambda_{GPC}^Q]}{\sum_{k=1}^{5} \exp[\lambda_k^Q]} = \frac{e^{-2.33}}{e^{-2.33} + e^{-1.25} + e^{-0.34} + e^{0.63} + e^{0}} \simeq 0.025
\]
Similarly, the predicted probabilities that a typical Québécois chooses the five parties (when all parties locate at the joint mean) are

$$\mathbf{\rho}^{Q} = (\mathbf{\rho}_{CP}, \mathbf{\rho}_{LP}, \mathbf{\rho}_{NDP}, \mathbf{\rho}_{GPC}, \mathbf{\rho}_{BQ})^{Q} = (0.178, 0.252, 0.072, 0.025, 0.473).$$

This compares to the sample vote shares, in Table 4:

$$\mathbf{s}_{CP}, \mathbf{s}_{LP}, \mathbf{s}_{NDP}, \mathbf{s}_{GPC}, \mathbf{s}_{BQ})^{Q} = (0.091, 0.251, 0.080, 0.027, 0.551)$$

and to the actual vote shares, in Table 3, in Québec:

$$\mathbf{v}_{CP}, \mathbf{v}_{LP}, \mathbf{v}_{NDP}, \mathbf{v}_{GPC}, \mathbf{v}_{BQ})^{Q} = (0.088, 0.339, 0.046, 0.032, 0.489).$$

We can use (9) to compute the convergence coefficient for Québec. Since the probability that a generic voter votes for the GPC (if all parties locate at the origin) is $\mathbf{\rho}_{GPC} = 0.025$ in (22),

$$2\beta^{Q}(1 - 2\mathbf{\rho}_{GPC}) = 2 \times 0.23 \times 0.95 = 0.44$$

and the trace($\nabla_{Q}^{2}$) = 2.46, we find

$$c^{Q} = 2\beta^{Q}(1 - 2\mathbf{\rho}_{GPC})\text{trace}(\nabla_{Q}^{2}) = 0.44 \times 2.46 = 1.08$$

The convergence coefficient therefore does not indicate whether the joint origin is an LNE.

Whether the GPC have an incentive to move is determined by GPC’s Hessian. We find the GPC Hessian to be

$$C_{GPC}^{Q} = (0.44) \begin{bmatrix} 1.48 & -0.57 \\ -0.57 & 0.98 \end{bmatrix} - I = \begin{bmatrix} -0.35 & -0.25 \\ -0.25 & -0.57 \end{bmatrix}.$$

The trace of $C_{GPC}^{Q}$ (= −0.92) is negative and the determinant (= 0.137) is positive so both eigenvectors have negative eigenvalues. We therefore conclude that at the electoral mean the GPC vote share function has a local maximum. Simulation verified that the equilibrium was one with all parties at the electoral mean, (−1.11, −0.08) (see Table A3).

The next question we address is whether the joint electoral mean is stable.

The estimates of $(\mathbf{\rho}_{GPC}, \mathbf{\rho}_{NDP})$ = (0.025, 0.072) are below the sample vote shares of (0.027, 0.080). We find that, according to the pure spatial model, neither the GPC nor the NDP in Québec have a strong incentive to move from their constituency positions in Québec towards the electoral mean in order to increase vote share.

Appendix B shows that the lower 95% bounds on $\mathbf{\rho}_{GPC}$ and $\mathbf{\rho}_{NDP}$ are 0.01 and 0.04. Using these low estimates we find that there is some small probability that the joint origin fails to be an LNE. The model with separate $\beta$-coefficients led to the same conclusion.

We now return to the nature of the LNE outside Québec

### 4.4 The LNE positions of the spatial models outside Québec

We use the coefficients of the pure spatial model to estimate the local Nash equilibrium (LNE). To find this LNE, we simulated the corresponding model by
estimating each party’s best response to the positions given in Figures 1 and 2 and the party’s anticipated electoral outcome as given by the coefficients of the pure spatial model given in (14) and in Table 5 (column S). Reiterating this procedure many times, and taking the best response in turn of each party until no party can increase vote share further, we obtain the LNE of the pure spatial model. We label this vector $z_S$.

For Canada without Québec the simulated positions are given by

\[
\begin{array}{cccccc}
\text{Party} & \text{NDP} & \text{GPC} & \text{LPC} & \text{CP} \\
\text{Social} & 0.57 & -1.14 & 0.30 & 0.30 \\
\text{Decent} & 0.04 & -0.03 & 0.02 & 0.02 \\
\end{array}
\] (25)

However, since this the electoral mean, outside Québec, was $z_C^{C/Q} = (0.31, 0.02)$ (see Table A3), renormalizing $z_C^{C/Q}$ with respect to the sample mean we obtain

\[
\begin{array}{cccccc}
\text{Party} & \text{NDP} & \text{GPC} & \text{LPC} & \text{CP} \\
\text{Social} & 0.26 & -1.45 & -0.01 & -0.01 \\
\text{Decent} & 0.02 & -0.05 & 0.00 & 0.00 \\
\end{array}
\] (26)

These simulated LNE positions are shown in Figure 3.

\[\text{[Insert Figure 3 here]}\]

The vote shares of these four parties at the simulated equilibrium $z_C^{C/Q}$ were determined to be

\[(\rho_{CP}, \rho_{LPC}, \rho_{NDP}, \rho_{GPC})^{C/Q} = (0.365, 0.374, 0.216, 0.044).\] (27)

Using the central estimate of $\rho_{GPC}^{C/Q} = 0.0404$ for GPC in (16) we find the GPC could increase its vote share by $0.044 - 0.0404 = 0.0036$. As the theory suggests, by moving away from the electoral mean, $z_C^{C/Q}$, the GPC could potentially increase its vote share from 4.04% to 4.4%, generating a trivial increase in votes. This further indicates that the joint mean cannot be an equilibrium.

The theoretical analysis above has shown that the GPC Hessian at the joint electoral mean has an eigenvector $(1.45, -0.05)$ associated with a positive eigenvalue. The valence theorem then predicts that the normalized position of the GPC should be approximately aligned with this vector. In order to maximize its vote share the Greens should move along this vector. Since the normalized position of the GPC in equilibrium is $(-1.45, -0.05)$ in (26) we find this to be the case. Notice also that the GPC LNE position in $z_S^{C/Q}$ is $(-1.14, -0.03)$ which is closer to the constituency position $(-0.63, -0.13)$ than is the electoral mean $(0.31, 0.02)$. However, the other low valence party, the NDP, should locate away from the electoral mean at an LNE position $(0.57, 0.04)$. This is further from the constituency position $(-0.78, 0.05)$ than is the electoral mean.

