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The Mean Voter Theorem: Necessary and Sufficient Conditions for Convergent Equilibrium

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Formal models of elections have emphasized the convergence of party leaders towards the centre of the electoral distribution. This paper attempts to resolve the apparent disparity between the formal result and the perception of political divergence by considering a model incorporating valence. Valence can be interpreted as the non-policy basis of political judgement made by the electorate concerning the quality of political contenders. The theorem presented here shows that there is a necessary condition for convergence. The condition involves the difference in party valences and the electoral variance. When the condition fails, the low-valence parties will be forced to adopt policy positions far from the electoral centre. The inference appears to be substantiated by an empirical model of the Israel election in 1996.

1. INTRODUCTION

Most of the early work in formal political theory focused on two-party competition and generally concluded that there would be strong centripetal political forces causing parties to converge to the electoral centre (Hotelling, 1929; Downs, 1957; Riker and Ordeshook, 1973). The simplest model assumed a one-dimensional policy space, X, and "deterministic" voter choice, and showed the existence of a "Condorcet" point, unbeaten under majority rule vote, at the median of the electoral distribution. In higher dimensions, such two-party "pure-strategy Nash equilibria" (PNE) generally do not exist and instability may occur.[1] To guarantee existence of such a Nash equilibrium either requires that the electoral preferences have a distribution which is spherically symmetric or is restricted in some non-generic fashion (Caplin and Nalebuff, 1991). Alternatively, an equilibrium can be guaranteed as long as the decision rule requires a sufficiently large majority (Schofield, 1984; Strnad, 1985; Caplin and Nalebuff, 1988). While these results suggest that PNE fails to exist in two-party competition, there will generally exist *mixed* strategy Nash equilibria (Kramer, 1978) whose support lies within a subset of the policy space known as the *uncovered set*.[2] Such an "attractor" of the political process is centrally located with respect to the distribution of voters' ideal points. This theoretical result seems at odds with empirical evidence that political competition does not force candidates or parties to converge to the electoral centre (Poole and Rosenthal, 1984; Adams and Merrill, 1999; Merrill and Grofman, 1999; Adams, 2001).

The extreme contrast between the instability results and the convergence theorems suggest that a "non-deterministic" model is more appropriate for examining elections. Such a formal "stochastic" model is, in principle, compatible with current empirical models of voter choice

1. Plott (1967), McKelvey (1976, 1979), Schofield (1978, 1983, 1985), Cohen and Matthews (1980), McKelvey and Schofield (1986, 1987), Banks (1995), Saari (1997), and Austen-Smith and Banks (1999). Eaton and Lipsey (1975) also obtained instability results when there are three or more candidates.

2. McKelvey (1986), Cox (1987), and Banks, Duggan and Le Breton (2002).

(Schofield, Martin, Quinn and Whitford, 1998; Quinn, Martin and Whitford, 1999). A theoretical basis for such stochastic models is provided by the notion of "quantal response equilibria" (McKelvey and Palfrey, 1995). In such models, the behaviour of each voter is modelled by a vector of choice probabilities.[3] Again, a standard result in this class of models is that all parties converge to the political centre, in this case the electoral mean (Austen-Smith and Banks, 2005; Banks and Duggan, 2005; McKelvey and Patty, 2006).

This paper will show that the formal convergence result for the stochastic model need not hold if there is an asymmetry in the electoral perception of the "quality" of party leaders (Stokes, 1992). The average weight given to the perceived quality of the leader of the j-th party is called the party's *valence.* In empirical models this valence is independent of the party's position, and adds to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future (Penn, 2003). Formal models of elections incorporating valence have been developed recently.[4] The theorem presented here extends this earlier work and gives a necessary condition under which the joint electoral mean is a "local pure-strategy Nash equilibrium" (LNE) in the stochastic vote model with valence.

The recent empirical work on stochastic vote models for a number of countries has been based on the "multinomial logit" assumption that the stochastic errors had a "Type I extreme value distribution" (Dow and Endersby, 2004) and has concluded that divergence rather than convergence is typical.[5] With the same stochastic distribution assumption, the theorem shows that a "convergence coefficient", incorporating all the parameters of the model, can be defined. This coefficient, c, involves the differences in the valences of the party leaders and the spatial coefficient β. When the "policy space" is assumed to be of dimension w, then the necessary condition for existence of an LNE at the electoral origin is that the coefficient c is bounded above by w. When the necessary condition fails, parties in equilibrium will adopt divergent positions. In general, parties whose leaders have the lowest valence will take up positions furthest from the electoral mean. Moreover, because a PNE must be a local equilibrium, the failure of existence of the LNE at the electoral mean implies non-existence of such a centrist PNE. The failure of the necessary condition for convergence has a simple interpretation. If the variance of the electoral distribution is sufficiently large in contrast to the expected vote share of the lowest valence party at the electoral mean, then this party has an incentive to move away from the origin towards the electoral periphery. Other low-valence parties will follow suit, and the local equilibrium will be one where parties are distributed along a "principal electoral axis". The general conclusion is that, with all other parameters fixed, a convergent LNE can be guaranteed only when β is "sufficiently" small. Thus, divergence away from the electoral mean becomes more likely the greater β, the valence difference, and the variance of the electoral distribution.

To illustrate the theorem, an empirical study of voter behaviour for Israel for the election of 1996 is used to show that the condition on the empirical parameters of the model, necessary for convergence, was violated. The equilibrium positions obtained from the formal result under vote maximization were found to be comparable with, though not identical to, the estimated positions: the two highest valence parties were symmetrically located on either side of the electoral origin, while the lowest valence parties were located far from the origin.