The next question we address is whether this LNE is stable. A move by the GPC involves some electoral risk because of the assumption that voters’
choices are stochastic. In other words, the GPC runs the risk that its vote share may actually decrease rather than increase when it moves to the LNE. To find whether the move is worth the risk, we take the low vote share to be the lower 95% bound, \( \rho_{\text{low},j} \), of a party \( j \) of the predicted vote share at the LNE and then define the \textit{low vote margin} of party \( j \) to be the difference between the low vote share and the sample vote share, that is,

\[
\text{vm}_{\text{low},j} = \rho_{\text{low},j} - s_j
\]

where \( s_j \) is the \textit{sample} vote share.

Using Taylor’s Theorem we estimated the 95% bounds on the vote share of the GPC to be \( 0.044[1 \pm 0.4] = [0.026, 0.062] \) at this LNE. The lower vote share is below the sample vote share of 0.040 in (17). Thus a move by the GPC to the equilibrium does not satisfy our condition for the stability of the LNE.

Note also that according to the theory, the local equilibrium positions for the spatial model of the two high valence parties, CP and LPC, should be close to the electoral origin, and \( z_{C/Q}^{G/Q} \) shows this to be the case. We came to similar conclusions for the spatial model with separate \( \beta \)-coefficients.

### 4.4.1 Sociodemographic characteristics

It is also possible that sociodemographic characteristics may influence voters’ choices. We used the sociodemographic characteristics available in the survey: age, gender and education. The descriptive statistics of these variable by region and by party are given in Tables A3 and A4 in Appendix A.

We estimated pure socio-demographic (SD) as well as spatial sociodemographic (SSD) models for both outside Québec and in Québec (see Tables 5 and 6 respectively). Just as in the SD model, in both regions, the SSD model shows that those supporting the NDP and the GP are younger than those supporting the Liberals and the Conservatives. In addition, outside Québec CP voters tend to have less education than those supporting the other parties, while the reverse is true in Québec. It seems that NDP supporters in Québec tend to be younger than those voting for other parties.

Notice that the \( \beta \)-coefficient is similar in the pure spatial models (S) and and in the SSD models for both regions Age and education coefficients are of similar sign and magnitude in the SD and SSD models. This supports our assumption that the sociodemographic variables are independent of voters’ and parties’ positions, thus showing the importance of accounting for voter heterogeneity.

The log-likelihood tests given in Tables 7 and 8 suggest that the SSD model performs better than the S or SSD models in both regions, again showing the importance of the sociodemographic characteristics in voters’ choices.

[Insert Table 7 and 8 here]

We used the coefficients of the spatial sociodemographic model to simulate the LNE of the SSD model using the corresponding coefficients of the model and obtained the LNE, labelled \( z_{SSD} \).
For outside Québec, the simulation gave the following party positions

\[
\mathbf{z}_{C/Q}^{C/Q} = \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP \\
\text{Social} & 0.50 & -1.22 & 0.27 & 0.34 \\
\text{Decent} & 0.07 & 0.10 & 0.00 & 0.02 
\end{bmatrix}.
\]  

(29)

Normalizing with respect to the electoral mean \(z_0 = (0.31, 0.02)\) gives:

\[
\mathbf{z}_{oSSD}^{C/Q} = \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP \\
\text{Social} & 0.29 & -1.51 & -0.04 & 0.03 \\
\text{Decent} & 0.05 & -0.08 & 0.00 & 0.00 
\end{bmatrix}
\]

These equilibrium positions are very similar to those obtained by the pure spatial model (compare with 26).

Using the coefficients from Table 5 (SSD) we estimated the predicted probabilities for the spatial sociodemographic model in C/Q at \(z_{SSD}^{C/Q}\) to be

\[
(\rho_{CP}, \rho_{LPC}, \rho_{NDP}, \rho_{GPC})_{SSD}^{C/Q} = (0.40, 0.35, 0.22, 0.03).
\]

(30)

Notice that the estimated vote share of the GPC at \(z_{SSD}^{C/Q}\) is 0.03 which is below its sample vote share of 0.040, so it has no incentive to move to this LNE.

We find once more that the LNE of the SSD model cannot be a stable attractor.

Outside Québec, the SSD model predicts that the Liberals, Conservatives and NDP should position themselves so as to advocate fewer social programs and no change in decentralization. The GPC should position itself so as to promote more social programs than the other three parties. In the next section we conjecture that the reason parties do not move to these positions is because of the pull exerted by their core constituencies.

For Québec the LNE of the SSD model is only slightly perturbed from the joint mean \((-1.11, -0.08)\):

\[
\mathbf{z}_SSD^Q = \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP & BQ \\
\text{Social} & -1.31 & -1.99 & -1.05 & -1.23 & -1.13 \\
\text{Decent} & 0.03 & 0.48 & -0.12 & 0.03 & -0.08 
\end{bmatrix}.
\]

(31)

At the LNE of the SSD model in Québec the party’s vote shares were estimated to be

\[
(\rho_{CP}, \rho_{LPC}, \rho_{NDP}, \rho_{GPC}, \rho_{BQ})_{SSD}^Q = (0.20, 0.37, 0.05, 0.01, 0.37).
\]

The estimate for the GPC vote share of 0.01 is below that of its sample vote share, 0.027, in the region. Consequently, the estimated vote margins again suggest that the GPC has no incentive to move to the equilibrium position. Thus the LNE for the SSD model in Québec is not a stable attractor.
4.5 Activist and supporters influence

The analysis done in the previous sections allowed us to conclude that low valence parties that are risk-averse have no incentive to move from their partisan constituency position to the LNE position predicted by the pure spatial or the spatial sociodemographic models. We conjecture that the preference for what have termed partisan constituency positions can be explained by the pull exerted by the party supporters and party activists.

We use the difference between the LNE positions in the SSD model and the partisan constituency positions to estimate the centrifugal tendency pulling the parties away from the LNE. Taking the LNE and partisan constituency positions for BQ to be the one in Québec, we can compare the estimated positions with the LNE positions to get a measure of the pull activists exert on party’s positions.

The partisan constituency positions given in (1) and replicated here are:

\[
z^* = \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP & BQ \\
\text{Social} & -0.78 & -0.63 & -0.17 & 1.27 & -1.48 \\
\text{Decent} & 0.05 & -0.13 & -0.38 & 0.32 & 0.23 \\
\end{bmatrix}.
\]

From (31) we know that the party’s positions in the SSD model are

\[
z_{SSD}^C = \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP & BQ \\
\text{Social} & 0.50 & -1.22 & 0.27 & 0.34 & -1.13 \\
\text{Decent} & 0.07 & 0.10 & 0.00 & 0.02 & -0.08 \\
\end{bmatrix}.
\]

The difference is

\[
[z^* - z_{SSD}^C] \approx \begin{bmatrix}
\text{Party} & NDP & GPC & LPC & CP & BQ \\
\text{Social} & -1.28 & 0.59 & -0.44 & 0.93 & -0.35 \\
\text{Decent} & -0.02 & -0.23 & -0.38 & 0.30 & 0.31 \\
\end{bmatrix}.
\]

The magnitudes in \(z^* - z_{SSD}^C\) indicate the directions in which parties are pulled away from the LNE positions in the SSD model towards those favored by the party supporters and activists. The LNE would put the LPC, CP and BQ at fairly neutral positions on the decentralization axis, with the LPC and CP close to the electoral mean on the social axis, balancing the BQ on the left. Obviously these positions do not appeal to the party supporters and activists. As a consequence the BQ and the CP are pulled towards more decentralization. This is to be expected of the Bloc, a separatist party which in the election won close to 50% of the vote in Québec (see Table 2). This electoral support gave the BQ the mandate to fight for more decentralization for Québec. Moreover, the main base of support of the CP is Alberta where it gains over 60% of the vote (see Table 2) and where voters want more control over their natural resources. The BQ and CP are pulled in opposite directions on the social axis, as the CP is traditionally a conservative party on economic issues.

It is also clear why the LNE of the SSD model is unattractive for the supporters of the NDP and GPC. The logic of vote maximization as modelled by the LNE would put both of them at a fairly neutral position on the decentralization axis, and opposed on the social axis. The average supporter of these two parties is, however, on the left of the electoral mean on this social axis.
4.6 After 2004

In 2004, economic growth was strong, with consumer spending growing at 4.8% and exports at 6.3%, so economic differences were not profound. Nonetheless, scandals over corruption and sponsorship forced an election on January 23, 2006. The Conservative Party won a plurality of seats (40.5%) or 124 out of 308, with 36.3% of the votes (see Table 1). The Conservative leader Stephen Harper became the 22nd Prime Minister of Canada, leading a minority government with the (informal) support of the Bloc Québécois. Since this support proved quite unpopular among BQ activists, the Bloc began to oppose the Conservatives on issues such as the environmental and the military role in Afghanistan.

Stéphane Dion had become leader of the Liberals before the election, after a close fought leadership race. Unwilling to force the country to a new election, he also provided support to the Conservatives in the House of Commons. However, in the election of October 2008, the Conservatives increased the seats they controlled to 143 (46% of the total) with a slight increase of the vote share to 37.6%, while the Liberals dropped to 77 seats from 103. This led to a minority Conservative government, and shortly after, to the resignation of Dion. On December 10, Michael Ignatieff was formally declared the interim leader in a caucus meeting, and his position was ratified at the party’s May 2009 convention.

The election of 2008 brought about the third minority government in Canada. The election outcomes of 2004, 2006 and 2008 had all created different coalition possibilities. These changes in coalitions can be illustrated using the notion of the heart. The heart is the star shaped figure bounded by the median lines in the policy space generated by the party positions and seat strengths. These medians can be associated with various possible winning coalitions, and Schofield (1999) has suggested that coalition outcomes will lie within the heart.

For example, Figure 4 shows the heart after the 2004 election, using the estimates of partisan constituency positions. As the Figure indicates, the LPC needed the support of the Bloc to secure a majority. The CP together with the NDP and BQ constituted a majority. The LPC and NDP minority government seemed a reasonable compromise because of the proximity of the two parties.

In 2008, as Figure 5 indicates, the increase in the number of seats controlled by the CP meant that it could form a government with the support of the Bloc. In the 2008 election the CP with 143 seats lacked a majority.

The government of Stephen Harper was dissolved on March 26, 2011, and in the surprising election on May 2, the Conservatives won a majority of 167 seats out of 308 with 39.6% of the vote (see Table 1). The Bloc was almost wiped out, winning only 4 seats with 6% of the vote (a drop from 10%). The Liberals

\[ \text{insert figures 4 and 5 here} \]

The government of Stephen Harper was dissolved on March 26, 2011, and in the surprising election on May 2, the Conservatives won a majority of 167 seats out of 308 with 39.6% of the vote (see Table 1). The Bloc was almost wiped out, winning only 4 seats with 6% of the vote (a drop from 10%). The Liberals

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dropped from 77 seats to 34, while its vote share fell from 26% to 19%. The NDP jumped from 37 seats to 102, increasing its vote share from 12% to 30.6%. The GPC vote share also fell from 6.8% to 3.9%, but it took a single seat.

5 Convergence in different polities

Our analysis of the 2004 Canadian election allows us to compare our results with those obtained for other polities and to derive broader conclusions on the classification of various political systems. In addition, we can complement the classification proposed by the convergence coefficient and the Hessian of the low valence parties of various countries for various elections with an analysis of the degree of fractionalization experienced in these countries and their ability to form stable governing coalitions.

Empirical analysis show that the convergence coefficients were 6.82 for the 1997 election in Poland (Schofield et al. 2011f), 5.94 for the 2002 election in Turkey (Schofield et al. 2011d) and 3.98 for the 1996 election in Israel (Schofield et al. 2011b) (see Table 9). These three polities all have highly fragmented party systems. According to the Valence Theorem given in Section (4.1), parties should diverge from the electoral mean in these polities, and position themselves closer to their supporting electoral base. Simulation of the models, including the sociodemographic valences, gives a reasonable estimate of party position at these elections (see Table 9).

A standard way of estimating political fragmentation is in terms of the effective number of party vote strength ($env$) or effective number of party seat strength ($ens$) (see footnote 13). We can see from Table 9 that for these three polities with high convergence coefficients, both $env$ and $ens$ were also very high, and this is why we call them fragmented. For example, in Poland under a very proportional electoral system, the $env$ increased from about 5.5 in 1997 to 7.7 in 2005, while the the $ens$ increased from 3.1 to 5.0. To illustrate the degree of fragmentation, Figure 6 shows the heart for Poland in 1997. Obviously the heart in Poland reflects the complicated coalition possibilities in a polity with a high number of parties (and a very high convergence coefficient of 6.82).

We suggest that a way of interpreting the arguments of Duverger (1954) and Riker (1953) on the effects of proportional electoral methods is that under such an electoral rule, there is a strong centrifugal tendency pulling all parties away from the electoral mean towards their core constituency. This tendency will be particularly strong for small, or low valence parties. In particular, even small parties in such a polity can assign a non-negligible probability to becoming a member of a coalition government, and it is this phenomenon that maintains the fragmentation of the party system. In Poland, no party can obtain a majority,
and parties and coalitions form and dissolve rapidly, so that the convergence coefficients have been above 6.0 in the recent elections of 1997, 2001 and 2005. To illustrate this argument about fragmentation, small parties in Poland like the Polish Peoples Party (PSL), with about 6% of the seats or the Freedom Union (UW) with 13% of the seats, are located on the boundary of the 1997 Polish heart, and they both have every reason to expect to be invited into government.