3. Hinich (1977), Enelow and Hinich (1984), Coughlin (1992), and Lin, Enelow and Dorussen (1999).

4. Ansolabehere and Snyder (2000), Groseclose (2001), and Aragones and Palfrey (2002, 2005).

5. Recent work has examined elections in Britain (Alvarez and Nagler, 1998; Quinn *et al*, 1999; Alvarez, Nagler and Bowler, 2000; Schofield, 2004, 2005*a*,*b*), Netherlands and Germany (Schofield *et al.*, 1998; Quinn and Martin, 2002; Schofield and Sened, 2005*a*), Israel (Schofield, Sened and Nixon, 1998; Schofield and Sened, 2005*b*), Italy (Giannetti and Sened, 2004), and the U.S. (Miller and Schofield, 2003; Schofield, Miller and Martin, 2003). A review of these models is given in Schofield and Sened (2006).

Since vote maximization is a natural assumption to make for political competition under an electoral system based on proportional representation, the result suggests that there are two natural non-centrist political configurations:

(1) If there are two or more dimensions of policy, but there is a principal electoral axis associated with higher electoral variance, then all parties will be located on this axis. In particular, if there are two competing high-valence parties, then they will locate themselves at vote-maximizing positions on this axis, but on opposite sides of the electoral mean. Low-valence parties will be situated on this axis, but far from the centre. The unidimensionality of the resulting configuration will give the median party on the axis the ability to control government and thus policy.

(2) If both policy dimensions are more or less equally important, then there will be no principal axis, and parties can locate themselves throughout the policy space. Again, high-valence parties will tend to position themselves nearer the mean. To construct a winning coalition, one or other of the high-valence, centrist parties must bargain with more "radical" low-valence parties. As the example of Israel illustrates, this may induce a degree of coalitional instability.

One theoretical contribution of this paper is to exploit the idea of a local Nash equilibrium, as earlier utilized by Patty (2005, 2006). The theorem is presented in terms of a model based on vote maximization. For models of two-party competition it should be possible to develop a similar classification result under the assumption of maximizing probability of winning. However, computation of equilibria for this different model is much more difficult (Duggan, 2000; Patty, 2001). Nonetheless, because local Nash equilibria will generally exist under reasonable conditions (Schofield, 2001), it should be possible to develop more complex models of party competition based on this equilibrium notion.

2. THE FORMAL STOCHASTIC MODEL OF ELECTIONS

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$, of voter ideal points for the members of the electorate, N, of size n. As usual we assume that X is an open convex subset of Euclidean space, $\mathbb{R}^w$, with w finite.

Each of the parties, or agents, in the set $P = \{1, \dots, j, \dots, p\}$ chooses a policy, $z_j \in X$, to declare to the electorate prior to the election. Let $\boldsymbol{z} = (z_1, \dots, z_p) \in X^p$ be a typical vector of agent policy positions. Given $\boldsymbol{z}$, each voter, i, is described by a vector

$$\boldsymbol{u}_i(x_i, \boldsymbol{z}) = (u_{i1}(x_i, z_1), \dots, u_{ip}(x_i, z_p)) \in \mathbb{R}^p,$$

where

$$u_{ij}(x_i, z_j) = u^*_{ij}(x_i, z_j) + \varepsilon_{ij}$$
$$\text{and } u^*_{ij}(x_i, z_j) = \lambda_j - \beta||x_i - z_j||^2.$$

Here, $u^*_{ij}(x_i, z_j)$ is the "observable" utility for i, associated with party j. The term λ_j is the *valence* of agent j, which I assume is exogenously determined. The term β is a positive constant and $||\cdot||$ is the usual Euclidean norm on X. The terms $\{\varepsilon_{ij}\}$ are the stochastic errors. I shall assume that each ε_{ij} is drawn from the same probability distribution. The cumulative distribution of the errors is denoted by Ψ.

Because of the stochastic assumption, voter behaviour is modelled by a probability vector. The probability that a voter i chooses party j is

$$\rho_{ij}(\boldsymbol{z}) = \Pr[(u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)), \quad \text{for all } l \neq j].$$

Here Pr stands for the probability operator associated with Ψ. The expected vote share of agent j at the vector $\boldsymbol{z}$ is

$$V_j(\boldsymbol{z}) = \frac{1}{n}\sum_{i\in N}\rho_{ij}(\boldsymbol{z}).$$

In the vote model it is assumed that each agent j chooses z_j to maximize V_j, conditional on $\boldsymbol{z}_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p)$. The function

$$V\colon X^p \to \mathbb{R}^p \text{ given by } V(\boldsymbol{z}) = (V_1(\boldsymbol{z}), \ldots, V_p(\boldsymbol{z}))$$

is termed the *profile function*. We seek equilibria of this function.

The theorem presented here assumes that the exogenous valences are given by the vector $\boldsymbol{\lambda} = (\lambda_p, \lambda_{p-1}, \ldots, \lambda_2, \lambda_1)$ and the valences are ranked $\lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1$.

We implicitly assume that the valence of party j, as perceived by voter i is $\lambda_{ij} = \lambda_j + \varepsilon_{ij}$. Thus we can regard λ_j as the "average" weight given by a member of the electorate to the perceived competence or quality of agent j. In the model, the cumulative distribution, Ψ, is assumed to be the C^2-differentiable, so that the individual probability functions $\{\rho_{ij}\}$ will be C^2-differentiable in the strategies $\{z_j\}$. Thus, the vote share functions will also be C^2-differentiable, and Hessians can be computed.