In Israel, the small religious parties have gained ground over the larger parties, Labor, Likud and Kadima, while in Turkey, there are at least six small parties, and though the electoral system has a minimum cut-off, small parties continue to gain enough votes to keep the convergence coefficient high.

For Canada without Québec, we have computed the convergence coefficient to be 1.94 in 2004, with 95% bounds of [1.51, 2.25]. The central value of for Canada is much lower than the values for the three polities just mentioned, and implies that the centrifugal tendency on small parties such as the NDP or Greens is relatively weak. The Canadian electoral system benefited the high valence parties, such as the Conservative and Liberal Parties in the 2004 election, over smaller parties such as New Democratic Party and Green Party. On the other hand, the pure spatial model indicates that in 2004 the Bloc Québécois has very high valence in Québec, thus allowing it to obtain a significant share of the seats in that Province in the 2004 election, gaining a much higher share of the seats than its vote share warrants. Between the elections of 2004 and 2008, the for all of Canada increased from 4.0 to 4.1, while the increased from about 3.1 in 2004 to 3.4 in 2006 and 3.5 in 2008. These figures suggest that Canada’s fractionalization increased, but only slightly, between these two elections, due in part to the different economic and social issues affecting Québec and Alberta.

Table 9 also shows the values of the convergence coefficients for the partial democracies or ‘anocracies,” Russia, Georgia and Azerbaijan. For Russia, Schofield and Zakharov (2010) obtain a value of 1.7 for the Duma election in 2007. There is a dominant pro-Kremlin party, United Russia, with 64% of the vote and 70% of the seats, giving low and . Schofield et al. (2011e, show that elections in the two presidential polities of Georgia and Azerbaijan have coefficients between 2.4 and 2.9. These values of the convergence coefficients in these three ”anocracies” are below those of the three ”fragmented” polities, mentioned above, because of the electoral dominance of the presidential parties, and the difficulty that small parties face in securing votes as a result.

Finally, consider the results on majoritarian polities using a first past the post electoral rule. Schofield et al. (2011a,b) show that in the United States the convergence coefficients for the last three U.S. presidential elections lay in the range [0.45, 1.0]. For presidential elections in the U.S. the is 2.0 and the can be regarded as 1.0 (for the presidential election) or close to 2.0 (for Congress). In related empirical work, Schofield et al. (2011c) found the convergence coefficient in the United Kingdom increased from 0.84 for the 2005

The term "partial democracy" or "anocracy" has been applied to new democracies without the full array of democratic institutions. See Epstein et al. (2006).
election to 0.98 for the 2010 election (both lower than that of Canada). The env for 2005 election was 2.7, while the ens was about 2.5, but the hung Parliament after the election of 2010 meant that the env increased to 3.8 while the ens also increased to 3.3. By our measure the electoral system in the United Kingdom has become more fragmented.

By our interpretation of the nature of the convergence coefficient, the convergence effect in presidential elections in the United States is stronger than in Parliamentary elections in Great Britain and Canada. These analysis also presented evidence that Presidential candidates tended not to converge to the electoral mean in the United States. Instead it was suggested that activists exerted a very powerful influence on US party positioning (see also Miller and Schofield, 2008; Schofield and Miller, 2007).

Table 9 suggests that there is a degree of correlation between the convergence coefficient in various polities and the fragmentation of the political configuration. Later work will attempt to combine empirical and theoretical models in an attempt to better understand the balance between electoral and activist effects under different electoral systems.

6 Conclusion

In this paper we used the Valence Theorem together with a logit model of the 2004 Canadian election. We found that parties did not locate themselves at vote maximizing positions as suggested by Downsian models of elections. Instead, there was evidence to support the hypothesis that in this election all parties positioned themselves at their partisan constituency positions.

We also discussed how the Valence theorem can be applied to various polities and offers a way to classify political configurations. We relate the notion of the convergence coefficient to the effective number of parties according to both vote and seat shares. In addition we introduced the theoretical idea of the political Heart as a way of delineating the the intricate bargaining situations that may occur when many parties are involved in the bargaining process.

This argument suggests that inferences made by Riker (1980) on the degree of instability depends on the context of the differing levels of conflict between electoral incentives and the influence of political groups in polities with different electoral systems. The relationship between the estimated convergence coefficients, the degree of political fragmentation, and the nature of the centrifugal tendency in different polities may go some way to answer the questions recently raised by Dow (2011) and Ezrow (2008, 2011) about the fundamental characteristics of electoral systems.

Acknowledgement

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Versions of this paper have been presented at the Public Choice Meeting, San Antonio March 11-13, 2011 at the Research Meeting, Bilgi University, Istanbul, Turkey., May 13-14, 2011, at the Workshop in Social Choice, Udine, Italy June 1-3, 2011, and the Priorat Workshop in Theoretical Political Science, Spain, June 16-18.

7 References


States and Israel. Soc Ch Welf 36, 483-518.
8 Appendix A

<table>
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<td><strong>Gun only police/military</strong></td>
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<td><strong>Iraq War</strong></td>
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<td><strong>Left-Right</strong></td>
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Table A3: Descriptive Statistics by Region

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Canada outside Québec (n = 675)

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Québec (n = 187)

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Table A5: Descriptive Statistics for the Bloc Québécois (n = 103)

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Table A6: 2004 Canada Separate dimension MNL Models, baseline LPC

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<td>[t-value]</td>
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<td>β Dec</td>
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* prob<0.05; ** prob<0.01; *** prob<0.001
9 Appendix B

In this Appendix we derive the confidence intervals in order to show whether the LNE is or not a stable attractor for Canada without Québec and for Québec.

9.1 Outside Québec

The convergence coefficient for Canada without Québec is given in (18)
\[ c_{C/Q} = 2b_{C/Q}(1 - 2\rho_{GPC})\text{trace}(\nabla_0^{C/Q}) = 1.94. \]

To construct a 95% confidence interval for \( c_{C/Q} \) we need the 95% confidence intervals for \( \beta_{C/Q} \) given in (14) and \( \rho_{GPC} \) in (15). Given that the standard error on \( \beta_{C/Q} \) is 0.02, we know that the 95% bounds of \( \beta_{C/Q} \) are given by 0.27 ± 0.04 = [0.23, 0.31]. Since the standard error in \( \lambda_{GPC}^{C/Q} \) is 0.20, the 95% confidence interval for \( \lambda_{GPC}^{C/Q} \) is \(-2.23 \pm 0.4 = [-2.63, -1.83] \).

Taylor’s theorem gives the 95% bounds on \( \rho_{GPC}^{C/Q} \) to be 0.041[1 ± 0.4] = [0.025, 0.057]. We can then estimate very conservative 95% bounds on \( c_{C/Q} \) as given by

\[ \{2 \times 0.23 \times (1 - (2 \times 0.057)), 2 \times 0.31 \times (1 - (2 \times 0.025))\} \times 3.88 \]
\[ = [0.41, 0.59] \times 3.88 = [1.59, 2.29] \]

The central estimate of \( c_{C/Q} = 1.94 \) with 95% bounds of [1.94 ± 0.35] = [1.59, 2.29]. Given that the lower bound of \( c_{C/Q} \) is 1.59, we conclude that, with a high degree of certainty, the convergence coefficient for C/Q is above 1. Moreover, the 95% confidence interval on \( c_{C/Q} \) does not show that the convergence coefficient is below 2. Thus, we cannot conclude immediately from the Valence Theorem whether the parties with the smallest valences should diverge away from the electoral origin to increase vote share.

To do so we need to examine whether the GPC has an incentive to move from the LNE position. From (19) we have
\[ C_{GPC}^{C/Q} = 2\beta_{C/Q}(1 - 2\rho_{GPC})\nabla_0^{C/Q} - I. \] (32)

We compute the the 95% confidence interval of \( C_{GPC}^{C/Q} \) to be:

\[ [0.41, 0.59] \begin{bmatrix} 2.70 & 0.12 \\ 0.12 & 1.18 \end{bmatrix} - I, \]

\[ = \begin{bmatrix} 1.10 & 0.02 \\ 0.05 & 0.48 \end{bmatrix} - I, \begin{bmatrix} 1.59 & 0.07 \\ 0.07 & 0.70 \end{bmatrix} - I \]

\[ = \begin{bmatrix} 0.10 & 0.02 \\ 0.02 & -0.52 \end{bmatrix}, \begin{bmatrix} 0.59 & 0.07 \\ 0.07 & -0.30 \end{bmatrix}. \]
Since both low and high estimates of $C_{GPC}^{C/Q}$ have one positive and one negative eigenvalue, the estimates in both cases give a saddlepoint. We conclude then that the GPC has an incentive to move away from the electoral mean. As a consequence, we infer that the probability exceeds 95% that the electoral origin cannot be an LNE.

9.2 In Québec.

In Québec,

$$\rho_{GPC}^Q = \frac{\exp[\lambda_{GPC}^Q]}{\sum_{k=1}^5 \exp[\lambda_j^Q]} = \frac{e^{-2.33}}{e^{-2.33} + e^{-1.25} + e^{-0.34} + e^{0.63} + e^0} \simeq 0.025.$$ \[ \beta^Q = 0.23 \]

Then $2\beta^Q(1 - 2\rho_{GPC}^Q) = 2(0.23)(0.95) = 0.44$

$$C_{GPC}^Q = (0.44) \begin{bmatrix} 1.48 & -0.57 \\ -0.57 & 0.98 \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.35 & -0.25 \\ -0.25 & -0.57 \end{bmatrix}$$

The convergence coefficient for Québec is

$$c^Q = 2\beta^Q(1 - 2\rho_{GPC}^Q)\text{trace}(\nabla^Q_0) = 2(0.95)(0.56) = 1.08.$$ In this case the trace is negative and the determinant is positive, so both eigenvectors have negative eigenvalues, and we have a local maximum.

Given that the standard error on $\beta^Q$ is 0.04, we know that the 95% bounds of $\beta^Q$ are given by $0.23 \pm 0.08 = [0.15, 0.31]$. Since the standard error in $\lambda_{GPC}^Q$ is 0.47, the 95% bounds on $\lambda_{GPC}^Q$ given in (15) are $-2.33 \pm 0.94 = [-3.27, -1.39]$ implying that the 95% bounds on $\rho_{GPC}^Q$ are $[0.01, 0.06]$. We can then estimate very conservative 95% bounds on $c^Q$ as given by

$$\{2 \times 0.15 \times (1 - (2 \times 0.06)), 2 \times 0.31 \times (1 - (2 \times 0.01))\} \times 2.46$$

$$\{2 \times 0.15 \times 0.88, 2 \times 0.31 \times 0.98\} \times 2.46$$

$$= [0.26, 0.61] \times 2.46 = [0.64, 1.49]$$

Thus the central estimate of $c^Q = 1.08$ with 95% bounds $[1.08 - 0.44, 1.08 + 0.41]$. The 95% confidence interval on $c^Q$ does show that the convergence coefficient
is below 2. We can also estimate very conservative 95% bounds on $C^Q_{GPC}$, to be

$$\begin{bmatrix} 0.26, 0.61 \end{bmatrix} | \nabla^Q_0 - I $$

$$= \begin{bmatrix} 1.48 & -0.57 \\ -0.57 & 0.98 \end{bmatrix} - I$$

$$= \begin{bmatrix} 0.38 & -0.15 \\ -0.15 & 0.25 \end{bmatrix} - I, \begin{bmatrix} 0.91 & -0.35 \\ -0.35 & 0.60 \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.62 & -0.15 \\ -0.15 & -0.75 \end{bmatrix}, \begin{bmatrix} -0.09 & -0.35 \\ -0.35 & -0.40 \end{bmatrix}$$

The determinant of the first matrix is positive and the trace negative, so both eigenvalues are negative. However, the determinant of the second matrix, corresponding to the lower estimate of $\rho^Q_{GPC} = 0.01$, is negative, so this matrix gives a saddlepoint.

In the same way, the standard error in $\lambda^Q_{NPD}$ is 0.24, the 95% bounds on $\lambda^Q_{NPD}$ are $-1.24 \pm 0.48 = [-1.70, -0.76]$ implying that the 95% bounds on $\rho^Q_{NDP}$ are $0.072 \pm 0.045 = 0.072 \pm 0.032 = [0.04, 0.10]$. Very conservative 95% bounds on $C^Q_{NPD}$ are

$$\{2 \times 0.15 \times (1 - (2 \times 0.10)), 2 \times 0.31 \times (1 - (2 \times 0.04))\} | \nabla^Q_0 - I $$

$$= \begin{bmatrix} 0.24, 0.57 \end{bmatrix} \nabla^Q_0 - I $$

$$= \begin{bmatrix} 0.24, 0.57 \end{bmatrix} \begin{bmatrix} 1.48 & -0.57 \\ -0.57 & 0.98 \end{bmatrix} - I$$

$$= \begin{bmatrix} 0.35 & -0.14 \\ -0.14 & 0.24 \end{bmatrix} - I, \begin{bmatrix} 0.84 & -0.32 \\ -0.32 & 0.56 \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.65 & -0.14 \\ -0.14 & -0.76 \end{bmatrix}, \begin{bmatrix} -0.16 & -0.32 \\ -0.32 & -0.44 \end{bmatrix}$$

Again the second matrix gives a saddlepoint, but we can assert that there is only a low probability that the joint origin fails to be a LNE.