Let $x^* = \frac{1}{n}\sum_i x_i$ be the electoral mean. Then the *mean voter theorem* for the stochastic model asserts that the "joint mean vector" $\boldsymbol{z}^* = (x^*, \ldots, x^*)$ is a PNE. Lin *et al.* (1999) used C^2-differentiability of the expected vote share functions, in the situation with zero valence, to show that the validity of the theorem depended on the concavity of the vote share functions. Because concavity cannot, in general, be assured, I shall utilize a weaker equilibrium concept, that of "local strict Nash equilibrium" (LSNE). A strategy vector $\boldsymbol{z}^*$ is an LSNE if, for each j, z_j^* is a critical point of the vote function $V_j : X \to \mathbb{R}^p : z_j \to V_j(z_1^*, \ldots, z_{j-1}^*, z_j, z_{j+1}^*, \ldots, z_p^*)$ and the eigenvalues of the Hessian of this function (with respect to z_j), are negative at z_j^*. Definition 1 gives the various technical definitions used here.

Definition 1. Equilibrium concepts

(i) A strategy vector $\boldsymbol{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile function $V : X^p \to \mathbb{R}^p$ iff, for each agent $j \in P$, there exists a neighbourhood X_j of z_j^* in X such that

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) > V_j(z_1^*, \ldots, z_j, \ldots, z_p^*) \text{ for all } z_j \in X_j - \{z_j^*\}.$$

(ii) A strategy vector $\boldsymbol{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *local weak Nash equilibrium* (LNE) if, for each agent j, there exists a neighbourhood X_j of z_j^* in X such that

$$V_j(z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \geq V_j(z_1^*, \ldots, z_j, \ldots, z_p^*) \text{ for all } z_j \in X_j$$

(iii) and (iv) A strategy vector $\boldsymbol{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*)$ is a *strict, respectively, weak pure, strategy Nash equilibrium* (PSNE, respectively, PNE) if X_j can be replaced by X in (i), (ii), respectively.

(v) The strategy z_j^* is termed a *local strict best response, a local weak best response, a global strict best response, or a global weak best response* to $\boldsymbol{z}_{-j}^* = (z_1^*, \ldots, z_{j-1}^*, z_{j+1}^*, \ldots, z_p^*)$, respectively, depending on which of the conditions (i)–(iv), hold for z_j^*.

Obviously if $\boldsymbol{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. I use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\boldsymbol{z}^* = (x^*, \dots, x^*)$ to be an LNE. This gives a necessary condition for PNE, without having to invoke concavity. In dimension w, a corollary shows that for $\boldsymbol{z}^*$ to be an LNE, a "convergence coefficient" must be weakly bounded above by w. When this condition fails, then the joint mean vector $\boldsymbol{z}^*$ cannot be an LNE and therefore cannot be a PNE. In two dimensions a sufficient condition for $\boldsymbol{z}^*$ to be an LSNE is that the convergence coefficient is strictly bounded above by 1. Of course, even if the sufficient condition is satisfied, and $\boldsymbol{z}^* = (x^*, \dots, x^*)$ is an LSNE, it need not be a PNE.

To parallel the empirical applications we assume that the cumulative distribution, Ψ, is the *Type I extreme value distribution*, where Ψ takes the closed form

$$\Psi(h) = \exp(-(\exp - h)).$$

The variance of this distibution is $\frac{1}{6}\pi^2$. It readily follows (Train, 2003) for the choice model given above that, for each voter, i, the probability of voting for j is given by

$$\rho_{ij}(\boldsymbol{z}) = \frac{\exp(u^*_{ij}(x_i, z_j))}{\sum_{k=1}^{p} \exp(u^*_{ik}(x_i, z_k))}.$$

This implies that the model satisfies the "independence of irrelevant alternative property", namely, that for each voter i, and for each pair of candidates, j, k, the ratio

$$\frac{\rho_{ij}(\boldsymbol{z})}{\rho_{ik}(\boldsymbol{z})}$$

is independent of a third candidate l (Train, 2003).

We first transform coordinates so that in the new coordinates, $x^* = 0$. I shall refer to $\boldsymbol{z}^*_0 = (0, \dots, 0)$ as the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends on the distribution of voter ideal points. These are encoded in the voter variance/covariance matrix.

Definition 2. The electoral covariance matrix ∇^*.

(i) Let X have dimension w and be endowed with an orthogonal system of coordinate axes $(1, \dots, s, \dots, t, \dots, w)$. For each coordinate axis let $\xi_t = (x_{1t}, x_{2t}, \dots, x_{nt}) \in \mathbb{R}^n$ be the vector of the t-th coordinates of the set of n voter ideal points. Let $(\xi_s, \xi_t) \in \mathbb{R}$ denote scalar product.

(ii) The symmetric $w \times w$ *electoral covariation matrix* ∇ is defined to be $[(\xi_s, \xi_t)]^{s=1,\dots,w}_{t=1,\dots,w}$. The *electoral covariance matrix* is defined to be $\nabla^* = \frac{1}{n}\nabla$.

(iii) The covariance between the s-th and t-th axes is denoted $(v_s, v_t) = \frac{1}{n}(\xi_s, \xi_t)$ and $v_s^2 = \frac{1}{n}(\xi_s, \xi_s)$ is the electoral variance on the s-th axis.

(iv) The total electoral variance is

$$v^2 = \sum_{r=1}^{w} v_r^2 = \frac{1}{n}\sum_{r=1}^{w}(\xi_r, \xi_r) = \operatorname{trace}(\nabla^*).$$

In empirical applications the electoral data is encoded by ∇^*. With the assumption of the Type I extreme value distribution, the coefficients $\boldsymbol{\lambda}, \beta$ can then be estimated.

The formal model with coefficients $(\boldsymbol{\lambda}, \beta)$ and covariance matrix ∇^* is denoted $M(\nabla^*; \boldsymbol{\lambda}, \beta; \Psi)$, though the reference to ∇^* will be suppressed when it is understood.