### 9.3 Models with separate $\beta$-coefficients.

From Table A6, outside Quebec, we have $\lambda^C_{NDP} = -0.54$, $\lambda^C_{CP} = -0.02$, $\lambda^C_{GPC} = -2.22$, $\lambda^C_{LPC} \equiv 0.0$, $\beta = (\beta_{soc}, \beta_{dec}) = (0.25, 0.48)$. As before, $\rho_{GPC} = 0.041$. 


Then $\beta \nabla_0 \beta$

\[
= \begin{bmatrix}
(0.25)^2(2.70) & (0.25)(0.48)(0.12) \\
(0.25)(0.48)(0.12) & (0.48)^2(1.18)
\end{bmatrix}
= \begin{bmatrix}
0.169 & 0.014 \\
0.014 & 0.272
\end{bmatrix}
\text{with trace 0.441.}
\]

\[C_{GPC}^{C/Q} = 2(1 - 2\rho_{GPC})\beta \nabla_0 \beta - \beta\]

\[
= 2(0.92) \begin{bmatrix}
0.169 & 0.014 \\
0.014 & 0.272
\end{bmatrix} - \begin{bmatrix}
0.25 & 0.0 \\
0.0 & 0.48
\end{bmatrix}
= \begin{bmatrix}
0.06 & 0.02 \\
0.02 & 0.02
\end{bmatrix}
\]

\[c^{C/Q}(\lambda, \beta) = \frac{2(1 - 2\rho_{GPC})\text{trace}(\beta \nabla_0 \beta)}{\frac{1}{2}(0.25 + 0.48)}\]

\[
= \frac{2[(1 - 2(1 - 0.041))(0.441)}}{0.365}
= 2.22 > 2.0
\]

\[d(\lambda, \beta) = 2(1 - 2\rho_1)[\beta_1 \sigma_1^2 + \beta_2 \sigma_2^2]
= 2(0.92)[(0.25)(2.70) + (0.48)(1.18)]
= 2.28 > 1.0.
\]

The determinant and trace are both positive so both eigenvalues are positive, giving a minimum at the electoral mean. One eigenvalue is $+0.068$ with eigenvector $(1, 0.4)$ and the other is $+0.012$ with eigenvector $(1, -2.5)$. For this model, there are two equilibria:

\[
z_{S,1}^{C/Q} = \begin{bmatrix}
\text{Party} & \text{NDP} & \text{GPC} & \text{LPC} & \text{CP} \\
Social & 0.46 & -0.84 & 0.31 & 0.31 \\
Decent & 0.07 & -0.48 & 0.03 & 0.03
\end{bmatrix}
\]

with vote shares

\[
(\rho_{CP}, \rho_{LPC}, \rho_{NDP}, \rho_{GPC})_{S,1}^{C/Q} = (0.367, 0.373, 0.217, 0.043).
\]

and

\[
z_{S,2}^{C/Q} = \begin{bmatrix}
\text{Party} & \text{NDP} & \text{GPC} & \text{LPC} & \text{CP} \\
Social & 0.06 & 2.38 & 0.24 & 0.24 \\
Decent & -0.03 & 0.50 & 0.01 & 0.01
\end{bmatrix}
\]

with vote shares

\[
(\rho_{CP}, \rho_{LPC}, \rho_{NDP}, \rho_{GPC})_{S,2}^{C/Q} = (0.363, 0.369, 0.215, 0.053).
\]

The first LNE is very similar to $z_{S}^{C/Q}$, with LPC and CP close to the electoral mean $z_0^{C/Q} = (0.31, 0.02)$ and NDP and GPC on opposite sides of the
The interval on the trace is we obtain a confidence interval of to a maximum. Furthermore, the 95% confidence interval on $d$ that it is highly probable that the joint mean is an LNE.

To determine confidence intervals for this model, we note that the confidence intervals on $0$ indicating that it is unlikely that the electoral mean is an LNE.

For Quebec, we have $\lambda^Q_{NDP} = -1.51, \lambda^Q_{CP} = -0.24, \lambda^Q_{GPC} = -2.26, \lambda^Q_B = 0.75, \lambda^Q_{LPC} \equiv 0.0, \beta = (0.19, 0.53)$. As before,

\[
\rho^Q_{GPC} = \frac{\exp[\lambda^Q_{GPC}]}{\sum_{k=1}^{\infty} \exp[\lambda^Q_j]} = \frac{e^{-2.26}}{e^{-2.26} + e^{-1.51} + e^{-0.24} + e^{0.75} + e^0} \simeq 0.025.
\]

\[
C^Q_{GPC} = 2(1 - 2\rho^Q_{GPC})\beta\nabla_{\beta} - \beta
\]
\[
= 2(0.95) \begin{bmatrix}
(0.19)^2(1.48) & -(0.19)(0.53)(0.57) \\
-(0.19)(0.53)(0.57) & (0.53)^2(0.98)
\end{bmatrix} - \begin{bmatrix}
0.19 & 0.0 \\
0.0 & 0.53
\end{bmatrix}
\]
\[
= 2(0.95) \begin{bmatrix}
0.053 & -0.057 \\
-0.057 & 0.275
\end{bmatrix} - \begin{bmatrix}
0.19 & 0.0 \\
0.0 & 0.53
\end{bmatrix}
\]
\[
= \begin{bmatrix}
0.0 & 0.57 \\
0.0 & 0.53
\end{bmatrix}
\]

\[
c^Q(\lambda, \beta) = 2(1 - 2\rho^Q_{GPC})\text{trace}(\beta\nabla_{\beta}) = \frac{2(0.95)(0.328)}{0.36} = 1.73.
\]

\[
d^Q(\lambda, \beta) = 2(0.95)(|0.19|(1.48) + (0.53)(0.98)) = 2(0.95)(0.8) = 1.52.
\]

The determinant of $C^Q_{GPC}$ is positive and the trace is negative, corresponding to a maximum. Furthermore, the 95% confidence interval on $c^Q(\lambda, \beta)$ suggests that it is highly probable that the joint mean is an LNE.

The confidence intervals on $\beta_{soc} = [0.19 \pm 0.08]$ and $\beta_{dec} = [0.53 \pm 0.33]$ so the interval on the trace is [0.06, 0.82]. Using the interval of [0.01, 0.06] on $\rho^Q_{GPC}$ we obtain a confidence interval of [0.68, 2.87] on $c^Q(\lambda, \beta)$ and of [0.62, 2.43] on $d^Q(\lambda, \beta)$.

The 95% confidence intervals on $c^Q(\lambda, \beta)$ and $d^Q(\lambda, \beta)$ suggest that it is highly probable that the joint mean is an LNE.
### Table 1: Canadian elections

<table>
<thead>
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<th></th>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
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<td>Party</td>
<td>Vote (%)</td>
<td>Seat ( % )</td>
<td>Vote (%)</td>
<td>Seat ( % )</td>
<td>Vote (%)</td>
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<td>66 (22)</td>
<td>31.2</td>
<td>86 (32)</td>
<td>37.7</td>
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<td>12 (4)</td>
<td>29.6</td>
<td>99 (32)</td>
<td>36.3</td>
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<td></td>
<td></td>
<td>29.6</td>
</tr>
<tr>
<td>LPC</td>
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<td>172 (57)</td>
<td>37.7</td>
<td>124 (40)</td>
<td>37.7</td>
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<td>BQ</td>
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<td>38 (13)</td>
<td>12.4</td>
<td>54 (18)</td>
<td>10.5</td>
</tr>
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<td>NDP</td>
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<td>13 (4)</td>
<td>15.7</td>
<td>19 (6)</td>
<td>17.5</td>
</tr>
<tr>
<td>GPC</td>
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<td></td>
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<td></td>
<td>0.8</td>
</tr>
<tr>
<td>Ind</td>
<td>0.5</td>
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<td>0.5</td>
<td>1 (0.3)</td>
<td>0.5</td>
</tr>
<tr>
<td>Total</td>
<td>98.5</td>
<td>301</td>
<td>99.2</td>
<td>304</td>
<td>99.5</td>
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### Table 2: Provincial Votes (%) and Seats in the 2004 Canadian Election

#### Western Provinces

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<thead>
<tr>
<th>Party</th>
<th>BC</th>
<th>AB</th>
<th>SK</th>
<th>MB</th>
<th>ON</th>
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<tbody>
<tr>
<td>CP</td>
<td>36.3</td>
<td>22</td>
<td>61.7</td>
<td>26</td>
<td>41.8</td>
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<tr>
<td>LPC</td>
<td>28.6</td>
<td>8</td>
<td>22.0</td>
<td>2</td>
<td>27.2</td>
</tr>
<tr>
<td>BQ</td>
<td>26.6</td>
<td>5</td>
<td>9.5</td>
<td>23.4</td>
<td>23.5</td>
</tr>
<tr>
<td>NDP</td>
<td>6.3</td>
<td>1</td>
<td>6.1</td>
<td>2.7</td>
<td>2.7</td>
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<tr>
<td>GPC</td>
<td>6.3</td>
<td>1</td>
<td>6.1</td>
<td>2.7</td>
<td>2.7</td>
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<tr>
<td>Ind</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>1 (0.3)</td>
<td>0.5</td>
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<tr>
<td>Total</td>
<td>98.1</td>
<td>36</td>
<td>99.3</td>
<td>28</td>
<td>99.7</td>
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#### Atlantic Provinces

<table>
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<th>QC</th>
<th>NB</th>
<th>NS</th>
<th>PEI</th>
<th>NL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>8.8</td>
<td>31.1</td>
<td>2</td>
<td>28.0</td>
<td>3</td>
</tr>
<tr>
<td>LPC</td>
<td>33.9</td>
<td>21</td>
<td>44.7</td>
<td>7</td>
<td>39.7</td>
</tr>
<tr>
<td>BQ</td>
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<td>54</td>
<td>48.9</td>
<td>54</td>
<td>48.9</td>
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<tr>
<td>NDP</td>
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<td>20.6</td>
<td>1</td>
<td>28.4</td>
<td>2</td>
</tr>
<tr>
<td>GPC</td>
<td>8.3</td>
<td>7.4</td>
<td>3.3</td>
<td>4.2</td>
<td>1.6</td>
</tr>
<tr>
<td>Ind</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.6</td>
<td>7</td>
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<td>Total</td>
<td>99.4</td>
<td>75</td>
<td>99.9</td>
<td>10</td>
<td>99.5</td>
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---

*AP=Alliance, PC=Progressive Conservative, CP=Conservative, LPC=Liberal, BQ=Bloc Québécois, NDP=New Democratic, GPC=Greens, Ind=independent.

*Other parties are not reported so total may not add to 100%.

---

**Table 3**

<table>
<thead>
<tr>
<th>Region</th>
<th>Provinces</th>
</tr>
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<tbody>
<tr>
<td>Western</td>
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<tr>
<td></td>
<td>BC</td>
</tr>
<tr>
<td></td>
<td>36.3</td>
</tr>
<tr>
<td></td>
<td>28.6</td>
</tr>
<tr>
<td></td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td>6.3</td>
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<tr>
<td></td>
<td>0.5</td>
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<tr>
<td>Atlantic</td>
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<tr>
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<td>QC</td>
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<td>8.8</td>
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<td>33.9</td>
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<td>48.9</td>
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<td>4.6</td>
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<tr>
<td></td>
<td>8.3</td>
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<td></td>
<td>0.1</td>
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<tr>
<td>Total</td>
<td>99.4</td>
</tr>
</tbody>
</table>


*AP=Alliance, CP=Conservatives, LP= Liberals, BQ=Bloc Québécois, NDP=New Democratic, GPC=Greens, Ind=independent.

*Three seats go to the Territories.*
Table 3: Votes and percentages by Region in the 2004 Canadian Election

<table>
<thead>
<tr>
<th>Party</th>
<th>Canada</th>
<th>Canada w/o Québec</th>
<th>Québec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vote</td>
<td>% Vote</td>
<td>Vote</td>
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<tr>
<td>CP</td>
<td>4,013,491</td>
<td>29.66</td>
<td>3,711,952</td>
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<tr>
<td>LPC</td>
<td>4,967,361</td>
<td>36.71</td>
<td>3,801,716</td>
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<tr>
<td>BQ</td>
<td>1,680,109</td>
<td>12.42</td>
<td>1,680,109</td>
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<tr>
<td>NDP</td>
<td>2,117,794</td>
<td>15.65</td>
<td>1,959,367</td>
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<td>GPC</td>
<td>580,845</td>
<td>4.29</td>
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<tr>
<td>All other parties</td>
<td>171,654</td>
<td>1.27</td>
<td>147,779</td>
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<td>Total</td>
<td>13,531,254</td>
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<td>10,092,999</td>
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</table>

a CP= Conservatives, LPC= Liberals, BQ=Bloc Québécois, NDP=New Democratic Party, GPC= Green Party.

Table 4: 2004 Sample Vote Shares

<table>
<thead>
<tr>
<th>Party</th>
<th>Canada</th>
<th>Canada w/o Québec</th>
<th>Québec</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Votes</td>
<td>%</td>
<td>Votes</td>
</tr>
<tr>
<td>CP</td>
<td>272</td>
<td>31.55</td>
<td>255</td>
</tr>
<tr>
<td>LPC</td>
<td>296</td>
<td>34.34</td>
<td>249</td>
</tr>
<tr>
<td>BQ</td>
<td>103</td>
<td>11.95</td>
<td>103</td>
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<tr>
<td>NDP</td>
<td>159</td>
<td>18.45</td>
<td>144</td>
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<tr>
<td>GPC</td>
<td>32</td>
<td>3.71</td>
<td>27</td>
</tr>
<tr>
<td>Total</td>
<td>862</td>
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<td>675</td>
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</table>

a CP= Conservative, LPC= Liberal, BQ=Bloc Québécois, NDP=New Democratic Party, GPC= Green Party.