Definition 3. The convergence coefficient of the model $M(\nabla^*; \boldsymbol{\lambda}, \beta; \Psi)$.

(i) Define

$$\rho_j = \left[1 + \sum_{k \neq j} \exp(\lambda_k - \lambda_j) \right]^{-1}. \tag{1}$$

(ii) The coefficient A_j for party j is

$$A_j = \beta(1 - 2\rho_j).$$

(iii) The characteristic matrix for party j is the $w \times w$ matrix

$$C_j = [2A_j \nabla^* - I] \tag{2}$$

where I is the $w \times w$ identity matrix.

(iv) The convergence coefficient of the model $M(\nabla^*; \boldsymbol{\lambda}, \beta; \Psi)$ is

$$c(\boldsymbol{\lambda}, \beta; \Psi) = 2\beta(1 - 2\rho_1)v^2 = 2A_1 v^2. \tag{3}$$

It is readily shown that at $\boldsymbol{z}_0 = (0, \ldots, 0)$, the probability, $\rho_{ij}(\boldsymbol{z}_0)$, that i votes for party j is independent of i. Indeed, $\rho_{ij}(\boldsymbol{z}_0) = \rho_j$, where ρ_j is given by Definition 3(i). Obviously if all valences are identical then $\rho_1 = \frac{1}{p}$, as expected. The effect of increasing λ_j, for $j \neq 1$, is clearly to decrease ρ_1, and therefore to increase A_1, and thus $c(\boldsymbol{\lambda}, \beta; \Psi)$.

We now examine the first- and second-order conditions at $\boldsymbol{z}_0$ for each vote share function, which are necessary and sufficient for local equilibria. The first-order condition is obtained by setting $dV_j/dz_j = 0$ (where we use this notation for full differentiation, keeping $z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p$ constant). This allows us to show that $\boldsymbol{z}_0$ satisfies the first-order condition. The second-order condition for an LSNE is that the Hessians $\{d^2V_j/dz_j^2 : j = 1, \ldots, p\}$ are negative definite at the joint origin. The Appendix shows that, at $\boldsymbol{z}_0$, the Hessian of V_1 is given by the matrix

$$2\beta(\rho_1 - \rho_1^2)C_1. \tag{4}$$

Since $2\beta(\rho_1 - \rho_1^2) > 0$, the eigenvalues of this Hessian will be negative if those of C_1 are negative. Thus the necessary condition for an LSNE at $\boldsymbol{z}_0$ can be expressed as a condition on the characteristic matrix, C_1, of the lowest valence party.

Theorem 1. *The joint origin satisfies the first-order necessary condition to be an LSNE for the model $M(\boldsymbol{\lambda}, \beta; \Psi)$. The further second-order necessary condition for the joint origin to be an LSNE is that every eigenvalue of the characteristic matrix $C_1 = [2A_1 \nabla^* - I]$ is negative.*

The proof of the theorem is given in the Appendix, together with proofs of the following corollaries.

Corollary 1. *In the case that X is w-dimensional then the necessary condition for the joint origin to be an LNE for the model $M(\boldsymbol{\lambda}, \beta; \Psi)$ is that $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$.*

Ceteris paribus, an LNE at the joint origin is "less likely", the greater are the parameters β, $\lambda_p - \lambda_1$, and v^2.

Corollary 2. *In the two-dimensional case, a sufficient condition for the joint origin to be an LSNE for the model $M(\boldsymbol{\lambda}, \beta; \Psi)$ is that $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$. Moreover, the two eigenvalues of*

C_1 are given by the real numbers

$$A_1[v_1^2+v_2^2 \pm \{(v_1^2-v_2^2)^2+4(v_1,v_2)^2\}^{1/2}]-1. \tag{5}$$

It is evident that sufficient conditions for existence of an LSNE at the joint origin in higher dimensions can be obtained using standard results on the determinants, $\det(C_j)$, and traces, $\mathrm{trace}(C_j)$, of the characteristic matrices.

Notice that the case with two parties of equal valence immediately gives a situation with $2\beta[1-2\rho_1]v^2=0$, irrespective of the other parameters. However, if $\lambda_2 \gg \lambda_1$, then the joint origin will fail to be an LNE if βv^2 is sufficiently large.

Corollary 3. *In the case that X is w-dimensional and there are two parties, with $\lambda_2 > \lambda_1$, then the joint origin fails to be an LNE if $\beta > \beta_0$ where*

$$\beta_0 = \frac{w[\exp(\lambda_2-\lambda_1)+1]}{2v^2[\exp(\lambda_2-\lambda_1)-1]}. \tag{6}$$

Proof. This follows immediately using $\rho_1 = [1+\exp(\lambda_2-\lambda_1)]^{-1}$. ‖

It follows that if $\lambda_2=\lambda_1$ then $\beta_0=\infty$. Since β is finite, the necessary condition for an LNE must be satisfied.

Example. *We can illustrate Corollary 3, in the case where the necessary condition fails, by assuming that X is a compact interval, $[-a,+a] \subset \mathbb{R}$. Suppose further that there are three voters at $x_1=-1, x_2=0$, and $x_3=+1$. Then $v^2=\frac{2}{3}$. Suppose that $\lambda_2>\lambda_1$ and $\beta>\beta_0$ where β_0 is as above. Then $\mathbf{z}_0$ fails to be an equilibrium, and party 1 must move z_1 away from the origin, either towards x_1 or x_3.*

To see this suppose $\lambda_2=1$ and $\lambda_1=0$, so that $\beta_0=1{\cdot}62$. If $\beta=2{\cdot}0$, then the condition fails, since we find that at $\mathbf{z}_0=(0,0)$, for each i,

$$\rho_{i1}(\mathbf{z}_0)=[1+[\exp(1)]]^{-1}=0{\cdot}269,$$

so that $V_1(\mathbf{z}_0)=0{\cdot}269$.

Now consider $\mathbf{z}=(z_1,z_2)=(+0{\cdot}5,0)$. We find

$$\rho_{11}=[1+[\exp(3{\cdot}5)]]^{-1}=0{\cdot}029$$

while

$$\rho_{21}=[1+[\exp(1{\cdot}5)]]^{-1}=0{\cdot}182$$

and

$$\rho_{31}=[1+[\exp(1-1{\cdot}5)]]^{-1}=0{\cdot}622.$$

Thus

$$V_1(\mathbf{z})=\frac{1}{3}[0{\cdot}029+0{\cdot}182+0{\cdot}622]=0{\cdot}277.$$

Hence candidate 1 can slightly increase vote share by moving way from the origin. Obviously the joint origin cannot be an equilibrium. The gain from such a move is greater the greater is $\lambda_2-\lambda_1$ and β.

In the two-dimensional case there is one situation where computation of eigenvalues is particularly easy. If the electoral covariance (v_1, v_2) is (close to) 0, then the voter covariance matrix is (approximately) diagonal and the two policy dimensions can be treated separately. In this case we obtain two separate necessary conditions

$$2\beta(1-2\rho_1)v_t^2 \leq 1$$

for $t = 1, 2$, for $\boldsymbol{z}_0^*$ to be an LNE.

The more interesting case is when the covariance (v_1, v_2) is significant. By a transformation of coordinates, we can choose v_t, v_s to be the eigenvectors of the Hessian matrix for agent 1, and let these be the new "principal components" of the electoral covariance matrix. If $v_t^2 \leq v_s^2$ then the s-th coordinate can be termed the "principal electoral axis". The two-dimensional empirical analysis of Israel, discussed below, shows that the valence differences implied that, for the lowest valence party, the eigenvalue associated with the principal electoral axis was large and positive, while the eigenvalue on the minor axis was negative. This immediately implies, with other agents at the electoral origin, that the position $z_1 = 0$ is a saddlepoint of the vote share function for agent 1. More generally, the eigenspace associated with the positive eigenvalue could be identified with the principal electoral axis, while the negative eigenvalue could be associated with the orthogonal minor axis. The gradient of agent 1's vote share function near the origin points in a direction away from the origin and is aligned with the principal axis. Since a similar argument holds for all parties, it follows that, in local equilibrium, all parties will be located on (or close to) the principal axis, with the lowest valence agents farthest from the origin. This formal result is matched by the simulation of the electoral model for this election.

Previous formal analyses of the stochastic vote model (Banks and Duggan, 2005; McKelvey and Patty, 2006) have focused on conditions sufficient for the mean vector, $\boldsymbol{z}^* = (x^*, \ldots, x^*)$ to be a Nash equilibrium. Generally this has involved assuming that the vote share functions are concave. Obviously, if the necessary condition given in Corollary 1 fails at the joint origin, then so must concavity. This casts doubt on the existence of PNE in these vote models. The natural question is whether there can be multiple LSNE, but no Nash equilibria. Generic existence of LSNE can be shown by more abstract arguments as long as a certain boundary condition is satisfied (Schofield, 2001). This suggests that, in general, there will exist many different, non-convergent LSNE. In the simulation of the model for the Israel election, discussed below, the various LSNE that were found were essentially permutations of one another. Most importantly, none of these equilibria involved parties adopting positions at the electoral origin.

3. EMPIRICAL MODEL FOR ISRAEL

Figure 1 shows the estimated positions of the parties in the Israel Knesset, and the electoral distribution, at the time of the 1996 election, while Table 1 presents summary statistics of the 1996 election.[6] The table also shows the valence estimates, based on a multinomial logit model, and therefore on the Type I extreme value distribution on the errors. The two dimensions of policy deal with attitudes to the Palestine Liberation Organization (the horizontal axis) and religion (the vertical axis). The policy space was derived from a voter survey (obtained by Arian and Shamir, 1999) and the party positions from analysis of party manifestos (Schofield *et al.*, 1998; Schofield and Sened, 2006).

Using the formal analysis, we can readily show that one of the eigenvalues of the low-valence party, the NRP (also called Mafdal), is positive. Indeed, it is obvious that there is a

6. The model correctly predicts 63·8% of the voter choices. Log marginal likelihood of the model is −465.

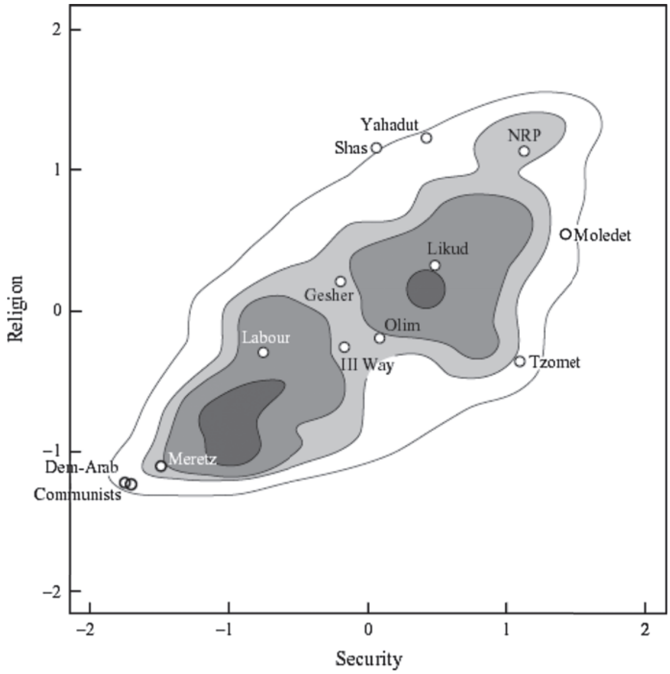


FIGURE 1

Estimated party positions and electoral distribution (showing the 95%, 75%, 50%, and 10% contours) in the Knesset at the election of 1996