Table 5: 2004 Canada Outside Québec MNL Models, baseline LPC

<table>
<thead>
<tr>
<th>Party</th>
<th>Var</th>
<th>Spatial (S)</th>
<th>Socio-dem (SD)</th>
<th>Spatial+Socio. (SSD)</th>
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<td>β</td>
<td>Est. a</td>
<td>t-stat</td>
<td>Est. a</td>
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<tr>
<td>CP</td>
<td>-0.02</td>
<td>0.24</td>
<td>1.91***</td>
<td>3.65</td>
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<td></td>
<td>-0.01</td>
<td>0.24</td>
<td>2.04</td>
<td>-0.01</td>
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<td></td>
<td>-0.28</td>
<td>0.24</td>
<td>1.53</td>
<td>0.15</td>
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<td>-0.16***</td>
<td>0.24</td>
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<td>NDP</td>
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<td>5.18</td>
<td>0.90</td>
<td>1.45</td>
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<td></td>
<td>-0.02**</td>
<td>5.18</td>
<td>3.00</td>
<td>-0.02*</td>
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<td>-0.19</td>
<td>5.18</td>
<td>0.91</td>
<td>0.06</td>
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<td>-0.06</td>
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<td>GPC</td>
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<td>10.97</td>
<td>1.11</td>
<td>0.97</td>
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<td></td>
<td>-0.04**</td>
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<td>n</td>
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<td>675</td>
<td>675</td>
<td>675</td>
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<tr>
<td>Log Like LL</td>
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<td>-673</td>
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<td>1559</td>
<td>1385</td>
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<td>BIC</td>
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<td>1590</td>
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a *prob<0.05; **prob<0.01; ***prob<0.001.
Table 6: 2004 Canada only Québec MNL Models, baseline LPC

<table>
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<tr>
<th>Party</th>
<th>Var</th>
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<th>Socio-dem (SD)</th>
<th>Spatial+Socio.(SSD)</th>
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<td>t-stat</td>
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<td>CP</td>
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<td>1.15</td>
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<td>age</td>
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<td>-0.38</td>
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<td></td>
<td>0.15</td>
<td>1.04</td>
<td>0.18</td>
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<td>3.36</td>
<td>2.23*</td>
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<td>-0.02</td>
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<td>educ</td>
<td>0.15</td>
<td>1.04</td>
<td>0.00</td>
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<td>λ_{NDP}</td>
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<td>1.53</td>
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<td>2.51</td>
<td>-0.04*</td>
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<td>educ</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.02</td>
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<td>GPC</td>
<td>λ_{GPC}</td>
<td>-2.33***</td>
<td>4.95</td>
<td>-0.46</td>
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<td>age</td>
<td>-0.03</td>
<td>2.71</td>
<td>-0.03</td>
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<td></td>
<td>gender(f)</td>
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<td>1.57</td>
<td>-1.92</td>
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<td></td>
<td>educ</td>
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<td>0.34</td>
<td>0.07</td>
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<td>n</td>
<td>187</td>
<td>187</td>
<td>187</td>
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<td>Log Like LL</td>
<td>-204</td>
<td>-107</td>
<td>-197</td>
<td></td>
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<tr>
<td>AIC</td>
<td>428</td>
<td>434</td>
<td>419</td>
<td></td>
</tr>
<tr>
<td>BIC</td>
<td>447</td>
<td>460</td>
<td>448</td>
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</table>

*a: prob < 0.05; **: prob < 0.01; ***: prob < 0.001.

Table 7. Comparison of Log Likelihood for Canada outside Québec 2004

| For M₂ Spatial Socio-Demographic Joint |
|--------------------------------------|-----------------|
| M₁ Spatial Socio-Demographic Joint   |
| na                                   | 107             |
| -107                                 | na              |
| 10                                   | 117             |

*a: Joint=spatial model with sociodemographics

Table 8. Comparison of Log Likelihood for Québec 2004

| For M₂ Spatial Socio-Demographic Joint |
|--------------------------------------|-----------------|
| M₁ Spatial Socio-Demographic Joint   |
| na                                   | 11              |
| -11                                  | na              |
| 8                                    | 19              |

*a: Joint=spatial model with sociodemographics
Table 9 Convergence coefficients and Fragmentation

<table>
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<th>Britain</th>
<th>Canada</th>
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</thead>
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<tr>
<td>Conv. Coef.</td>
<td>[0.40,1.1] (2000-08)</td>
<td>[0.84,0.98] (2005-2010)</td>
<td>1.94 (2004)</td>
</tr>
<tr>
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<td>Parl.(^a) PL.(^b)</td>
<td>Parl.(^a) PL.(^b)</td>
</tr>
<tr>
<td>env</td>
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<td>2.7 (2005)</td>
<td>4.0 (2004)</td>
</tr>
<tr>
<td>Political system</td>
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<td>Anoc Pres.(^d) PL.(^b)</td>
<td>Anoc Pres.(^d) PL.(^b)</td>
</tr>
<tr>
<td>env</td>
<td>2.3</td>
<td>2.9 (2008)</td>
<td>2.27</td>
</tr>
<tr>
<td>ens</td>
<td>2.0</td>
<td>1.0 (2008)</td>
<td>1.3</td>
</tr>
<tr>
<td>Political system</td>
<td>Frag.(^e) PR(^b)</td>
<td>Frag.(^e), PR(^b), cut off</td>
<td>Frag.(^e) PR(^b)</td>
</tr>
<tr>
<td>env</td>
<td>10.0 (2009)</td>
<td>4.0 (2007)</td>
<td>7.7 (2005)</td>
</tr>
<tr>
<td>ens</td>
<td>10.0 (2009)</td>
<td>2.3 (2007)</td>
<td>5.0 (2005)</td>
</tr>
</tbody>
</table>

\(^a\) Parl= parliamentary; Pres.= presidential. \(^b\) PL= plurality; PR= proportional representation.

\(^c\) Convergence coefficient modified for two dim

\(^d\) Anoc.Pres= Anocratic presidential.

\(^e\) Frag. = fragmented
Figure 1: The electoral distribution in Canada outside Québec in 2004, with party positions estimated by voter means.

Figure 2: The electoral distribution in Québec in 2004, with party positions estimated by voter means.
Figure 3: A simulated equilibrium in Canada without Québec

Figure 4: The heart in Canada in 2004
Figure 5: The heart in Canada in 2006

Figure 6: The heart in Poland in 1997