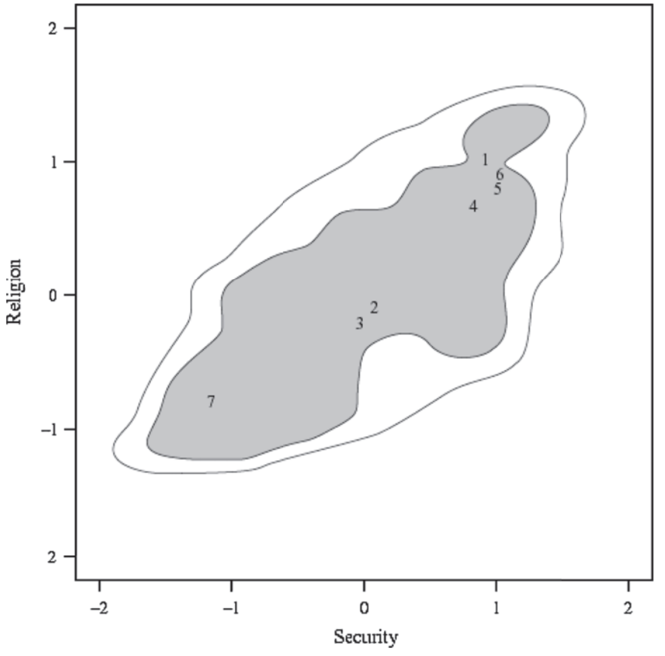
TABLE 1

Seats and votes in the Knesset

Party	1996 National (%)	1996 Sample (%)	1996 Seats	1996 Valence
Others left	7·3	0	9	−
Meretz	7·6	6·0	9	0
Labour	27·5	44·0	34	4·15
Olim, Third Way	90	1·8	13	−2·34
Likud	25·8	43·0	30	3·14
Shas	8·7	2·0	10	−2·96
NRP (Mafdal)	8·0	5·1	9	−4·52
Moledet	2·4	1·8	2	−0·89
Others right	3·7	0	4	−

principal component of the electoral distribution, and this axis is the eigenspace of the positive eigenvalue. It follows that low-valence parties should position themselves close to this principal axis, as illustrated in the simulation given below in Figure 2.

In 1996, the lowest valence party was the NRP with valence $-4{\cdot}52$. The spatial coefficient is $\beta = 1{\cdot}12$, so to use the theorem we note that the valence difference between the NRP and Labour is $4{\cdot}15-(-4{\cdot}52) = 8{\cdot}67$, while the difference between the NRP and Likud is $3{\cdot}14-(-4{\cdot}52) = 7{\cdot}66$. Since the electoral variance on the first axis is $1{\cdot}0$, and on the second axis it is $0{\cdot}732$, with covariance $0{\cdot}591$, we can compute the characteristic matrix of the NRP at the origin



Notes: 1, Shas; 2, Likud; 3, Labour; 4, NRP; 5, Moledet; 6, Third Way; 7, Meretz.

FIGURE 2

A simulated local Nash equilibrium in the vote-maximizing game in Israel in 1996

as follows:

$$\rho_{\text{NRP}} \simeq \frac{1}{1+e^{4\cdot 15+4\cdot 52}+e^{3\cdot 14+4\cdot 52}} \simeq 0$$

$$A_{\text{NRP}} \simeq \beta = 1\cdot 12,$$

$$C_{\text{NRP}} = 2(1\cdot 12)\begin{pmatrix} 1\cdot 0 & 0\cdot 591 \\ 0\cdot 591 & 0\cdot 732 \end{pmatrix} - I = \begin{pmatrix} 1\cdot 24 & 1\cdot 32 \\ 1\cdot 32 & 0\cdot 64 \end{pmatrix}.$$

From the estimate of C_{NRP} it follows that the two eigenvalues are 2·28 and −0·40, giving a saddlepoint and a value of 3·88 for the convergence coefficient. This exceeds the necessary upper bound of 2. The major eigenvector for the NRP is (1·0, 0·8), and along this axis the NRP vote share function increases as the party moves away from the origin. The minor, perpendicular axis associated with the negative eigenvalue is given by the vector (1, −1·25). Figure 2 gives one of the local equilibria in 1996, obtained by simulation of the model. The figure makes it clear that the local equilibrium positions of all parties lie close to the principal axis through the origin and the point (1·0, 0·8). In all, five different LNE were located. However, in all the equilibria, the two high-valence parties, Labour and Likud, were located close to the positions given in Figure 2. The only difference between the various equilibria was that the positions of the low-valence parties were perturbations of each other.

It is evident that if the high-valence party occupies the electoral mean, then all parties with lower valence can compute that their vote share will increase by moving up or down the principal electoral axis. In seeking local maxima of the vote shares all parties other than the highest valence party should vacate the electoral centre. Then, however, the first-order condition for the high-valence party to occupy the electoral centre would not be satisfied. Even though this party's vote share will be little affected by the other parties, it too should move from the centre. The simulations illustrated in Figure 2 make it clear that there is a weak correlation between a party's valence and the distance of the party's equilibrium position from the electoral mean. A similar analysis is given in Schofield and Sened (2006) for the elections of 1992 and 1988.

The simulations are compatible with the formal analysis: low-valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral centre. Their optimal positions will lie either in the "north-east" quadrant or the "south-west" quadrant. The vote-maximizing model, without any additional information, cannot determine which way the low-valence parties should move.

In contrast, since the valence difference between Labour and Likud was relatively low, their local equilibrium positions are close to, but not identical to, the electoral mean. Intuitively, it is clear that once the low-valence parties vacate the origin, then high-valence parties, like Likud and Labour should position themselves almost symmetrically about the origin and along the principal axis.

Clearly, the configuration of equilibrium party positions will fluctuate as the valence differences of the parties change in response to exogenous shocks. The logic of the model remains valid, however, since the low-valence parties will be obliged to adopt relatively "radical" positions in order to maximize their vote shares.

There is a disparity between the estimated party positions in 1996 given in Figure 1 and the simulated equilibrium positions given in Figure 2. The two religious parties, Shas and Yahadut, are estimated to be far from the principal axis, seeming in contradiction to the prediction of the stochastic model. Moreover, the high-valence parties, Labour and Likud appear further from the origin than suggested by the simulation. This disparity may be accounted for by modifying the assumption that valence is exogenous, and by allowing for the influence of activists on party position (Miller and Schofield, 2003; Schofield, 2006; Schofield and Sened, 2006).

The location of the religious parties so far from the origin has meant that until recently the governing coalition led by Ariel Sharon, of Likud, was constrained in its policy choices. This apparently caused Sharon to leave Likud to set up a new centrist party, Kadima ("Forward") with Shimon Peres, previously leader of Labour. The reason for this reconfiguration was the victory on 10 November, 2005 of Amir Peretz over Peres for leadership of the Labour Party and Peretz's move to the left down the principal electoral axis.

Consistent with the model presented here, we can infer that Sharon's intention was to position Kadima near the electoral centre on both dimensions, to take advantage of his relatively high valence among the electorate. Sharon's hospitalization in January 2006 had an adverse effect on the valence of Kadima, under its new leader, Ehud Olmert. Even so, in the election of 28 March, 2006, Kadima took 29 seats, against 19 seats for Labour, while Likud only won 12. Because Kadima with Labour and the other parties of the left had 70 seats, Olmert was able to put together a majority coalition on 28 April, 2006, including Shas. We can infer that Shas was still centrist on the security dimension (as in Figure 1), indicating that this was the key issue of the election.

4. CONCLUDING REMARKS

Although the non-cooperative stochastic electoral model presented here may be able to provide some insight into the relationship between electoral preferences and beliefs (regarding the valences of party leaders), it is still incomplete. The evidence suggests that party leaders pay attention not only to electoral responses, but also to activist support and to the post election consequences of their policy choices (Cox, 1984, 1997; Schofield and Sened, 2006). Indeed, it appears that the centrifugal logic of activist support and coalition bargaining will cause parties to move further from the electoral centre than indicated by the vote-maximizing model.

In an attempt to begin the modelling of the complex multiparty game, this paper has introduced the idea of local Nash equilibrium. The underlying premise of this notion is that party principals will not consider "global" changes in party policies, but will instead propose small changes in the party position, possibly accompanied by changes in party leadership, in response to changes in beliefs about electoral response and the likely consequences of policy negotiations. A key element of the electoral model presented here is of the notion of valence. The empirical models provide a justification for the inclusion of this variable, while the formal model suggests that its inclusion changes the usual presumption about the centripetal tendency of electoral politics. The models suggest that a voter's choice is partly determined by policy preference and partly by the exercise of judgement.

It was James Madison's argument in *Federalist 10* of 1787 that judgement rather than preference would prevail in the choice of the Chief Magistrate in the extended Republic. Schofield (2005*c*) has argued that Madison derived his argument concerning the "probability of a fit choice" from Condorcet's Jury theorem as expressed in Condorcet's treatise of 1785 (Condorcet, 1994). However, Madison was concerned that preferences could intrude on electoral choice and that the resulting "factions" would invalidate his argument. The model presented here includes both judgements and preferences and suggests that a candidate of low valence or quality will be obliged to adopt a non-centrist position. Conversely, only a candidate or party leader of high valence will be attracted to the electoral origin. This may be one way of interpreting Madison's notion of a "fit choice".

If candidates move away from the origin in different directions, then it is plausible that the resulting asymmetry in policy positions will induce differential support by activist groups. Such support could then maintain non-convergence in party positions. Miller and Schofield (2003) have proposed a model of this kind to account for the slow process of partisan realignment that seems to have occurred in the U.S. political system over the past 40 years.

APPENDIX

Proof of Theorem 1. At $\mathbf{z} = (z_1, z_2, \ldots, z_p)$, the probability that i picks j is

$$\rho_{ij}(\mathbf{z}) = \Pr[(\lambda_j - \beta||x_i - z_j||^2 - \lambda_k + \beta||x_i - z_k||^2 > \varepsilon_{ik} - \varepsilon_{ij}), \text{ for all } k \neq j].$$

Using the extreme value distribution Ψ we obtain

$$\rho_{ij}(\mathbf{z}) = \left[1 + \sum_{k \neq j} (\exp(f_k))\right]^{-1}$$

$$\text{where } f_k = \lambda_k - \lambda_j + \beta||x_i - z_j||^2 - \beta||x_i - z_k^2||.$$

$$\text{Thus } \frac{d\rho_{ij}}{dz_j} = 2\beta(z_j - x_i)(\rho_{ij}^2 - \rho_{ij}).$$

At $\boldsymbol{z}_0 = (0, \ldots, 0)$, $\rho_{ij} = \rho_j$ is independent of i, so we obtain

$$\frac{d\rho_{ij}}{dz_j} = 2\beta(z_j - x_i)(\rho_j^2 - \rho_j)$$

$$\text{and } \frac{dV_j}{dz_j} = \frac{1}{n}\sum_i \frac{d\rho_{ij}}{dz_j} = \frac{2\beta}{n}(\rho_j^2 - \rho_j)\sum_{i=1}^{n}(z_j - x_i).$$

$$\text{The condition } \frac{dV_j}{dz_j} = 0 \text{ is satisfied at } z_j = \frac{1}{n}\sum_i x_i.$$

By choice of coordinates, $\frac{1}{n}\sum_i x_i = 0$, so $z_j = 0$ satisfies the first-order condition when $\boldsymbol{z}_{-j} = (0, \ldots, 0)$. Since this holds for each j, it follows that $\boldsymbol{z}_0 = (0, \ldots, 0)$ satisfies the first-order condition.

At $\boldsymbol{z}_{-1} = (0, \ldots, 0)$ the Hessian of ρ_{i1} is

$$\frac{d^2\rho_{i1}}{dz_1^2} = (\rho_{i1} - \rho_{i1}^2)((1 - 2\rho_{i1})(\nabla_{i1}(z_1)) - 2\beta I).$$

Here $(\nabla_{i1}(z_1)) = 4\beta^2((x_i - z_1)^{\boldsymbol{T}}(x_i - z_1))$ where superscript $\boldsymbol{T}$ is used to denote a column vector. Clearly $[\nabla_{i1}(z_1)]$ is a w by w matrix of cross product terms. Now $\sum_i(\nabla_{i1}(0)) = 4\beta^2\nabla$, where ∇ is the electoral covariation matrix given in Definition 2. Since $\rho_{i1} = \rho_1$ is independent of i at $z_1 = 0$, we see that the Hessian of V_1 at $\boldsymbol{z}_0 = (0, \ldots, 0)$ is given by

$$\frac{1}{n}\sum_i \frac{d^2\rho_i}{dz_1^2} = (\rho_1 - \rho_1^2)(4\beta^2(1 - 2\rho_1)(\nabla^*) - 2\beta I)$$

$$= 2\beta(\rho_1 - \rho_1^2)C_1,$$

where $C_1 = (2A_1\nabla^* - I)$ and $A_1 = \beta(1 - 2\rho_1)$.

Because β and the term $(\rho_1 - \rho_1^2)$ are both positive, the eigenvalues of this matrix will be negative if the eigenvalues of C_1 are negative. If the eigenvalues of C_1 fail to be negative, then obviously $\boldsymbol{z}_0$ cannot be an LSNE.

Proof of Corollary 1. In the same fashion, a necessary condition for $\boldsymbol{z}_0$ to be an LNE is that all eigenvalues of C_1 be non-positive. Since trace(C_1) equals the sum of the eigenvalues the necessary condition is trace$(C_1) \leq 0$. But

$$\text{trace}(C_1) = 2\beta(1 - 2\rho_1)v^2 - w,$$

giving the necessary condition $2\beta[1 - 2\rho_1]v^2 \leq w$ or $c(\boldsymbol{\lambda}, \beta; \Psi) \leq w$. ||

Proof of Corollary 2. In dimension 2, the condition that both eigenvalues of C_1 are negative is equivalent to the condition that $\det(C_1)$ is positive and trace(C_1) is negative. Using Definition 2 for the variance terms v_t^2, $t = 1, 2$, and (v_1, v_2) for the covariance we see that

$$\det(C_1) = (2A_1)^2((v_1, v_1)\cdot(v_2, v_2) - (v_1, v_2)^2)$$

$$+1 - (2A_1)((v_1, v_1) + (v_2, v_2)).$$

By the triangle inequality, the term $[(v_1, v_1)\cdot(v_2, v_2) - (v_1, v_2)^2]$ is non-negative. Thus $\det(C_1)$ is positive if

$$2A_1(v_1^2 + v_2^2) - 1 < 0$$

$$\text{or } 2\beta(1 - 2\rho_1)v^2 < 1.$$

If this is satisfied, then the necessary condition is clearly satisfied. Thus we obtain the sufficient condition $c(\boldsymbol{\lambda}, \beta; \Psi) < 1$ for the Hessian of V_1. Moreover, as in the proof of the theorem the Hessian of V_j at $\boldsymbol{Z}_0$ is given by the matrix

$$2\beta(\rho_j - \rho_j^2)C_j.$$

so the Hessian of agent j is negative definite if C_j is. Moreover,

$$\lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1$$

$$\text{implies that } \rho_p \geq \rho_{p-1} \geq \cdots \geq \rho_2 \geq \rho_1$$

$$\text{so that } A_1 \geq A_2 \geq \cdots \geq A_p.$$

$$\text{Thus trace } (C_j) = 2\beta(1 - 2\rho_j)v^2 - 2 \leq 2\beta(1 - 2\rho_1)v^2 - 2 < -1, \quad \text{for all } j.$$

Thus the condition on the trace(C_j) is satisfied. Moreover, as with C_1, we obtain

$$\det(C_j) = (2A_j)^2[(v_1,v_1)\cdot(v_2,v_2)-(v_1,v_2)^2] + 1 - 2A_j[(v_1,v_1)+(v_2,v_2)].$$

Thus $\det(C_j) > 0$ if $2A_j v^2 < 1$. But $2A_j v^2 \leq 2A_1 v^2 < 1$, by the assumption on $c(\boldsymbol{\lambda},\beta;\Psi)$. Thus $c(\boldsymbol{\lambda},\beta;\Psi) < 1$ is a sufficient condition for the eigenvalues of all Hessians at $\mathbf{z}_0$ to be negative. The computation of the eigenvalues is standard and follows immediately, using the expression for $\det(C_1)$ in the above proof, together with the identity $a_1 + a_1 = \text{trace}(C_1) = c(\boldsymbol{\lambda},\beta;\Psi) - 2$. ‖

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